Wake symmetry, subharmonic and quasi-periodic instability modes

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The long-wavelength mode A and the shorter wavelength mode B are the best-known secondary wake instability modes. These instabilities each grow in synchronicity with vortex shedding which is the primary instability. However, it is also known that other secondary instability modes can arise that do not share the underlying period of the vortex street wake; these are either subharmonic modes (double the underlying period, with negative real Floquet multipliers) or quasi-periodic (arbitrary secondary period, complex-conjugate-pair Floquet multipliers). For circular cylinder wake, the quasi-periodic mode is the third to bifurcate from the two-dimensional basic state as Reynolds number is increased (Re_c=377), and the same is true for the square cylinder wake (Re_c=200) [1,2]. On the other hand, for closely related systems (the wake of a moderate aspect ratio ring with circular cross section, the wake of a square cylinder that has been rotated slightly about its axis of symmetry) the equivalent secondary instability is subharmonic [4,5,6].

The key to understanding which type of non-synchronous mode could arise in a new case lies in the symmetry properties of the two-dimensional vortex street. The two-dimensional wake of a circular cylinder, or a square cylinder, or any body with reflection symmetry such as a symmetric airfoil generating no net lift, has the property that it remains invariant under evolution by half a shedding period, i.e. $T/2$, and a spatial reflection (see Figure 1). This spatio-temporal symmetry is a generalized reflection, and temporal evolution over a full period is equivalent to two sequential applications (i.e. a squaring) of this operation. This has the consequence that period-doubling or subharmonic instabilities cannot generically occur for this kind of wake, as originally pointed out from theoretical considerations by Swift and Wiesenfeld [7]. Instead, any non-synchronous Floquet instability must be quasi-periodic, and the associated theory for these kinds of wakes has been fully developed [3].

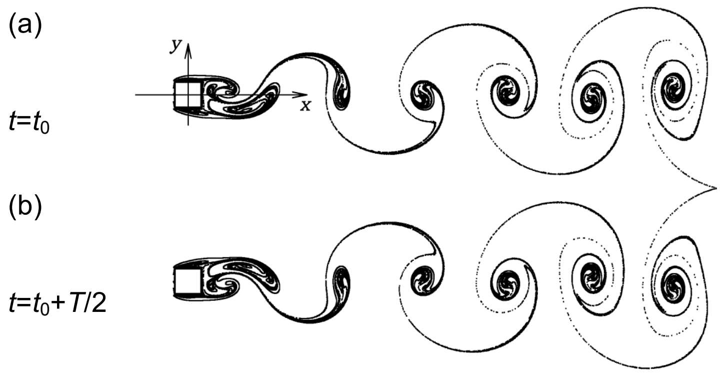

Figure 1 Two-dimensional square cylinder wake at two times, half a shedding period apart. Note that the wake is symmetrical under the operation of a $T/2$ temporal evolution and a reflection about the x axis.

Now if the geometric symmetry of the system is broken, e.g. by a small rotation of the body, then this spatio-temporal symmetry is destroyed (see Figure 2), although superficially the wake may look the same. Other means of geometric symmetry breaking (e.g. by distorting an infinite circular cylinder into a ring) has a similar effect. The result is that period-doubling

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Floquet instabilities are no longer suppressed, and indeed they are observed when a moderate amount of distortion is introduced [5,6].

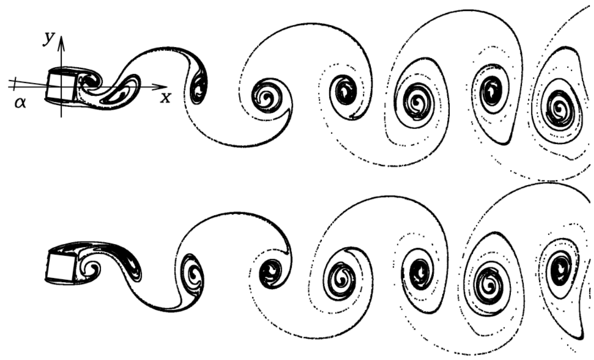

Figure 2 Two-dimensional square cylinder wake at two times, half a shedding period apart. Here the square has been rotated a small amount (α=7.5°) about its axis of symmetry, and the wake loses its symmetry characteristics.

The subharmonic and quasi-periodic modes are definitely related when the amount of symmetry breaking is small, as they are both non-synchronous modes. The question we examine here is: how do the quasi-periodic modes for the symmetric system evolve into the sub-harmonic modes for the system with broken symmetry? We address this by examining the evolution of Floquet multipliers for the square cylinder as it is rotated slightly away from symmetric alignment to the freestream flow (as in the change between Figure 1 and Figure 2) and also for the circular cylinder as it is distorted into a ring. For these two cases we find similar behaviour: as distortion is introduced, the complex-conjugate-pair of Floquet multipliers associated with the quasi-periodic modes evolve continuously towards the negative real axis, coalesce, then split into a pair of subharmonic modes, one which evolves towards the origin in the complex plane (i.e. becomes more stable), the other which moves outwards along the negative real axis (becomes less stable). This behaviour is illustrated for the square cylinder at *Re*=225 in Figure 3: a finite rotation of α=5.85° is required to change the quasi-periodic multiplier pair to a pair of subharmonic multipliers. The initial effect of distortion in this case is to stabilize the system, but ultimately it is destabilizing: at α=8° one Floquet multiplier is of larger modulus than that which obtains at α=0.

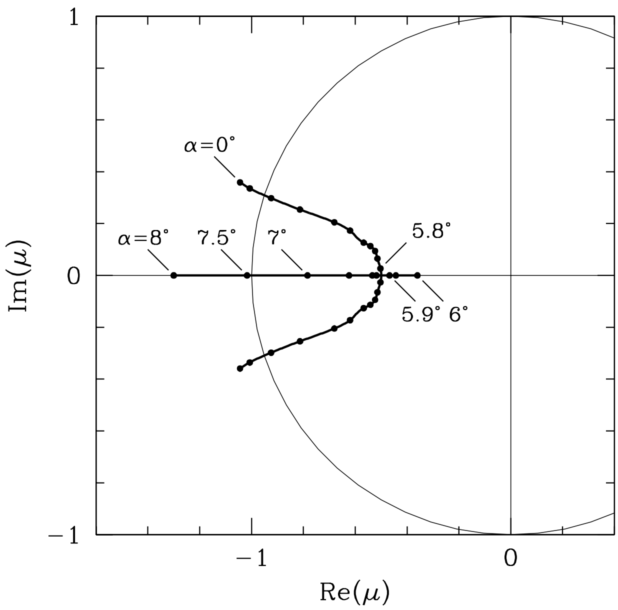

Figure 3 Floquet multiplier locus for the rotated square cylinder wake at *Re*=225.

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