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Structure of Vortex Breakdown in a Torsionally Driven Cylinder at Low Reynolds Number

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Abstract

Flow visualisation using tracer and particle techniques is well known to be susceptible to fallacious interpretation for *unsteady* shear flows. Recent experimental observations using these types of techniques have led to the conclusion that even for low Reynolds number flows, the vortex breakdown bubble in a torsionally driven cylinder is not closed and has an inflow/outflow asymmetry at its tail. These results would suggest that this type of vortex structure is more similar to that observed in the open vortex tube than previously thought and is therefore a "true" vortex breakdown. In this article, an alternative explanation of the observed asymmetry is offered. It is demonstrated that the dye visualisation technique can lead to spurious interpretations even for a *steady* axisymmetric flow field, due to tracer diffusion and minute errors in tracer injection location. It is concluded that there is no evidence that bubble structure in a confined cylinder flow is asymmetric at these lower Reynolds number.

1. Introduction

The essential nature of vortex breakdown has been a matter of debate since the early days of its observation in flow over delta wings (Peckham & Atkinson [20]). The classic photograph by Lambourne & Bryer [16] of vortex breakdown on a delta wing shows both a bubble-type and a spiral-type instability occurring on opposite sides of the wing simultaneously under apparently identical conditions. Since then, vortex breakdown has been studied in a variety of other situations: vortex tubes (Harvey [12]; Sarpkaya [21-23]; Faler & Leibovich [5,6]; Garg & Leibovich [8]; Escudier & Zehnder [4]; Brücker & Althaus [2]); confined cylinder with rotating disk lid (Vogel [25]; Escudier [3]; Hourigan et al. [13]; Spohn et al. [24]); tornado generator (Khoo et al. [15]); free swirling air jet (Farokhi et al. [7]; Panda & McLaughlin [19]); and the swirling water jet (Billant et al. [1]). The differing environments in which vortex breakdown occurs and the variety of breakdown characteristics observed have led to dispute over the fundamental nature of the breakdown process. In this article, we focus on the issue of whether vortex breakdown can be purely axisymmetric and the issue of whether flow visualization of vortex breakdown in *steady* flow can be misleading.

The two main types of enclosed experimental rigs employed to visualise vortex breakdown are the vortex tube and the confined cylinder with a torsionally driven end (see Figure 1); other types, such as the tornado chamber and free swirling jets, produce structures similar to those observed in the vortex tube. Recent experiments by Spohn et al. [24] suggest that the vortex breakdown patterns observed at low Reynolds numbers in confined cylinders and vortex tubes are more similar than previously thought. The implications of this are: vortex breakdown bubbles in the confined case are not closed but are *steady* open bubbles with permanent inflow and outflow at the tail. Furthermore, it is shown that the flows are not perfectly symmetric. It is claimed that the observed asymmetry is real and is not caused by inadequacies of the visualisation techniques; the imperfections in their installation were so small that even reassembling of the rig led to no differences in the flow structures observed. They conclude that it is not justified to assume that unsteadiness is a basic ingredient of vortex breakdown and that the flow in the confined cylinder leads to breakdown bubbles similar to those observed in open tubes. In addition, they conclude that the formation of the streaks on the rear side of the recirculation bubbles in the confined cylinder is due to asymmetric flow separations on the container walls.

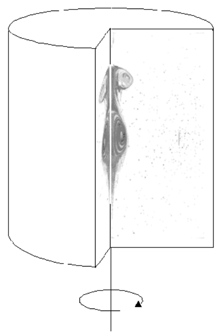

Figure 1. Schematic of the confined cylinder with rotating lower lid.

In this article, we demonstrate how, through a combination of *diffusion of the marker* and the *error in injection location,* "asymmetry" can appear for a steady axisymmetric flow involving vortex breakdown.

2. Numerical Method

The strategy employed to investigate tracer visualisation by numerical means was as follows. First, the purely axisymmetric steady base fluid flow was predicted computationally. Then, numerical particles representing the tracer were projected onto the flow solution through their introduction near the top of the confined cylinder, and convected following the streamlines of the base flow plus a random walk to mimic mass transfer.

