A numerical survey of wake modes and energy transfers for an oscillating cylinder at Re=200

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Abstract

A lack of information regarding the connections between energy transfer and wake mode of oscillating bodies in low Reynolds number flow prompted a two-dimensional numerical investigation at Re = 200. The region in which vortex shedding synchronized with the cylinder oscillation frequency was defined and the energy transfers calculated. The mode of Kármán vortex shedding, in which two vortices of opposite sign were shed each cycle (2S), displayed a gradual change from positive to negative energy transfer with increasing amplitude of motion. This change in energy transfer was directly related to the relative velocity of the fluid, which altered the position of the stagnation point and upper and lower shear layers on the body. As amplitude was increased the 2S shedding mode evolved into an asymmetric mode of shedding in which a pair and a single vortex (P+S) were shed each motion cycle. Energy transfers were calculated in the region of primary lock-in. The P+S mode occurred only in the region of negative energy transfer.

Introduction

A numerical investigation was conducted into the wake states and energy transfers experienced by a cylinder, undergoing forced oscillations transverse to the flow at Re=200. By investigating the parameter space defined by reduced velocity, V_r, and amplitude of the motion, A, the resulting lift trace was recorded and flow visualisations obtained. Changes in the form or phase of the lift trace, relative to the cylinder displacement result in a change in the direction or magnitude of the energy transfer of the system (Carberry *et al.*, 2001; Blackburn and Henderson, 1999). The forced oscillations experienced by the cylinder supress the three-dimensionality of the flow and oscillations near the natural shedding frequency, f_n, may extend the laminar flow range from Re = 150 up to approximately 350 (Koopmann, 1967; Griffin, 1971). Subsequently, all simulations were restricted to two-dimensions. Particular attention was given to the region in which the vortex shedding frequency synchronised with the imposed cylinder motion.

Carberry *et al.* (2001) found experimentally that changes in the lift force, and hence the energy transfer, of a cylinder undergoing forced oscillation were intrinsically linked to the characteristics of the near wake. When traversing the parameter space defined by frequency and amplitude of the body motion, a discontinuity in the phase between the lift force and cylinder displacement was observed. This discontinuity was noted by Bishop and Hassan (1964) and has since been observed at frequencies both above and below the natural shedding frequency at a range of Reynolds numbers (Lu and Dalton, 1996; Hover *et al.*, 1998). Difficulties in detecting the phase jump at low Reynolds number led Blackburn and Henderson (1999) to conclude that for flows with Re < 400, there exists a viscous dissipation inhibiting the switch and only for Reynolds number greater than this can the phenomenon be accurately observed.

Currently, a lack of information exists concerning the link between energy transfer and wake modes throughout the wider parameter space at lower Reynolds numbers. The current investigation attempts to fill this gap. Additionally, the calculation of energy transfer throughout the parameter space may be useful in predicting flow regimes likely to result in vortex-induced vibration (Hover *et al.*,1998).

Methodology

The two-dimensional numerical code utilized a spectral-element scheme with three-step time-splitting to solve the incompressible Navier-Stokes equations with an additional forcing term. Eighth-order Lagrangian interpolating polynomials were used to approximate the solution variables within the macro-elements of the mesh. Both the freestream flow speed, U, and diameter, D, were normalised. For a more detailed description of the numerical scheme, see Thompson *et al.* (1996). The mesh (shown in figure 1) consisted of quadrilateral elements with the domain extending 23D downstream and 15D upstream and to each of the transverse boundaries. The mesh was attached to the non-inertial frame of reference of the cylinder and the free-stream flow and cylinder motion were started impulsively from rest. A no-slip condition was enforced at the cylinder body.

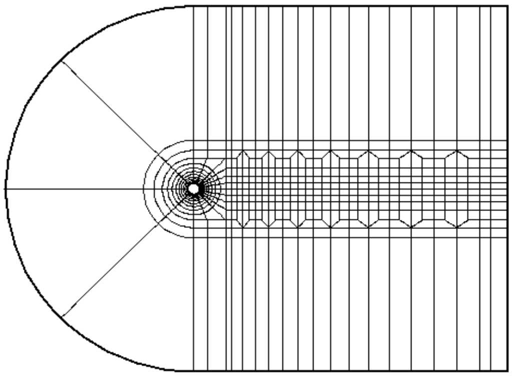

Figure 1. Macro-element mesh used during investigation.

