## Natural convection in enclosures filled with a vapour and a non-condensing gas

D. J. CLOSE and J. SHERIDAN<sup>†</sup>

Division of Building, Construction and Engineering, Commonwealth Scientific and Industrial Research Organization, Highett, Victoria 3190, Australia

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Abstract—In this paper, a similarity relationship is derived for enclosures filled with a saturated gasvapour mixture. Interest in the process arises from the need to cool electronic equipment in sealed enclosures. Firstly, an argument is presented showing that the mixture will be saturated or nearly so when vapour is generated at the highest temperature surface. Then the equations derived from the conservation of mass, energy and momentum are manipulated to a form similar to those for a single component fluid, and the approximations required are discussed. Finally, an example is given showing how the net heat transfer rate in a cavity is enhanced using this process.

### INTRODUCTION

IN RECENT papers [1-3], the problem of natural convection in wet porous media has been addressed. In ref. [1], equations were derived which showed that if the interstitial fluid was a saturated mixture of a gas and vapour, then convection would be modified considerably over the case of a single fluid. Data in ref. [2] confirmed the analysis in ref. [1]. Davidson [3] showed how the non-constant properties affected the similarity between transfer of heat only and the coupled heat and mass transfer case described in refs. [1, 2], and also how the temperature and flow fields were modified in the coupled case.

Of equal interest is the open cavity problem. The situation visualized here is that of a cavity with wetted walls and the fluid within the cavity comprising the liquid's vapour and a second non-condensing gas. As in the case of the porous bed, it is postulated that the wetted walls result from a liquid source at the highest temperature surface. In Fig. 1(a), the situation is that of isothermal vertical surfaces with liquid covering the hot surface.

Trevisan and Bejan [4] have dealt with the general problem of cavities in which the buoyancy flows are driven by both temperature and concentration differences. However, they have treated the problem as essentially an uncoupled one. In the case of a cavity with wetted walls, the situation is likely to give rise to coupled equations since the mixture of gas and vapour will be saturated or nearly so.

The reason for interest in this process arises from the need for high thermal conductances between heat sources and enclosure walls in sealed electronic equipment. The process being analysed here offers the possibility of using the heat pipe-like feature of a vaporizing fluid but, with the addition of a second gas, maintaining the system at 1 atm pressure, with consequent savings in box construction costs.

This paper extends the analysis contained in refs. [1, 2] to the open cavity.

## PROPERTIES OF A GAS-VAPOUR MIXTURE WITHIN A CAVITY

Consider the situation shown in Fig. 1(a). The heated and cooled surfaces are both covered with a liquid film, which may also occur on the adiabatic surfaces.

Using air and water vapour as the example mixture, states within the enclosure are shown on the temperature-absolute humidity diagram (Fig. 1(b)).

Pressure within the enclosure is assumed constant, and other properties are assumed to be constant also with the usual Boussinesq assumption applying.

Fluid states adjacent to the heated and cooled surfaces are shown in Fig. 1(b) as states 1 and 2, both being saturated. Now consider the various regimes discussed by Bejan [5].

In the conduction and diffusion regime I (no significant fluid motion), the temperature and concentration profiles are both linear. Hence all states within the cavity should be along a straight line joining states 1 and 2. Owing to the curvature of the saturation line, the states will be in the supersaturated region, so that in practice all states will be saturated. This situation is close to the tall system, regime II, with some possible departures near the heated and cooled walls as discussed below for the boundary layer regime.

When distinct boundary layers form on the vertical walls, regime III, the states in the region away from

<sup>&</sup>lt;sup>†</sup> Present address: Department of Mechanical Engineering, Monash University, Clayton, Victoria 3168, Australia.

