

## LETTERS

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### Metastable states of a cylinder wake adjacent to a free surface

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For the same Froude number and depth of submergence beneath a free surface, the wake of a cylinder exhibits two admissible states. The first state involves a jet-like flow generally attached to the free surface; it gives rise to a large-amplitude, quasistationary wave. In the second state, the jet is detached from the free-surface, which exhibits only mild distortion. The critical feature of the transformation between these two states involves the formation of a separated vorticity layer from the free-surface and its interaction with the vorticity layer from the surface of the cylinder. This transformation can occur spontaneously over a time scale much longer than the Kármán period. © 1995 American Institute of Physics.

Formation of large-scale Kármán vortices from a fully submerged cylinder, in the absence of free-surface effects, has been a topic of major interest in recent decades. The origin of this classical vortex formation is now viewed to be a global (absolute) instability of the near-wake region, as described in the range of investigations assessed by Huerre and Monkewitz<sup>1</sup> and Oertel.<sup>2</sup> In addition to this instability giving rise to Kármán vortices, the Kelvin–Helmholtz instability in the shear layers separating from the cylinder becomes apparent at sufficiently high Reynolds number. It leads to small-scale vortices, which feed into the Kármán vortices as visualized, for example, by Gerrard,<sup>3</sup> Sheridan *et al.*,<sup>4</sup> and Lin *et al.*<sup>5</sup>

When the cylinder is located close to a free surface, one expects substantial alteration, even attenuation, of the classical Kármán vortex formation. Indeed, the stability analysis of Triantafyllou and Dimas<sup>6</sup> shows, for the limiting case of a cylinder piercing the free surface, the potential for existence of a convective versus an absolute instability of the wake. Even if Kármán-type vortex formation does not occur in the wake beneath a free surface, Kelvin–Helmholtz vortices are expected to arise; they may coexist with new types of large-scale structures in the near-wake region. In turn, these features of the flow structure may couple with distortion of the free-surface to produce complex types of wave–wake interaction phenomena, which have remained unexplored.

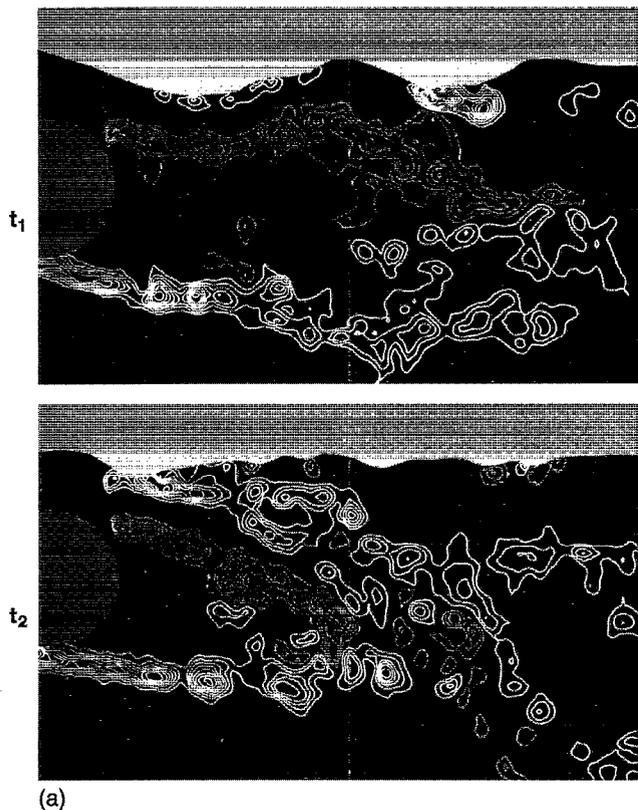
Experiments were carried out in the test section of a free-surface water channel having a width of 210 mm and a water depth of 527 mm; this section was preceded by two successive 3:1 contractions. The cylinder of diameter  $D=25.4$  mm was mounted horizontally at a distance  $h=11.4$  mm beneath the free surface, measured at a streamwise location corresponding to the axis of the cylinder. The dimensionless depth of submergence  $h^*=h/D$  was therefore 0.45.

The free-stream velocity, Froude number, and Reynolds number had values, respectively, of  $U=297$  mm/s,  $Fr=0.60$ , and  $Re=7550$ .

