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Post-Newtonian Collapse Calculations

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Introduction

If computer speed and storage keeps increasing at the present rate a three dimensional numerical code modelling the exact equations governing general relativistic collapse will soon be possible; however, at present it is necessary to use simplifying approximations. Provided the general relativistic effects are limited to a 'small' perturbation, the post-Newtonian equations of Chandrasekhar (1965) should be adequate. These equations take the form of the Newtonian equations for a self gravitating fluid with extra terms to incorporate the $1/c^2$ contributions, where c is the speed of light. The numerical solution of these equations can be achieved using any method appropriate for three dimensional Newtonian hydrodynamics.

Two applications come to mind. Consider firstly a binary system in which one of the members is a white dwarf. If this star can accrete enough matter from its companion star so that it reaches the Chandrasekhar limiting mass (M_{crit}), approximately 1.4 $M_\odot$ for a carbon-oxygen core, then the pressure forces can or may not lead to complete disruption of the body depending on details of the initial structure (Canal *et al*, 1982). The second case is the core collapse of a star of eight of more solar masses. Again, the 'burnt out' core grows to a stage when degenerate electron gas pressure can no longer support it, and then gravitational collapse must ensue. Both of these cases can be modelled as the collapse of a body slightly in excess of M_{crit}. Except in the spherically symmetric case little is known about the evolution of the core. If the core is rotating the collapse will be non-axisymmetric and it has been suggested (Wheeler and Ruffini 1971) that the core will flatten and fragment. Numerical computations are needed to determine whether or not the core will evolve in this way. The purpose of this paper is to examine the dynamical collapse of a rotating body, (with $M > M_{crit}$), from white dwarf densities down towards neutron star densities, hoping to resolve some of the effects of rotation on the evolution.

Method

The post-Newtonian equations for continuity and momentum (Chandrasekhar 1964) for a perfect fluid are modelled with the smoothed particle hydrodynamics (SPH) of Gingold and Monaghan (1982). The details will be given elsewhere. The method has been well tested by the above mentioned authors. They have used the method to model such things as polytropic gas spheres (Gingold and Monaghan 1977), fission of damped, rotating barotropic stars (Gingold and Monaghan 1979) and collapse of rotating, isothermal gas clouds (Gingold and Monaghan 1981). The version of the code used conserves both linear and angular momentum exactly.

In order to test the SPH method for post-Newtonian hydrodynamics two cases were examined. Firstly general relativistic polytropes were modelled. The exact solutions were derived by Tooper (1964). The relativity content is measured by σ, the ratio of central pressure to central energy density, and this is closely related to the specific thermal energy and potential through the virial theorem. It is found that the SPH and the true solutions are very close to each other if σ is less than 0.10. For $\sigma = 0.10$, the maximum difference between the SPH and exact solution is about ten percent of the difference between the exact relativistic and non-relativistic solution. However, with a σ greater than about 0.15 the method will not even converge to give a solution. This indicates that results will be valid if the effects of general relativity are limited to be less than a ten percent perturbation.

Perihelion shift provides a dynamical test. To allow the precession to show up numerically it is necessary to use a moderately relativistic case. Typically, a point mass of 0.0025 $M_\odot$ was placed in an elliptical orbit around a 0.25 $M_\odot$ neutron star at a distance 4.0×10^6cm. The ellipticity was 0.2. For such a case the classical general relativistic approximation gives a precession rate of 9 degrees per revolution while the measured rate was 8 degrees per revolution. Relativistic effects here are quite large so neither result is exactly right.

Collapse Calculations

The equation of state used for the collapse calculations is that of 'cold', catalysed matter, where the pressure is due to electron and neutron degeneracy only, i.e. the zero point momentum of these species. The form of the equation used is due to Baym, Pethick and Sutherland (see Canuto 1974). There is no thermal pressure included. This is an accurate representation of the physics provided the temperature is low enough. For white dwarf densities it needs to be limited to $10^{8.5}$K, while the limit for neutron stars is around $10^{11.5}$K. (e.g. Misner, Thorne and Wheeler page 599) These limits will not be strictly adhered to during the collapse but it represents a first approximation.