## NOMENCLATURE

- $D \qquad \text{diffusivity of gas-vapour mixture} \\ [m^2 s^{-1}]$
- g gravitational acceleration  $[m s^{-2}]$
- Gr Grashof number of single component fluid [---]
- Gr<sub>s</sub> Grashof number of saturated mixture [---]
- H height of enclosure [m]
- $h_c$  heat transfer coefficient for single component fluid  $[Wm^{-2}K^{-1}]$
- $h_{cs}$  heat transfer coefficient for saturated mixture [W m<sup>-2</sup> K<sup>-1</sup>]
- $h_d$  enthalpy of non-condensing component in saturated mixture  $[J kg^{-1}]$
- $h_{\rm fg}$  latent heat of vaporization of vapour component [J kg<sup>-1</sup>]
- $h_{\rm i}$  enthalpy of liquid [J kg<sup>-1</sup>]
- $h_{\rm m}$  enthalpy of saturated mixture [J kg<sup>-1</sup> non-condensing component]
- $h_v$  enthalpy of vapour  $[J kg^{-1}]$
- k thermal conductivity of single component fluid  $[Wm^{-1}K^{-1}]$
- $k_{\rm m}$  thermal conductivity of saturated mixture excluding mass diffusion  $[W m^{-1} K^{-1}]$
- $k_s$  thermal conductivity of saturated mixture including mass diffusion  $[W m^{-1} K^{-1}]$
- L width of enclosure [m]
- Le Lewis number,  $\alpha_m/D$  [—]
- $M_{\rm c}$  molecular weight of vapour [kg kg-mol<sup>-1</sup>]
- $M_d$  molecular weight of non-condensing component [kg kg-mol<sup>-1</sup>]
- m mass ratio of vapour to non-condensing component [---]
- $\dot{m}_1$  mass flux of liquid per unit width of heated surface [kg m<sup>-1</sup>s<sup>-1</sup>]
- $\Delta(m/(1+m))$  reference mass ratio difference [--]
- Nu Nusselt number for single component fluid [--]
- NusNusselt number for saturated mixture [---]Ppressure [Pa]

- Pr Prandtl number for single component fluid [---]
- Pr<sub>s</sub> Prandtl number for saturated mixture [--]
- $\dot{Q}$  heat flux [W m<sup>-2</sup>]
- $\dot{Q}_{c}$  heat flux when fluid is stagnant [W m<sup>-2</sup>]
- R universal gas constant [J kg-mol<sup>-1</sup> K<sup>-1</sup>]
- Ra Rayleigh number for single component fluid [---]
- Ra<sub>s</sub> Rayleigh number for saturated mixture [---]
- *Re* Reynolds number [—]
- T temperature of fluid [K]
- $T_0$  reference temperature [K]
- $\Delta T$  reference temperature difference [K]
- u, v fluid velocities in the x- and y-directions  $[m s^{-1}]$
- $u_0$  reference velocity [m s<sup>-1</sup>]
- $u_1, v_1$  liquid velocities in the x- and y-directions  $[m s^{-1}]$
- x, y coordinates [m].

Greek symbols

- $\alpha$  thermal diffusivity of single component fluid  $[m^2 s^{-1}]$
- $\alpha_{\rm m}$  thermal diffusivity of mixture [m<sup>2</sup> s<sup>-1</sup>]
- $\beta$  buoyancy of single component fluid  $[K^{-1}]$
- $\beta_s$  buoyancy of saturated mixture,  $\beta [1 - (m/(1+m))(M_c - M_d)(h_{fg}/RT)]$  $[K^{-1}]$
- $\mu_{\rm m}$  dynamic viscosity of saturated mixture [N s m<sup>-2</sup>]
- v kinematic viscosity of single component fluid [m<sup>2</sup>s<sup>-1</sup>]
- $v_m$  kinematic viscosity of saturated mixture  $[m^2 s^{-1}]$
- $\rho$  density of single component fluid  $[kg m^{-3}]$
- $\rho_d$  density of non-condensing component [kg m<sup>-3</sup> mixture]
- $\rho_1$  liquid droplet concentration [kg m<sup>-3</sup> mixture]
- $\rho_{\rm m}$  density of saturated mixture [kg m<sup>-3</sup>]
- $\rho_{m0}$  density of saturated mixture corresponding to temperature  $T_0$  [kg m<sup>-3</sup>].

the boundary layers tend to be nearly the same. From the similarity of the heat and mass transfer processes at the two walls, the region in the centre of the enclosure will be close to state 3 (Fig. 1(b)). Within the boundary layers, however, the value of the Lewis number will determine the states. The distributions of states through the boundary layers and the cavity are shown in Fig. 1(b) for the cases of Le < 1, Le > 1 and Le = 1. For regions where the states are supersaturated, saturation conditions would apply in practice. The same argument applies to the shallow enclosure case, regime IV.