2.1 Finite Element Method

The Galerkin finite-element method was used to obtain the solution of the Navier-Stokes equations for axisymmetric flow in cylindrical coordinates. The numerical mesh consisted generally of uniform rectangular elements with sinusoidal compression of the elements near the boundaries in order to capture the boundary layers including the Ekman layer. The penalty formulation with biquadratic Lagrangian interpolation (9-node quadratic quadrilateral elements) for the velocity field and under-integration of the continuity constraint was employed (Zienkiewicz [26]). Formally this method leads to a third-order accurate velocity field. The nonlinear set of equations was solved by Newton iteration, with the stopping criterion being when the

L2-norm of the velocity differences was less than 10^{-6}. The code has been tested successfully against a variety of benchmark problems, including driven cavity, backward-facing step and flow around a circular cylinder, and previously for the torsionally driven cylinder (Hourigan et al. [13]).

2.2 Tracer Diffusivity and Injection Offset

Numerical experiments simulating dye tracing have been undertaken by introducing particles near the centre of the top non-spinning lid. This mimics the way dye traces have been introduced by a variety of researchers. The effect of diffusion is incorporated through the application of random walk spreading at each timestep. The equivalent diffusion coefficient is mathematically related to the standard deviation of the random step and the timestep (Ghoniem & Sherman [9]). From this, a molecular Schmidt number (ratio of momentum diffusivity to mass diffusivity) can be calculated. In the three-dimensional computations, the tracer particles are released randomly within a small circular cross-section, analogous to dye injection or precipitated particle generation in the experiments.

The effect of the two parameters on the visualisation results were quantified in this study: the offset of the injection point relative to the centre axis of the cylinder; and the mass diffusivity (or Schmidt number) of the tracer in the fluid.

3. Results and Discussion

The main hypothesis to be tested is that even for a purely steady axisymmetric flow field, namely the closed bubble of vortex breakdown in a torsionally driven cylinder, passive tracer visualisation can suggest that the bubble is open and asymmetrical together with "Christmas tree" patterns downstream.

The hypothesis was tested as follows. First, a purely axisymmetric flow field featuring vortex breakdown in a confined cylinder was numerically predicted. Then, the release of a passive tracer was simulated in the three-dimensional axisymmetric flow field by releasing tracer particles across a small finite circular cross-section, analogous to dye injection or precipitated particle generation in the experiments. The two parameters associated with passive tracer and its injection included in the study were the offset of the injection position relative to the centre axis of the cylinder and the mass diffusivity (via the random velocity fluctuation of the particles) of the tracer.

The flow field was predicted for a Reynolds number of 1850 and an aspect ratio of 1.75, after Spohn et al. [24]. The streamline pattern of the predicted axisymmetric flow is shown in Figure 2. A recirculation bubble is found on the central axis, similar in general shape to that observed experimentally except that it has a closed form and is by design axisymmetric.

The effect of feeding in tracer particles from a circular cross-section offset from the central axis was then investigated. First, the tracer particles were allowed to follow the streamlines of the general flow. The offset of the circular cross-section through which particles were released was varied between 0 and 0.004 times the cylinder diameter, *D*. The predicted visualisation patterns are shown in Figure 2. When the tracer particles are released from within a circle, of radius 0.01*D*, concentric with the cylinder, the predicted flow pattern of the tracer is symmetrical everywhere in the flow (see Figure 3). However, offsetting the particle release positions by even minute amounts is seen to introduce an increasingly distinct asymmetry to the flow, although the bubble remains closed.

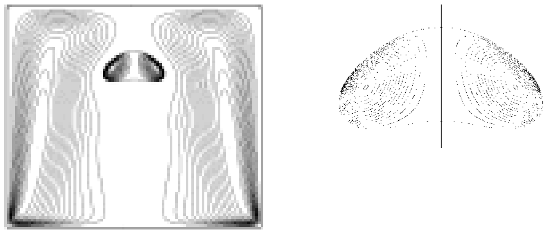

Figure 2. The streamlines of predicted axisymmetric flow for Reynolds number 1850 and aspect ratio 1.75. Full diametral plane is shown at the left and the enlarged view of the recirculation bubble with centreline axis is shown at the right.