The cylinder motion was described by the relationship for displacement,

$$y(t) = A\cos(2\pi f_o t)\,, \qquad (1)$$

where A is the amplitude of displacement, f_o is the forcing frequency and t is time. Results are described in terms of the scaled amplitude, A/D, and the reduced velocity

$$V_r = \frac{U}{f_o D}\,, \qquad (2)$$

which is equivalent to the wavelength of the cylinder motion scaled by D. Throughout the investigation, the scaled amplitude was varied between 0.1 and 1.0 and the reduced velocity between 1.0 and 8.0. The effect of the drag force was neglected throughout this study.

Detailed resolution and convergence tests have previously been conducted for this system at a variety of Reynolds numbers and were found to give results that compared well with accepted values (Thompson *et al.*, 1996). A time step of $\Delta\tau = 0.001$ was used throughout the investigation, where $\tau = tU/D$ is the normalised time. A fixed cylinder simulation was run at Re = 200 and the Strouhal number

$$S_t = \frac{f_n D}{U}. \qquad (3)$$

was calculated to be St = 0.198. Data analysis was carried out using the lift coefficient, C_L, and non-dimensional energy transfer per motion cycle,

$$E = -\frac{1}{D}\int_0^T A\sin(2\pi f_o t)C_L(t)dt\,, \qquad (4)$$

as defined by Blackburn and Henderson (1999). E indicates the component of C_L in phase with the cylinder velocity over one period of motion T and positive E indicates work done by the fluid on the cylinder. The energy integral given by equation (4) was evaluated using a composite Simpson's Rule.

Results

For certain forcing frequencies near f_n, synchronization was difficult to obtain. A small variation in the frequency altered the lift data from a steady synchronized form to a slowly varying series with a small constant change in E each cycle. This behaviour continued for $\tau > 500$ with no sign of stabilising. Such sensitivity of cylinder wakes near the Strouhal frequency has been observed in previous numerical investigations (Blackburn and Henderson, 1999; Meneghini and Bearman, 1995).

At reduced velocities below V_r=4.5, wake visualisations showed an apparently stable Kármán wake; however a prominent beating was present between the two constituent frequencies, f_o and f_n. Although the forcing frequency was dominant, the interaction of these two frequencies led to a periodic beating in the energy transfer per motion cycle and E consequently varied about a mean value. Areas in which this non-synchronised nature was particularly apparent were at $3f_n$ and $3/2f_n$. Near these frequencies, the energy transfer oscillated about a mean value that was becoming increasingly negative. Figure 2a) shows a wake in this region for which the interaction between f_n and f_o was clearly apparent. The low frequency modulation produced by f_n caused the timing of vortex shedding to vary slightly each period resulting in the wake shown.

As V_r increased to values near 4.5, the vortex shedding locked on. This occurred first at amplitudes above A/D=0.5 where the effects of cylinder motion were more pronounced. As f_o approached f_n synchronisation occurred at all values of A/D investigated. This lock-on was observed throughout the parameter space until V_r > 6.5, at which point the wake pattern became disordered. This synchronisation boundary coincided closely with that established by Williamson and Roshko (1988), defining the limits of the fundamental synchronization region.

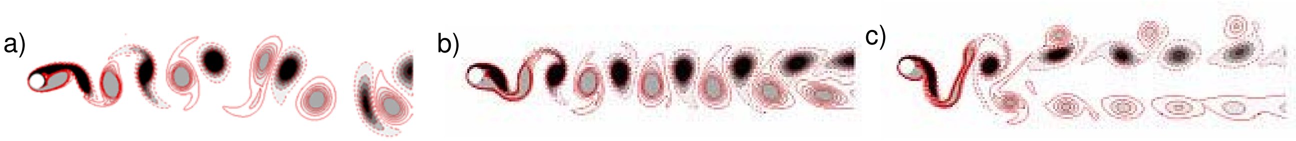

Figure 2. Vorticity contours for a) the unstable wake at V_r=3, A/D=0.179 showing low frequency modulation, b) synchronised 2S or Kármán street wake at V_r=5, A/D=0.199.and c) the synchronised, asymmetric P+S shedding mode at V_r=5, A/D=0.696