From the foregoing, the assumption of saturation conditions throughout the cavity is acceptable when Le = 1, and is only invalid for one of the boundary layers when Le departs from 1. Considering that for



FIG. 1. Diagrams showing the system considered and the mixture states within a cavity with humid air as the example.

a number of gas-vapour mixtures *Le* is close to 1, the assumption of a saturated mixture is a reasonable basis on which to conduct preliminary analyses.

Finally, while the case of the cavity with vertical isothermal walls has been chosen as the example case, the flow phenomena are general enough for the argument to be valid for other cavity geometries as well.

## **DEVELOPMENT OF EQUATIONS**

#### Case of a single fluid

The equations describing this case are well known see for example Bejan [5]. They are repeated here so that the similar set for the saturated gas-vapour mixture can be compared. The system is shown in Fig. 1(a), with the properties constant and the Boussinesq assumption applying

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) -g[1 - \beta(T - T_0)] \quad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right).$$
 (4)

By cross-differentiating equations (2) and (3) and subtracting to eliminate pressure terms, then nondimensionalizing using

$$u' = \frac{u}{u_0}, \quad v' = \frac{v}{u_0}, \quad x' = \frac{x}{L'},$$
$$y' = \frac{y}{L}, \quad T' = \frac{T - T_0}{\Delta T}$$

we obtain

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0$$

$$u' \left( \frac{\partial^2 u'}{\partial x' \partial y'} - \frac{\partial^2 v'}{\partial x'^2} \right) + v' \left( \frac{\partial^2 u'}{\partial y'^2} - \frac{\partial^2 v'}{\partial x' \partial y'} \right)$$

$$v \left[ -\frac{\partial^3 v'}{\partial x'^2} - \frac{\partial^3 v'}{\partial x'^2} - \frac{\partial^3 v'}{\partial x' \partial y'} \right]$$
(5)

$$= \frac{v}{u_0 L} \left[ \frac{\partial^3 u'}{\partial x'^2 \partial y'} + \frac{\partial^3 u'}{\partial y'^3} - \frac{\partial^3 v'}{\partial x'^3} - \frac{\partial^3 v'}{\partial x' \partial y'^2} \right] -g \frac{\beta \Delta T L}{u_0^2} \frac{\partial T'}{\partial x'} \quad (6)$$

$$u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \frac{\alpha}{u_0 L} \left(\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2}\right).$$
(7)

The dimensionless groups  $v/u_0L$ ,  $g(\beta\Delta TL/u_0^2)$  and  $\alpha/u_0L$  can be manipulated to give the familiar groupings

$$Re = \frac{u_0 L}{v}$$
$$Gr = g \frac{\beta L^3 \Delta T}{v^2}$$
$$Pr = \frac{v}{\alpha}.$$

To obtain a relationship for the Nusselt number, consider the heat flow rate at one of the isothermal surfaces in a geometry such as that shown in Fig. 1(a). Note that there will be no liquid, as we consider a single component fluid.

The heat flow rate

$$\dot{Q} = -\int_0^H k \left(\frac{\partial T}{\partial x}\right)_{x=0} \mathrm{d}y.$$

Under stagnant conditions

$$\dot{Q}_{\rm c}=\frac{k}{L}H\Delta T.$$

Defining

$$Nu = \dot{Q}/\dot{Q}_c$$

it follows that

$$Nu = -\int_{0}^{H/L} \frac{L}{H} \left(\frac{\partial T'}{\partial x'}\right)_{x'=0} \mathrm{d}y'.$$
 (8)

#### Case of a saturated gas-vapour mixture

At this stage it is helpful to visualize the system in particular the mass transfer processes which occur within the fluid.