An analogous result was observed in the case of the pre-breakdown flow in a torsionally driven cylinder by Hourigan et al. [13]. In that case, the offset of dye release was shown to result in the predicted visualisation of spiral structures, as observed in physical flows, even though the flow was strictly axisymmetric. In the present case, asymmetry is observed although it is noted that the predicted tracer visualisation still shows closed bubble configurations.

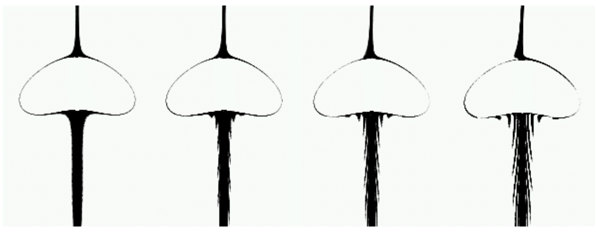

Figure 3 The particle trace plots as the offset of dye introduction from the centreline is increased: (a) no offset (b) offset = 0.00125*D* (c) offset = 0.0025*D* (d) offset = 0.0040*D*. Zero mass diffusion, Re = 1850, aspect ratio 1.75

material different to that of the carrier fluid was investigated (i.e., different Schmidt number). In the case of a zero offset of the tracer particle introduction, the effect of increasing the tracer diffusivity is shown in Figure 4.

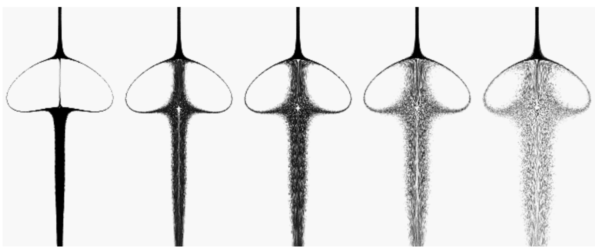

Figure 4. Plots showing the effect of increasing the tracer mass diffusivity (from left to right, Schmidt number of numerical tracer is infinite, 1.6 x 10^5, 4.0 x 10^4, 1.0 x 10^4, 2.5 x 10^3, respectively). There is no offset of injection from central axis. Re = 1850, aspect ratio 1.75

As the tracer diffusivity is increased (or Schmidt number decreased), particles are able to cross the closed streamlines around the recirculation bubble, both into and out of the breakdown region. Additional diffusivity of the tracer produces the image of an increasingly open vortex breakdown bubble. In the experiments of Spohn et al. [24], two types of tracer visualisation were used: dye injection and electrostatic precipitation. It is probable that the diffusivity properties of the two tracers would be different; together with differences in the injection locations, the varying diffusivities would explain the different visualisation patterns observed.

When the effects of dye offset and diffusivity are combined, it is possible to reproduce the asymmetrical and open bubbles observed by previous authors and in the current experiments. The predicted tracer particle distributions for different angle views in the case of combined injection offset and additional tracer diffusivity are shown in Figure 5. The patterns produced resemble closely those observed by Spohn et al. [24].

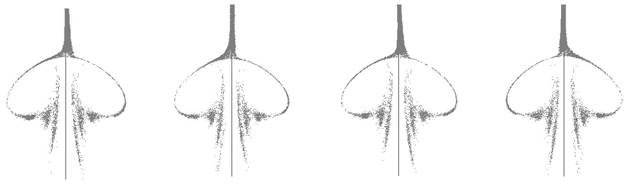

Figure 5. Diametral view of tracer at 4 azimuthal angles separated successively by 90^0.

Clearly, the above numerical experiments provide strong support for the hypothesis that the asymmetry and openness of the vortex breakdown bubble in the confined cylinder can be explained by the properties of the tracer and its introduction to the flow.

