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AXISYMMETRIC AND NON-AXISYMMETRIC FLOW AND WALL SHEAR STRESS IN A MODEL FUSIFORM ABDOMINAL AORTIC ANEURYSM

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ABSTRACT

Pulsatile flow through a sinusoidal bulge in an otherwise straight circular tube is used to model the fluid mechanics within a fusiform abdominal aortic aneurysm. Three-dimensional flow is computed using a high-order spectral-element/Fourier method, driven by an anatomically realistic heartbeat waveform. Model dimensions and parameters are chosen to describe human abdominal aortic aneurysms considered both low and high risk in terms of their likelihood of rupture. A Reynolds number of 330, a Womersley number of 10.7, and aneurysm length and diameter ranges of 2.9-5.2 and 1.3-2.1 times the vessel diameter, respectively, are investigated. Variation in wall shear stress with both time and as a function of aneurysm dimension is computed. From computations on a bulge with maximum diameter approximately twice the undilated tube diameter, the flow is found to be naturally three-dimensional under conditions consistent with the human abdominal aorta. However, the dominant feature of the flow remains an axisymmetric vortex ring, which is generated at the proximal end of the bulge during systole. Both three-dimensional flow and non-uniformity in azimuthal wall-shear-stress distribution are most pronounced in the vicinity of the distal end of the bulge during the resting phase of the heartbeat. The axial distribution of wall shear stress scales approximately with the length of the bulge. The flow is sensitive to changes in the bulge diameter: a bulge with maximum diameter 1.9 times the vessel diameter invokes significantly more complex dynamics than a modest bulge of 1.3 diameters.

Key Words: *blood flow, wall shear stress, aneurysm, three-dimensional transition, spectral-element method.*

1 INTRODUCTION

Aneurysms present as a localized enlargement of an artery, caused by weakness or degradation in the tissue integrity comprising the artery wall.[1,2] Recent attention has focused on the role of blood flow on aneurysm mechanics, characterizing the fluid mechanics within an aneurysm, and determining the fluid stresses imparted on the artery and aneurysm walls. Recent experiments[3] and axisymmetric simulation[4] have shown that in fusiform aneurysms, the flow is dominated by a strong vortex ring, which develops in the bulge during the systolic phase of the heartbeat waveform. This study employs three-dimensional analysis and simulation to investigate three-dimensional features of this complex flow system.

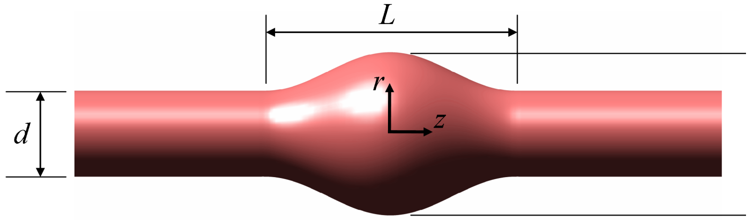

Figure 1: A schematic diagram of the fusiform aneurysm model, with cylindrical r-z coordinate system and dimensions shown.

2 METHODOLOGY

In this study, the aneurysm is modeled as a sinusoidal bulge in an otherwise straight pipe,[4] characterized by a length ratio $\text{LR} = L/d$ and a diameter ratio $\text{DR} = D/d$, where L is the aneurysm bulge length, d is the undilated tube diameter, and D is the maximum bulge diameter. Flow is driven by a pressure gradient derived to reproduce a physiologically realistic pulse waveform. A Reynolds number based on the time-averaged mean velocity through the model (U) is defined as $Re = Ud/\nu$, where ν is the kinematic viscosity of the fluid. In this study a Reynolds number of $Re = 330$ is chosen consistent with previous studies. Similarly, a Womersley number of $\alpha = 10.7$ is employed ($\alpha = \frac{d}{2}\sqrt{2\pi f/\nu}$, where f is the frequency of the heartbeat). Two-dimensional flow and linear stability analysis were computed using an incompressible Navier–Stokes solver[4,5] based on the spectral-element method.[6] Three-dimensional computations were efficiently calculated using a spectral-element/Fourier algorithm detailed in Blackburn & Sherwin.[7]

3 RESULTS: 3D FLOW DEVELOPMENT

Using a Floquet linear stability analysis technique formulated in cylindrical coordinates,[5] the axisymmetric flow computed and described in Sheard[4] was analysed at a range of Reynolds numbers to determine the stability of the pulsatile flow to non-axisymmetric perturbations. The fastest-growing wavenumber was found to change with Reynolds number: the flow was predicted to first become unstable with a wavenumber $m = 1$ at $Re \approx 270$. Inspection of the perturbation field arising from this analysis demonstrated that the non-axisymmetric flow features evolved in the distal region of the bulge during the resting phase of the hearbeat, before being flushed out of the bulge during the systolic phase. At $Re = 330$, matching Salsac *et al.*[3] and Sheard[4], who selected parameters relevant hemodynamics within a human abdominal aorta, the fastest-growing wavenumber was $m = 3$, though the location and behaviour of the three-dimensional structures was similar to that at onset of the transition.