Considering the arrangement in Fig. 1(a), there will be some liquid condensing on the top surface which will then drip, falling rapidly through the fluid and unlikely to exchange significant quantities of heat or mass with the fluid. Otherwise, there must be a source of liquid to ensure that the fluid remains saturated as it circulates within the cavity. We postulate that this is a fog which circulates with the gas. Some justification for this assumption has been obtained using ethanol and air, where fine fog droplets were observed to circulate with the buoyancy driven flow.

A guide to the liquid droplet concentration can be obtained from ref. [7], where maximum values of the order of  $3 \times 10^{-3}$  kg m<sup>-3</sup> have been observed in clouds. This should be compared with air densities of the order of 1 kg m<sup>-3</sup>. With these concentrations, it will be assumed that the liquid droplets' effects on density and viscosity will be ignored; also the liquid and gas velocities will be assumed equal.

The conservation of the condensing and non-condensing components yield, respectively

$$\frac{\partial}{\partial x}(\rho_{d}um) + \frac{\partial}{\partial y}(\rho_{d}vm) - \frac{\partial}{\partial x}\left[\rho_{m}D\frac{\partial}{\partial x}\left(\frac{m}{1+m}\right)\right].$$
$$-\frac{\partial}{\partial y}\left[\rho_{m}D\frac{\partial}{\partial y}\left(\frac{m}{1+m}\right)\right] + \frac{\partial}{\partial x}(\rho_{l}u_{l}) + \frac{\partial}{\partial y}(\rho_{l}v_{l}) = 0 \quad (9)$$
$$\frac{\partial}{\partial x}(\rho_{d}u) + \frac{\partial}{\partial y}(\rho_{d}v) - \frac{\partial}{\partial x}\left[\rho_{m}D\frac{\partial}{\partial x}\left(\frac{1}{1+m}\right)\right]$$
$$-\frac{\partial}{\partial y}\left[\rho_{m}D\frac{\partial}{\partial y}\left(\frac{1}{1+m}\right)\right] = 0. \quad (10)$$

With  $u_1 = u$  and  $v_1 = v$ , these can be combined to give

$$\frac{\partial}{\partial x} [(\rho_{\rm m} + \rho_{\rm l})u] + \frac{\partial}{\partial y} [(\rho_{\rm m} + \rho_{\rm l})v] = 0.$$
(11)

It is now important to note that for the saturated gas-vapour mixture at essentially constant pressure, m = m(T), so that the enthalpy  $h_m = h_m(T)$  only, and  $\rho_m = \rho_m(T)$  only.

Conservation of momentum yields

$$\frac{\partial}{\partial x}(\rho_{m}uu) + \frac{\partial}{\partial y}(\rho_{m}uv) + \frac{\partial}{\partial x}(\rho_{1}u_{1}u_{1}) + \frac{\partial}{\partial y}(\rho_{1}u_{1}v_{1}) = -\frac{\partial P}{\partial x} + \mu_{m}\left(\frac{\partial^{2}u}{\partial x^{2}} + \frac{\partial^{2}u}{\partial y^{2}}\right)$$
(12)

and

$$\frac{\partial}{\partial x}(\rho_{m}uv) + \frac{\partial}{\partial y}(\rho_{m}vv) + \frac{\partial}{\partial x}(\rho_{1}u_{1}v_{1}) + \frac{\partial}{\partial y}(\rho_{1}v_{1}v_{1}) = -\frac{\partial P}{\partial y} - g\rho_{m0}[1 - \beta_{s}(T - T_{0})] + \mu_{m}\left(\frac{\partial^{2}v}{\partial x^{2}} + \frac{\partial^{2}v}{\partial y^{2}}\right). \quad (13)$$