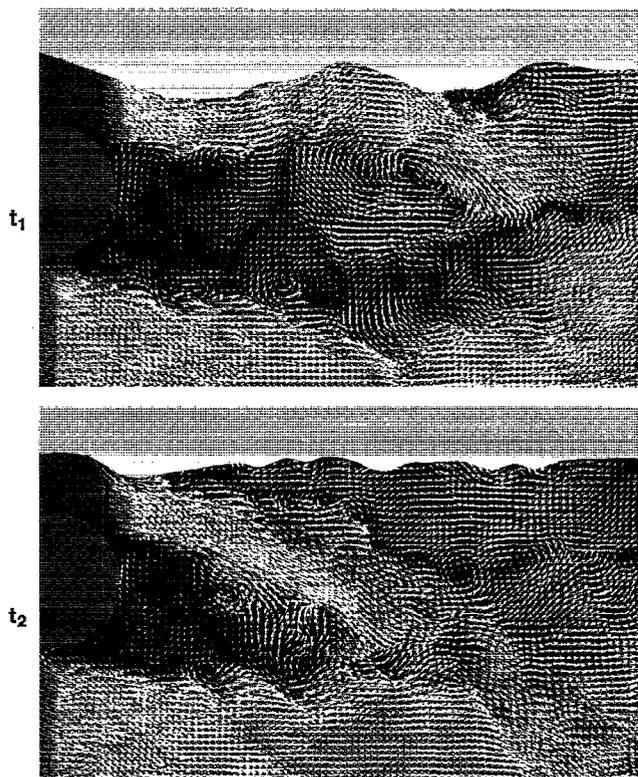
Instantaneous distributions of velocity and vorticity were obtained over a plane cutting across the near-wake of the cylinder using the technique of high-image-density particle image velocimetry described by Rockwell *et al.*<sup>7</sup> In essence, a continuous wavelength argon–ion laser (20 W beam) was deflected from a rotating polygonal mirror to generate a scanning laser sheet, which provided multiply exposed images of 12  $\mu\text{m}$  diam spheres. Use of a rotating bias mirror in front of the camera lens precluded difficulties associated with directional ambiguity and large dynamic range. The camera lens had a magnification of 1:3. Multiply exposed images were recorded on 35 mm film, digitized, then interrogated, and evaluated using a single frame cross-correlation technique. The effective grid size in the plane of the laser sheet was 1.1 mm.

The two admissible states of the wave are shown in the patterns of instantaneous velocity and vorticity in Figs. 1(a) and 1(b). The color black denotes water, light gray is air, and dark gray is the shadow region of the laser sheet; in this region above the cylinder, the velocity field and locus of the free surface could not be determined.

Transformation between the two states defined at instants  $t_1$  and  $t_2$  in Fig. 1 evolved spontaneously at a frequency of the order  $fD/U \sim 10^{-3}$ , which is two orders of magnitude smaller than the formation frequency of large-scale Kármán vortices. Fluctuations about a given state, without a complete transformation to the other state, occurred irregularly at an average frequency having an order of magnitude  $fD/U \sim 10^{-1}$ . In other words, the states shown at  $t_1$  and  $t_2$  are metastable, i.e., they show only a slight margin of stability. Transformation could be induced artificially by



(a)



(b)

FIG. 1. (a) Instantaneous velocity fields showing the two states that can exist at different times  $t_1$  and  $t_2$  for the same flow conditions. Froude number  $Fr=0.6$  and cylinder depth  $h^*=h/D=0.45$  for both images. (b) Instantaneous vorticity fields corresponding to the velocity fields in (a). White contours are positive vorticity levels and gray are negative. The minimum and incremental contour levels are  $|\omega_{\min}|=20 \text{ s}^{-1}$  and  $\Delta\omega=20 \text{ s}^{-1}$ , respectively.

transiently piercing the free surface in a region downstream of that shown in Fig. 1 to a depth of approximately 10 mm, or by causing hysteresis associated with decreasing versus increasing the flow velocity.

The velocity field at instant  $t_1$  shows a jet-like flow attached to the free surface, until small-scale flow separation from, then reattachment to, the free surface occurs at the second trough of the free-surface wave. The corresponding vorticity distribution at  $t_1$  in Fig. 1(b) shows that this region of separation is associated with high levels of vorticity in the separated mixer layer. It therefore exhibits the characteristics of the instantaneous structure of a quasisteady breaking wave, as determined experimentally by Lin and Rockwell.<sup>8</sup> They induced wave breaking at a large scale and in the absence of a cylinder by locating a hydrofoil well beneath the free-surface; no reattachment to the free surface was apparent. In view of the fact that the separated zone associated with the breaking process at instant  $t_1$  reattaches and is thereby confined to a small region, it might be designated as localized wave breaking. In addition to this separated vorticity layer formed from the free surface, vorticity layers from the top and bottom surfaces of the cylinder are clearly evident. All these layers exhibit small-scale Kelvin–Helmholtz vortices that eventually take the form of irregular distributions of vorticity. These complex patterns of vorticity at instant  $t_1$  exist beneath a quasistationary, free-surface wave having a relatively large amplitude. Although the uniform approach velocity  $U=297 \text{ mm/s}$  only slightly exceeds the minimum of  $230 \text{ mm/s}$  for the establishment of a stationary wave past a submerged obstacle in an otherwise irrotational flow, as described by Lighthill,<sup>9</sup> the maximum velocity of the jet-like flow beneath the free surface at  $t_1$  is  $390 \text{ mm/s}$ . This relatively high velocity, in conjunction with the impermeable boundary of the cylinder close to the free surface, allows establishment of the large-amplitude wave.