Other features of flows often observed in confined cylinders can also be explained by the vagaries of dye or tracer visualisation. For example, it is claimed in Spohn et al. [24] that "Neither in the horizontal plane adjacent to the cover, nor in a vertical diametral plane can we see any asymmetries. Clearly the electrolytic precipitation supplied tracer particles in a highly axisymmetric way." However, we have showed by the current study and the previous one on pre-vortex breakdown spirals (Hourigan et al. [13]) that this is not necessarily the case. By definition, a vortex breakdown bubble involves the rapid and relatively massive dilation of streamlines; the radial expansion of streamlines can be orders of magnitude near the central axis. Therefore, even asymmetries in the streaklines that are imperceptible to the eye are amplified to significant magnitudes at the breakdown.

If one inspects Fig. 5(b) of Spohn et al. [24], it would appear that the flow upstream of the vortex breakdown is not really perfectly symmetrical. In fact, the streaklines clearly progress from generally left of the central axis to right of axis upon reaching the breakdown.

An additional case for a cylinder aspect ratio of 2.5 and Reynolds number of 2119 was examined and is shown in Fig. 6. The experimentally observed structures, using dye, in the observations made by Dr Lachlan Graham at the CSIRO, Melbourne, are similar to the predicted dye patterns, again for purely asixsymmetric flow; both display open and asymmetrical vortex breakdown structures.

Spirals and Christmas Tree Patterns

Another feature often observed in confined cylinder flow is the occurrence of the asymmetrical ("Christmas tree") streaks downstream of vortex breakdown. Spohn et al. [24] claim that these are due to the flow separation from the side wall of the container. They state they only see it for the precipitation where the tracer is released from the solder wire on the sidewall below separation, and not when the tracer is released on the top fixed wall or injected along the central axis. If one looks at Fig. 4(c) of Spohn et al. [24], it is clear that the streak layers upstream of breakdown are very thick. In fact, clearly there has been a number of recirculations around the whole circuit of the container whereby the streaklines are carried further away from the central axis upon returning, leading to the "Christmas tree".

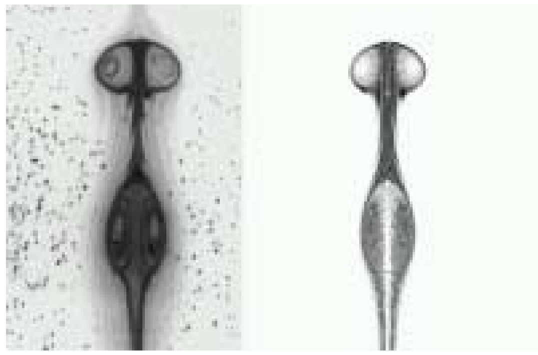

Figure 6. Observed dye pattern at left and predicted tracer pattern at right; Re = 2119, aspect ratio = 2.5, tracer injection offset = 0.0005*D*, Schmidt number = 9.5 x 10^3.

Similar "Christmas tree" streak patterns are observed with dye injection in the paper by Hourigan et al [13], even for a pre-vortex breakdown flow and where the dye was released close to the central axis of the fixed endwall. Such patterns, for both pre- and post-vortex breakdown, are shown in Figure 7 from dye observations made at the CSIRO. The prominence of these streaklines is a function of how long the tracer is allowed to evolve, how spread out it is originally, and how rapidly the tracer dissipates before completing successive circuits of the container. Corresponding numerical traces, obtained when the code was run long enough to allow the tracer to complete several circuits of the cylinder, showed similar patterns. When the tracer returns after completing a circuit, it has spread due to its additional diffusivity and follows a trajectory radially shifted from its original path. Each successive circuit completed leads to an additional shift and the appearance of another layer of the "Christmas tree".