For a saturated gas-vapour mixture the components of which follow perfect gas behaviour, and using the Clausius-Clapeyron equation, then, as shown in ref. [1], the buoyancy  $\beta_s$  is given by

$$\beta_{\rm s} = \beta \left[ 1 - \frac{m}{1+m} (M_{\rm c} - M_{\rm d}) \frac{h_{\rm fg}}{RT} \right]$$

Cross-differentiating, substituting from equations (9) to (11) and neglecting the density of the liquid droplets, we obtain

$$\rho_{\rm m} \left( u \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^2 u}{\partial y^2} - u \frac{\partial^2 v}{\partial x^2} - v \frac{\partial^2 v}{\partial x \partial y} \right) + \frac{d\rho_{\rm m}}{dT} \left[ \left( v \frac{\partial v}{\partial x} + u \frac{\partial u}{\partial x} \right) \frac{\partial T}{\partial y} - \left( u \frac{\partial u}{\partial y} + v \frac{\partial v}{\partial y} \right) \frac{\partial T}{\partial x} \right] = - g\rho_{\rm m0} \left[ \beta_{\rm s} + (T - T_0) \frac{d\beta_{\rm s}}{dT} \right] \frac{\partial T}{\partial x} + \mu_{\rm m} \left( \frac{\partial^3 u}{\partial x^2 \partial y} + \frac{\partial^3 u}{\partial y^3} - \frac{\partial^3 v}{\partial x^3} - \frac{\partial^3 v}{\partial x \partial y^2} \right).$$
(14)

Here we invoke the Boussinesq assumption of constant density  $\rho_m$ , except in the gravitational force term. Over the temperature range 10–95°C, the density of a saturated air-water vapour mixture at 1 atm varies from 1.242 to 0.657 kg m<sup>-3</sup>, while  $\rho$  for air varies from 1.257 to 0.956 kg m<sup>-3</sup>. As the molecular weight difference between vapour and gas increases, so does the variation in  $\rho_m$ . For example, a saturated mixture of water vapour and Freon 12 at 1 atm changes from 5.25 kg m<sup>-3</sup> at 10°C to 1.18 kg m<sup>-3</sup> at 95°C.

Non-dimensionalizing equation (14) shows that the second term on the left-hand side should only become significant for mixtures where the molecular weight difference between vapour and gas and the temperature difference are large. For a saturated airwater vapour mixture at 1 atm and 95°C, the coefficient of the term is  $0.0175\Delta T$ ; whereas for air, the term is  $0.00265\Delta T$ . Hence, for the air-water vapour mixture with a temperature difference of 5°C, neglecting this term is equivalent to neglecting it for an air only case with a temperature difference of 33°C. The same argument can be used to justify regarding  $\rho_m$  a constant in equation (11).

It is customary to neglect  $(T-T_0)(d\beta_s/dT)$  in single fluids, but this is only valid if  $T-T_0$  is sufficiently small. Taking the 5°C difference employed above, then for air-water vapour at 90°C,  $(T-T_0)(d\beta_s/dT)$  is about  $3 \times 10^{-3}$  K<sup>-1</sup>, as compared to  $1.6 \times 10^{-2}$  K<sup>-1</sup> for  $\beta_s$ . Values become progressively small as T decreases. This discrepancy is equivalent to a 68°C temperature difference for a perfect gas.

From the conservation of energy

$$\frac{\partial}{\partial x} \left( k_{\rm m} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_{\rm m} \frac{\partial T}{\partial y} \right) \\ + \frac{\partial}{\partial x} \left[ \rho_{\rm m} D(h_{\rm v} - h_{\rm d}) \frac{\partial}{\partial x} \left( \frac{m}{1 + m} \right) \right] \\ + \frac{\partial}{\partial y} \left[ \rho_{\rm m} D(h_{\rm v} - h_{\rm d}) \frac{\partial}{\partial y} \left( \frac{m}{1 + m} \right) \right] \\ = \frac{\partial}{\partial x} (\rho_{\rm d} u h_{\rm d}) + \frac{\partial}{\partial y} (\rho_{\rm d} v h_{\rm d}) + \frac{\partial}{\partial x} (\rho_{\rm d} m u h_{\rm v}) \\ + \frac{\partial}{\partial y} (\rho_{\rm d} m v h_{\rm v}) + \frac{\partial}{\partial x} (\rho_{\rm l} u_{\rm l} h_{\rm l}) + \frac{\partial}{\partial y} (\rho_{\rm l} v_{\rm l} h_{\rm l}).$$
(15)