Three-dimensional simulations were then conducted to confirm these predictions at $Re = 330$. Figure 2 plots contours of wall shear stress magnitude over single a period from the saturated three-dimensional simulation. The contours indicate that the non-axisymmetric effects are most visible in the resting phase of the pulse cycle (a). During the systolic phase (c), where wall shear stress levels are highest, there is little wall shear stress variation in the azimuthal direction: axisymmetric features dominate.

4 RESULTS: GEOMETRY VARIATION

Consideration was given to the effect of changing the aneurysm geometry on the distribution of wall shear stress magnitude ($|\text{WSS}|$, taken as the leading eigenvalue of the rate-of-strain tensor: the wall shear

(a) (b) (c) (d)

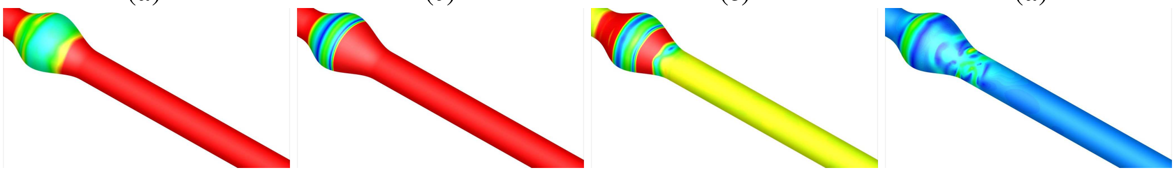

Figure 2: Flooded contours of wall shear stress magnitude (|WSS|) plotted on the surface of the aneurysm model with LR $= 2.9$ and DR $= 1.9$. The model is shown in isometric view with flow from left to right. The frames are taken at equi-spaced times during the pulse cycle: (a-b) displays the resting (diastolic) phase, and (c-d) shows the systolic phase. Contour levels from 0 (blue) through to $40U/d$ (red) are plotted.

stress is recovered by multiplying these dimensionless values by $\mu U/d$). Motivated by the previous observation that |WSS|is predominantly axisymmetric in this model aneurysm, axisymmetric simulations were undertaken to investigate how |WSS|throughout the model varied in time. The plots in figure 3 show contours of |WSS|in t-z space: that is, along any horizontal line the axial |WSS|distribution along the vessel and aneurysm wall is given at that instant in time. A single heartbeat is shown.

In figure 3(a), the length ratio is varied over $2.9 \leq$ LR ≤ 5.2 for a constant diameter ratio DR $= 1.3$. It is notable that the |WSS|is similarly distributed in these plots (peak systole occurs at approximately $t = 0$), and away from the aneurysm bulge, the wall shear stress is axially uniform and consistent irrespective of LR. Within the bulge ($|z|/d < 1.45$, 1.95 and 2.6 for top, middle and bottom, respectively), a high zone of |WSS|is detected in the vicinity of $z/d = 1.5$-2 at $t \approx 0$, followed by a localized peak of |WSS|which migrates upstream over $0.1 \lesssim t/T \lesssim 0.3$ from just upstream of the centre of the aneurysm bulge ($z \approx 0$). These correspond to the flushing of fluid into the distal artery at systole, and the development of a secondary vortex ring after peak systole (as described in Salsac *et al.*[3] and Sheard[4]). With increasing length ratio, the magnitude of |WSS|within the bulge decreases, and the distribution appears to scale in the axial direction with the length ratio.

To consider diameter-ratio variation, three models with LR $= 2.9$ and DR $= 1.3$, 1.9 and 2.1 were studied (figure 3(b)). It was found increasing the diameter ratio leads to a marked change in |WSS|distribution. At higher diameter ratios, the regions of highest |WSS|are consistently located in the distal half of the bulge and into the distal artery. In contrast to the smaller diameter ratio DR $= 1.3$, for DR $\gtrsim= 1.9$, the aneurysm bulge experiences |WSS|levels far higher than recorded in the undilated artery segments, and these persist for a majority of the heartbeat cycle. Even during the resting phase, elevated |WSS|levels are detected in the bulge at $z \approx -0.6$. A further striking difference is the appearance of small regions of low |WSS|during systole (e.g. see $t \approx 0$ and 0.2), in contrast to the DR $= 1.3$ cases. This indicates that at larger aneurysm diameters, vessel wall tissue is exposed to greater levels of peak |WSS|, and greater temporal and spatial fluctuations in |WSS|, all of which can erode the integrity of luminal tissue.[1,2]