A very important feature of the equation of state is the sudden increase in the compressibility at $10^{11.4}$g/cm^3. At this point neutrons spontaneously begin to drip out of the highly complex nuclear species until the fluid becomes just a sea of neutrons, electrons and protons. The compressibility drops to below 0.5 here, which should be compared to the limit for gravitational stability, requiring it to be greater than 4/3.

The starting model for the collapses was a 1.1 $M_\odot$ white dwarf which is just in excess of the limiting mass for this equation of state. This was artificially supported by raising the pressure by ten percent. At the start of the collapse the pressure was reduced to its correct value.

For the uniformly rotating case centrifugal mass shedding will occur at the equator if the ratio α, the rotational to potential energy, is greater than about 0.05. A case with $\alpha = 0.01$ will be examined. (This corresponds to a surface velocity of about 3000 km/s, which is certainly much greater than the observed rotation rate for white dwarfs, being typically less than 100 km/s, but should enable the effects of rotation to show themselves.) This rotation rate is imposed on the body at the start of the collapse phase.

The collapses are performed with 1000 particles. (An SPH particle is different from those found in the usual particle codes.) Initially the central density is approximately 2×10^8 g/cm^3 and the radius 3×10^8 cm. The collapse proceeds smoothly at first until the central density reaches the neutron drip point. Then the pressure no longer presents a barrier to collapse, so the central region of the star contracts much faster than the outer envelope, leading to a very centrally condensed configuration. The central density builds up until it reaches $10^{13.5}$ gm/cm^3, before particles begin to penetrate the plane of symmetry. This takes 80 s (real time) to occur compared to the spherically symmetric case which takes 70 s. At this stage the star is in the form of a pancake with the approximate width to height ration of 4:1. The parameter α is then slightly in excess of 0.30. The maximum potential and velocity squared are both limited to about 0.06, (in units where $G = c = 1$), thus the post-Newtonian equations are probably adequate to describe the evolution. The distended nature of the fluid body makes the modelling very difficult from here on. There are two length scales involved; that of the compact core, and that of the diffuse envelope. The difference in scales is about a factor of four yet the numerical method only uses a single resolution length. The scale length is mainly determined by the core size. This means that the outside envelope will be modelled as a cloud of particles rather than a continuous fluid unless the smoothing length again grows to a much greater value. (To some extent this does happen later on in the collapse as the core oscillates.) Timestep reduction due to the Courant condition means that the evolution almost stops after the maximum density is reached. On the VAX computers used for this calculation it takes about five hours processor time to reach the maximum density and then a further fifteen hours to evolve the system a further four seconds real time. In this time the pancake oscillates, but perturbations in the envelope do not show any sign of growing. It was pointed out by Wheeler (1966) that gravitational radiation damping will take place on a timescale of 1 s and unless the perturbations grown on a timescale smaller than this the star is unlikely to fragment. It seems much more likely that the star will die a quiet death, by gradual loss of angular momentum and energy, as perturbations develop slowly. This result is fare from conclusive with the current resolution of the scheme. To treat the problem properly one would need to look at the core and the envelope using a different scale length for each. Higher resolution in the core is also necessary to examine the formation of shock waves and to stop the SPH particles interpenetrating, while the diffuse nature of the other region neccessitates a much larger particle number to model the fluid well statistically. The major amount of processor time used in the calculation is due to the gravitational force calculation. At present it increases as the particle number squared since it is found by a direct summation over particles. An improvement to the code would be the use of a grid Poisson solver, in which the increase is more like the particle number. This should allow more reliable results to be obtained.

In the future the effects of differential rotation will be examined. This will enable the use of large rotational energies while keeping the surface velocities to more realistic values. Also the modelling should be considerably more accurate because the central regions should no longer collapse much faster than the outside, due to the different distribution of angular momentum.

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An Investigation of Fragmentation in Collapsing Magnetic Gas Clouds

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Introduction

In recent years, a large amount of work has been directed towards understanding the process of star formation. However, despite these efforts, there still remain areas largely unexplored, where theories are only justified by approximate qualitative arguments rather than fully three dimensional calculations.