Substituting from equations (9) and (10), and defining

$$k_{\rm s} = k + \rho_{\rm m}(1+m)Dh_{\rm fg}\frac{\rm d}{{\rm d}T}\left(\frac{m}{1+m}\right)$$
$$\left(\frac{{\rm d}k}{{\rm d}T}\right)' = \left[\frac{{\rm d}k}{{\rm d}T} + (1+m)h_{\rm fg}\frac{\rm d}{{\rm d}T}\left\{\rho_{\rm m}D\frac{\rm d}{{\rm d}T}\left(\frac{m}{1+m}\right)\right\}$$
$$+ \rho_{\rm m}D\frac{\rm d}{{\rm d}T}\left(\frac{m}{1+m}\right)\frac{\rm d}{{\rm d}T}(h_{\rm v}-h_{\rm d})\right]$$

$$C_{\rm s} = \frac{\mathrm{d}h_{\rm m}}{\mathrm{d}T} - h_{\rm l}\frac{\mathrm{d}m}{\mathrm{d}T} + \frac{\rho_{\rm l}}{\rho_{\rm d}}\frac{\mathrm{d}h_{\rm l}}{\mathrm{d}T}$$

then manipulation of equation (15) yields

$$k_{s}\left(\frac{\partial^{2}T}{\partial x^{2}} + \frac{\partial^{2}T}{\partial y^{2}}\right) + \left(\frac{\mathrm{d}k}{\mathrm{d}T}\right)' \left[\left(\frac{\partial T}{\partial x}\right)^{2} + \left(\frac{\partial T}{\partial y}\right)^{2}\right]$$
$$= \rho_{d}C_{s}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right). \quad (16)$$

Eckert and Faghri [8] have shown that for small temperature differences, the second term on the left-hand side of equation (16) can be neglected.

With the assumption of constant  $\rho_m$ , and neglect of the terms in equations (15) and (16) noted above, then non-dimensionalizing equations (11), (14) and (16) yields an equation set identical in form to equations (5)-(7), with modified dimensionless groups

$$g\frac{\beta_{s}\Delta TL}{u_{0}^{2}}, \frac{u_{0}L}{v_{m}} \text{ and } \frac{\rho_{d}C_{s}u_{0}L}{k_{s}}$$

which can be manipulated to

$$Re, \quad Pr_{\rm s} = \rho_{\rm d}C_{\rm s}v_{\rm m}/k_{\rm s},$$
$$Gr_{\rm s} = g\beta_{\rm s}\Delta TL^3/v_{\rm m}^2.$$

The form of the Prandtl number arises from the use of mixture enthalpy based on unit mass of the noncondensing component.

The Nusselt number is obtained as for the single component fluid, now deriving an expression for the heat transfer rate to or from one of the liquid covered isothermal plates. If the local liquid flow rate in the y-direction in Fig. 1(a) is  $\dot{m}_i$ , then at x = 0

$$\dot{Q} = -\int_{0}^{H} k_{m} \left(\frac{\partial T}{\partial x}\right)_{x=0} dy - \int_{0}^{H} \rho_{m} (1+m) Dh_{v}$$
$$\times \frac{\partial}{\partial x} \left(\frac{m}{1+m}\right)_{x=0} dy - \int_{0}^{H} \left(\frac{d\dot{m}_{1}}{dy}\right)_{x=0} h_{1} dy. \quad (17)$$

But since

$$\frac{\mathrm{d}\dot{m}_{\mathrm{i}}}{\mathrm{d}y} = \rho_{\mathrm{m}}(1+m)D\frac{\partial}{\partial x}\left(\frac{m}{1+m}\right)$$

it follows that

$$\dot{Q} = -\int_0^H k_s \left(\frac{\partial T}{\partial x}\right)_{x=0} \mathrm{d}y$$

with  $k_s$  corresponding to x = 0.