5 CONCLUSIONS

A stability analysis and high-resolution non-axisymmetric computations of a flow representative of pulsatile flow through a fusiform human abdominal aortic aneurysm have shown that while the flow is non-axisymmetric under these conditions, wall shear stress is predominantly axisymmetric. By systematically varying the length ratio and diameter ratio independently, it has been found that the axial wall-shear-stress distribution scales approximately with the length of the aneurysm bulge, and that increasing the bulge diameter invokes a marked increase in the spatio-temporal fluctuation of wall shear stress throughout the pulse cycle, due to the increased circulation in the vortex ring shed into the bulge during the systolic phase of the heartbeat.

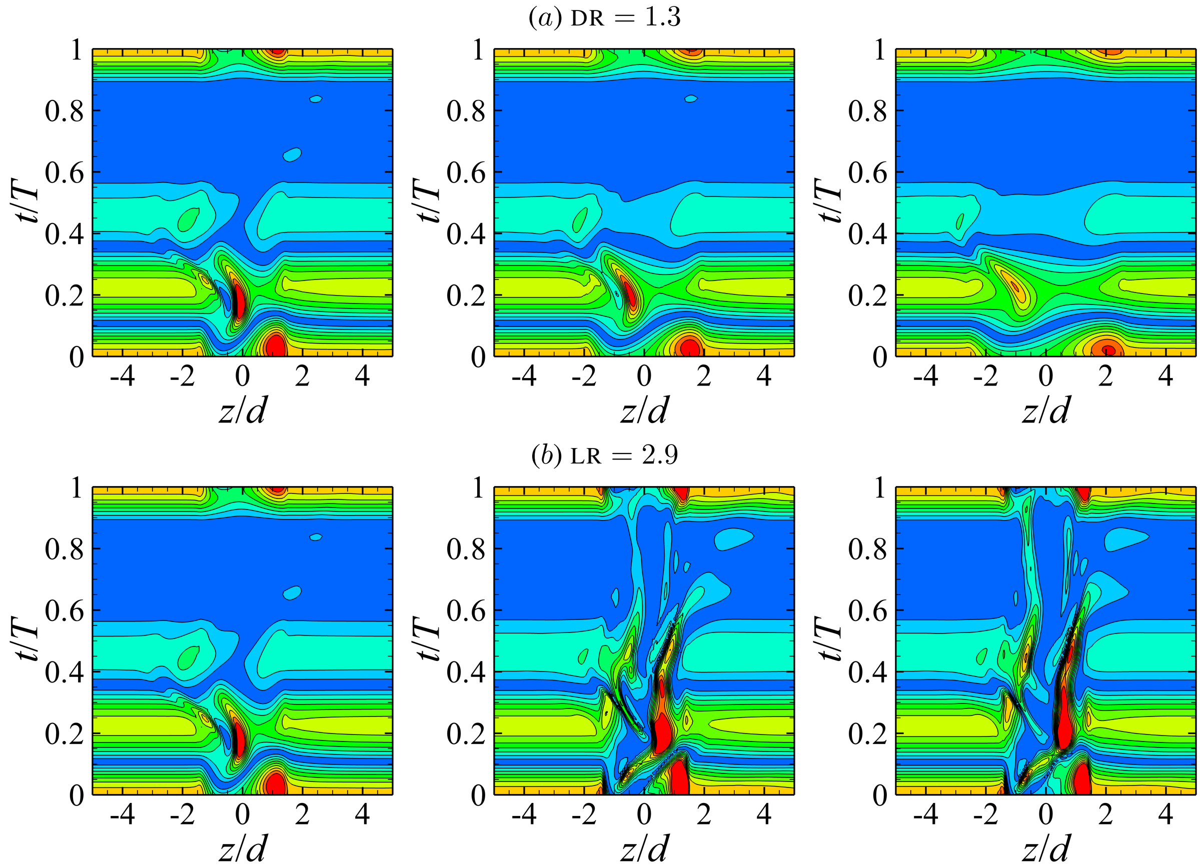

Figure 3: Contours of shear rate plotted on t-z axes to demonstrate the variation in wall shear stress over a single pulse cycle. (a) DR $= 1.3$, and from left to right LR $= 2.9$, 3.9 and 5.2. (b) LR $= 2.9$, and left to right DR $= 1.3$, 1.9 and 2.1. 10 equi-spaced contour levels are plotted between 0 (blue) and $100U/d$ (red).

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