As the amplitude of oscillation was increased from a value of A/D=0.2 in the synchronization region, the lift trace varied continuously from a sinusoidal trace, out of phase with the cylinder displacement, to an asymmetric mode with non-zero mean. This became apparent at values of A/D > 0.6. This represented the progression from the standard 2S or Kármán street wake, as shown in figure 2b), to the P+S mode of shedding in figure 2c). A a gradual change from positive to negative energy transfer was observed with increasing amplitude. P+S shedding was observed only in the region of negative energy. Figure 2c) shows the asymmetric pairing of a positive and negative vortex along the top of the wake, with the outermost vortex being convected downstream more rapidly than its negative signed partner.

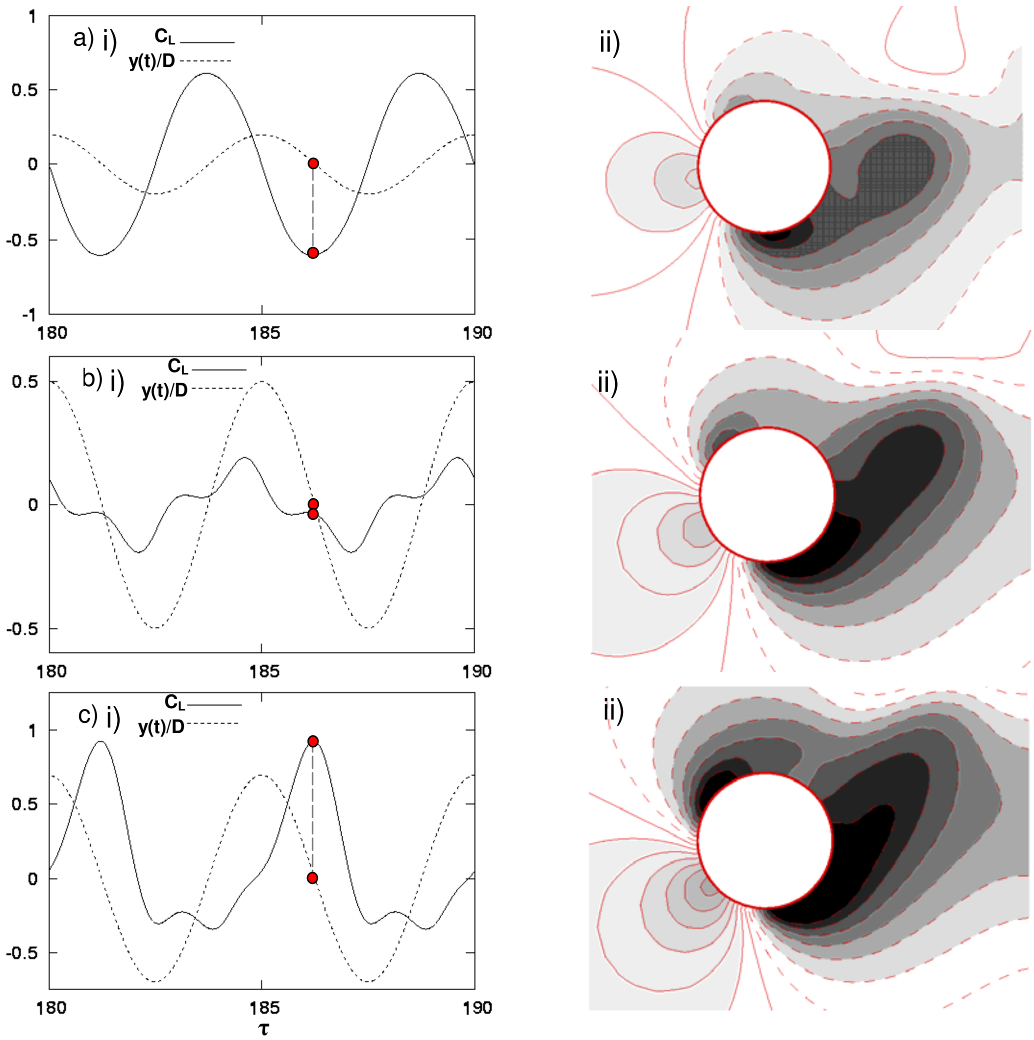

Figure 3. Lift coefficients and corresponding pressure contours for cylinder oscillating at V_r=5.0, a) A/D=0.199, b) A/D=0.497 and c) A/D=0.696. Red dots indicate point at which images were obtained. Negative pressure is shown in black and enclosed by dashed contours.