The stagnant heat transfer rate

$$\dot{Q}_{c} = k_{m} \frac{H}{L} \Delta T + \rho_{m} (1+m) Dh_{fg} \Delta \left(\frac{m}{1+m}\right) \frac{H}{L}$$
$$= k_{s} \frac{H}{L} \Delta T$$
(18)

and

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and

$$Nu_{\rm s}=\dot{Q}/\dot{Q}_{\rm c}$$

so

$$Nu_{\rm s} = -\int_{0}^{H/L} \frac{L}{H} \left(\frac{\partial T'}{\partial x'}\right)_{x'=0} \mathrm{d}y'. \tag{19}$$

From the similarity of the single component fluid and saturated gas-mixture equation sets noted above, functional relationships between the dimensionless groups for a single component fluid should apply to a saturated gas-vapour mixture.

Since heat transfer in enclosures is generally correlated by a relationship of the form Nu = f(Gr, Pr,geometry), then from the similarity of the governing equations, it is to be expected that a relation of the form

$$Nu_{\rm s} = f(Gr_{\rm s}, Pr_{\rm s}, \text{geometry})$$

will apply to the saturated mixture, where the functional relationships are the same in both cases.

When  $Nu = h_c L/k$ , the form of  $Nu_s$  is expected to be  $Nu_s = h_{cs} L/k_s$ .

#### INFLUENCE OF PROPERTIES

The modifications to the Grashof and Prandtl numbers are similar to those for the porous media Rayleigh number derived in refs. [1, 2]. Consequently, the conclusions drawn in refs. [1, 2], and partially verified experimentally in ref. [2], should apply to the open cavity.

(1) Significant increases in heat transfer rates under convecting conditions can be achieved using the same vapour but increasing the difference between the gas and vapour molecular weights.

(2) For those cases where the buoyancy and gravitational force vectors are collinear, convection will be initiated at much smaller temperature differences and cavity heights than for a single gas having the same density, viscosity and thermal conductivity. Also, upside-down convection (hot top, cold base) can occur under some conditions when the gas molecular weight is lower than that of the vapour. It should be noted that Hu and El-Wakil [9], when observing flows in a geometry similar to that in Fig. 1(a), noted that the flow direction reversed from a water-air case to an *n*-heptane-air case. From the theory presented above, this can occur since water has a lower molecular weight than air, and *n*-heptane a higher molecular

(3) The effective heat transfer coefficient  $h_{cs}$  can be very much higher than can be achieved with gases, and may be comparable with those obtainable with



FIG. 2. Predicted heat transfer coefficient between a heated source and an isothermal cavity as a function of mean temperature for various fluids:  $\Delta T = 5^{\circ}$ C.

liquids. However, this depends on the choice of mixture properties.

#### An example

Recently, Symons *et al.* [6] have shown that heat transfer from square heat dissipating plates in a large isothermal cube can be correlated by the function

$$Nu = 0.545 Ra^{0.25}$$

The length dimension used in Nu and Ra is the side length of the plates, and  $\Delta T$  in Ra refers to the temperature difference between plates and a cube.

Figure 2 compares predicted heat transfer coefficients for a plate side length of 150 mm and a temperature difference of 5°C. For the saturated gasvapour mixture  $Nu_s$  replaces Nu, and  $Ra_s$  replaces Rain the correlation.

The dip in the calculated curve for ethanol-air is due to a reversal in buoyancy from positive to negative as the temperature increases through  $31.6^{\circ}$ C.

While the increase in thermal conductances of the two gas vapour mixtures over air or Freon 12 was expected, their values when compared with water are encouraging. If used in applications such as cooling electronics, these gas-vapour mixtures appear to have significant advantages over liquids, including lighter weight, lower cost and much less mess due to leaks.