Viewing the overall structure of the wake–wave system at  $t_1$ , the layer of negative (gray) vorticity from the top surface of the cylinder initially remains parallel to the curved free surface. The eventual occurrence of localized wave breaking at the free surface and the corresponding formation of the layer of positive (white) vorticity displaces the neighboring layer of negative vorticity downward, relative to the free surface. This event promotes a metastable condition, and sets the stage for transformation to the other state.

The second state is defined at  $t_2$  in the velocity and vorticity fields in Figs. 1(a) and 1(b), respectively; the free surface exhibits only mild distortions. The pattern of velocity vectors shows that the jet-like flow is detached from the surface and eventually merges with the mixing layer formed from the bottom surface of the cylinder. The corresponding vorticity distributions indicate that the jet-like flow now involves layers of vorticity from two different sources: the free surface and the surface of the cylinder. The peak levels of positive (white) and negative (gray) vorticity on either side of the jet are of the same order of magnitude; maximum values as high as  $\omega_{\max}=180 \text{ s}^{-1}$  are attained. Moreover, the spatial development of the small-scale concentrations of vorticity, eventually leading to larger-scale clusters, occurs remarkably in phase on either side of the jet. In contrast to the

state at  $t_1$ , the free surface now exhibits only small-scale distortions. Immediately beneath the free surface, flow is in the upstream direction. It generates significant levels of positive (gray) vorticity at the small-scale troughs of the free surface. The form of these vorticity concentrations relative to the location of the troughs is similar to that predicted by Dimas and Triantafyllou<sup>10</sup> for the case of mean shear flow beneath a free surface.

As pointed out by a referee, one may employ linear theory to determine the wavelength of the wave train behind the cylinder. From Lighthill,<sup>9</sup> the phase speed of the free-surface wave, relative to still water, is

$$c = [(g/k) + (T/\rho)k]^{1/2}, \quad (1)$$

in which  $g$  is gravitational acceleration,  $T$  is surface tension,  $\rho$  is liquid density, and  $k = 2\pi/\lambda$ , where  $\lambda$  is the wavelength. Now, setting  $c = U = 29.7$  cm/s, the calculated wavelength is  $\lambda = 5.08$  cm, which is twice the diameter of the cylinder. According to the interpretation of the referee, this means that a resonance condition for wave generation by the cylinder is established for the conditions of Fig. 1. It should be emphasized, however, that this simplified approach does not account for the presence of the cylinder which modifies the effective value of  $c$ . Further experiments at other values of Froude number have shown that substantial amplitudes of the free-surface wave are attainable for wavelengths lying in the range  $0.96 \leq \lambda/D \leq 2.0$ .

On the other hand, when the jet-like flow is fully deflected from the free surface, the surface takes the form of either low amplitude, irregular distortions of the type shown at instant  $t_2$  in Fig. 1, or undetectable distortions. Regarding the irregular frequency of fluctuation of the free surface described in the foregoing, consideration of the group velocity of the wave system leads to a characteristic frequency of the order of  $fD/U \sim 10^{-1}$ , which is consistent with the experimental observations.

The delicate existence of the states defined at  $t_1$  and  $t_2$ , and their susceptibility to transform from one to the other, has important consequences for both the loading on the cylinder and free-surface signature immediately downstream of it. Detachment of the jet from the free surface simultaneously deflects the momentum of the jet-like flow and decreases the

streamwise extent of the separated flow behind the base of the cylinder. Substantial changes in both lift and drag are therefore expected. Correspondingly, the free surface undergoes, not simply a decrease in amplitude of the spatially periodic quasistationary wave, but the onset of irregular distortions. These changes in amplitude, form, and scale of the free surface provide a means for indirect detection of the altered vorticity field beneath it.

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