The transition from positive to negative E is further illustrated with the lift data and cylinder displacement at V_r = 5.0 for three different amplitudes within the lock-on region (figure 3a), b) and c)). It was apparent that the lift phase experienced a shift between the 2S shedding in figure 3a) and the P+S shedding in figure 3c). During the 2S shedding the lift progressed from a near sinusoidal trace to one that contained two smaller, secondary peaks per oscillation, shown in figure 3b)i). These peaks first appeared at 0.40 < A/D < 0.45D in the primary lock-on region. The flow remained symmetric but a further increase in amplitude caused the major peaks and troughs to reduce until the lift trace was effectively tripled.

Oscillations with A/D > 0.45 caused a growth in alternate peaks, creating a phase shift between the lift and displacement. At this time the wake was still symmetric and shedding in the 2S mode. One such peak transition occurred at the time indicated by dots in figure 3a)i), b)i) and c)i) at which wake images were obtained. This phase shift corresponded to a change in the direction of energy transfer. The switch occurred as the result of a continuous process but took place over a small amplitude range and was completed within a variation of A/D < 0.1. An increase in amplitude following this switch led to the disappearance of one of the remaining secondary peaks (figure 3c)i)) and the development of the asymmetric, P+S mode mentioned previously. The reason for development of this asymmetry was not clearly apparent.

Three dominant pressure regions were observed to affect the lift force on the cylinder. These were the high pressure area at the front stagnation point and low pressure regions in the top and bottom shear layers. As amplitude was increased these regions of positive and negative pressure were displaced further around the perimeter of the cylinder when velocity was at a maximum (shown in figures 3a)ii), b)ii) and c)ii)). In figure 3a)ii) the cylinder's downward motion caused accelerated motion of the lower shear layer which created an area of low pressure near the bottom of the cylinder and a net downward lift force (C_L negative). This was unlike the wake structure at amplitudes near 0.5 (figure 3b)ii)), when the front stagnation point was shifted towards the bottom of the cylinder. This high pressure region partially offset the low pressure in the lower shear layer, resulting in a near zero value for C_L.

At amplitudes in the region of 0.7 (figure 3c)), the P+S mode of shedding occurred and the elongated region of positive vorticity shedding from the bottom of the cylinder separated into two distinct vortices downstream. The elongation of the lower shear structure moved the concentration of low pressure further from the cylinder and the low pressure in the upper shear layer dominated, giving a maximum positive C_L. During the upwards motion of the cylinder in this asymmetric wake mode, negative vorticity from the upper shear layer formed a low pressure region much closer to the back of the cylinder. This contributed a larger component to the upward lift, resulting in the net downward lift being of much lower amplitude than the net upward lift generated in the previous half cycle.

Energy measurements throughout the lock-in region indicated that the energy transition took place at 0.45 < A/D < 0.55. For vortex-induced vibration (VIV) to occur a positive energy transfer is required to account for losses due to structural damping (Carberry *et al.*, 2001; Hover *et al.*, 1998), hence this range represents an approximate upper limit to the oscillation amplitude occurring in VIV.

Conclusions

Following an investigation of two-dimensional flow past an oscillating cylinder at Re = 200, information was obtained for the lift force and wake mode occurring at various frequencies and amplitudes of motion. The region in which vortex shedding synchronized with the cylinder motion was determined and energy transfer calculated for all points within this region. Following analysis of the wake modes, it was discovered that the 2S shedding mode displayed a gradual decrease in energy transfer as the reduced velocity and amplitude increased. This was brought about by the development of two secondary peaks in the lift force. At a certain value of A/D, these peaks switched dominance, affecting a shift in phase between lift and displacement, and a change in direction of energy transfer. This change in energy was closely related to the position of the stagnation region during the motion cycle. A further increase in amplitude, following the energy transition, saw the onset of asymmetric P+S shedding in regions of negative energy transfer at A/D > 0.6. The calculated energy transfers indicated that an energy transition took place over a fairly small range of amplitudes in the primary lock-on region with 0.45 < A/D < 0.55 and 4.5 < V_r < 6.5.

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