#### CONCLUSIONS

For the case of natural convection in an open cavity when the cavity contains a saturated gas-vapour mixture, equations have been derived which have the same form as those applying to a single fluid. The parameters governing the system resemble those applying to the single fluid, but the Grashof and Prandtl numbers for the saturated mixture include significant modifications to the buoyancy and specific heat terms.

These modifications can lead to significant increases or decreases in the buoyancy forces and in the heat transfer rates. Since data obtained with packed beds filled with a saturated or near saturated gas-vapour mixture support these conclusions, the main question remaining is whether a saturated or near saturated mixture can be achieved in an open cavity.

Arguments in this paper and preliminary observations of an ethanol-air mixture support the view that this will occur if there is a supply of liquid available to form vapour at the highest temperature surface, and if the Lewis number is near 1. For those mixtures where the Lewis number departs significantly from 1, unsaturated conditions may occur in some of the boundary layers. Which boundary layers this applies to depends on whether the Lewis number is greater than or less than 1.

This theoretical study requires experimental confirmation. Steps to obtain this are proceeding, but it should be noted that a similar approach has been verified for packed beds in ref. [2].

The stimulus for this work has been the need to dissipate heat from electronic components to the walls of a sealed box. While containing some assumptions, the analysis is useful in preliminary system design, suggesting desirable gas and vapour properties, and assisting in planning the experimental program now being followed.

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#### CONVECTION NATURELLE DANS DES CAVITES EMPLIES D'UNE VAPEUR ET D'UN GAZ INCONDENSABLE

Résumé—On obtient une relation de similitude pour des enceintes emplies d'un mélange gaz-vapeur saturante. L'intérêt vient du besoin de refroidir un équipement électronique dans des enceintes soudées. Un argument est présenté qui montre que de la vapeur soit dégagée à des températures de surface plus élevées. Ensuite on exploite les équations de bilan de masse, d'énergie et de quantité de mouvement, pour obtenir une forme semblable à celles d'un fluide pur, et on discute les approximations faites. On donne enfin un exemple qui montre comment le transfert net de chaleur dans une cavité est augmentée en utilisant ce procédé.

# NATÜRLICHE KONVEKTION IN EINEM MIT DAMPF UND NICHTKONDENSIERBAREM GAS GEFÜLLTEN HOHLRAUM

Zusammenfassung—In dieser Arbeit wird eine Ähnlichkeitsbeziehung für Hohlräume abgeleitet, die mit einem Gas-Dampfgemisch beim Sättigungszustand gefüllt sind. Diese Anordnung ist interessant für die Kühlung von elektronischen Bauteilen, die hermetisch abgeschlossen sind. Zuerst wird gezeigt, daß das Gemisch sich im Sättigungszustand befindet oder zumindest sehr nahe dabei, wenn der Dampf an der Fläche mit der höchsten Temperatur erzeugt wird. Dann werden die Gleichungen, die aus Masse-, Energieund Impulserhaltungssatz abgeleitet werden, in eine Form umgewandelt, wie sie für ein reines Fluid gilt. Die dazu notwendigen Annahmen werden diskutiert. Zum Schluß wird anhand eines Beispiels gezeigt, wie sich die Wärmeübertragungsleistung in einem Hohlraum mit Hilfe dieses Prozesses verbessern läßt.

#### ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ В ПОЛОСТЯХ, ЗАПОЛНЕННЫХ ПАРОМ И НЕКОНДЕНСИРУЮЩИМСЯ ГАЗОМ

Аннотация — Выводится уравнение подобия для полостей, заполненных насыщенной смесью газ пар. Интерес к данному процессу обусловлен необходимостью охлаждения электронного оборудования, находящегося в герметичных полостях. Сначала приводится доказательство того, что смесь будет находиться в состоянии насыщения или близком к нему, когда образование пара происходит при самой высокой температуре поверхности. Затем уравнения, выведенные из законов сохранения массы, энергии и импульса, приводятся к форме, сходной с уравнением для текучей среды, состоящей из элементарных компонент, и обсуждаются необходимые приближения. Наконец, дан пример, показывающий, как увеличить интенсивность суммарного теплопереноса в полости.