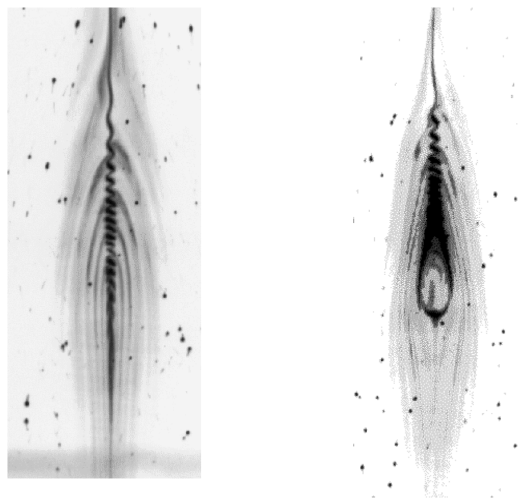

Figure 7. Observed spirals before (left, *Re*=2862) and after (right, *Re*=2957) vortex breakdown for an aspect ratio of 3.5.

Flow visualization using tracers such as dye or electrostatic precipitation is a useful means to elucidate structures in fluid flows. However, the technique still contains a number of pitfalls that can lead to erroneous conclusions. It is well known that traces in unsteady flows are not equivalent to instantaneous streamlines. We find that even in *steady* flows, traces can be misleading. The diffusivity of tracers allows them to cross streamlines and enter otherwise prohibited regions. The method of visualising a rotating flow with a plane of light adds a level of ambiguity; the patterns observed are no longer traces but a series of points at which tracer particles pass through the light beam on their spiral paths.

Is the Recirculation Bubble in the Confined Cylinder a Vortex Breakdown?

Vortex breakdown, according to Hall [11], is distinguished by three different characteristics: a free stagnation point in combination with reversed axial flow; the divergence of the stream tubes upstream of the bubble; and the existence of an adverse pressure gradient along the rotation axis. The recirculation zones within a confined cylinder flow observed at lower Reynolds numbers do satisfy these basic criteria of vortex breakdown. However, they do not satisfy the extended criteria according to Leibovich [17,18] (i.e., unsteady and asymmetric) nor those according to Keller [14] and Goldshtik and Hussain [10] (i.e., jumps and hysteretic transitions).

A motivation for highlighting similarities in the flow structures observed in confined cylinders and vortex tubes is to rebuff criticism that the confined cylinder recirculation bubble is not really a true vortex breakdown because it is symmetric and closed as well as not exhibiting hysteresis. However, it appears that the arguments and conclusions of Spohn et al. [24] cannot be sustained. We have shown that a purely axisymmetric solution can lead to particle traces that suggest erroneously that vortex breakdown bubbles in the confined case are not closed; even in steady flow, open bubbles with permanent in- and out-flow are observed. Very small deviations from axisymmetry of dye release create the incorrect impression that the flows are not perfectly symmetric.

Nonetheless, it is becoming clear that the flow structures that are observed in the confined cylinder as the Reynolds number is increased display the wide variety of characteristics that are observed for vortex breakdown in open tubes and delta wings; axisymmetric bubble, rotating or spiral modes, and unsteady. It appears that the difference is chiefly the order in which the different characteristics appear. For example, in the case of the vortex tube, it appears that the rotating modes ($m \neq 0$) usually appear before the symmetric ($m=0$) mode, whereas in the confined cylinder the axisymmetric bubble appears first. On this basis, excluding the rich variety of recirculation structures observed in the confined cylinder from being classified as vortex breakdown would seem too restrictive.

4. Conclusions

Although there have been documented previously a number of cases where streamlines and streaklines differ in *unsteady flows*, we have shown here that this can occur also in a *steady flow*. Tracer fluid can be transported across otherwise forbidden dividing streamlines as a result of mass diffusion. In the particular case of a vortex breakdown bubble, this can make the bubble appear to be an "open" rather than "closed" structure. The addition of even a minute offset from the central axis to the introduction of the "dye" resulted further in an apparent asymmetry to the flow. It is concluded that there is no compelling evidence to suggest that the vortex breakdown at low Reynolds number in the torsionally driven cavity is other than symmetrical and closed.

Acknowledgments

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