A Model for Solar Powered Aircraft Preliminary Design.

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Summary.

Solar powered aircraft are becoming of interest, especially for a long endurance flight (surveillance, telecommunications). The present paper presents a mathematical model for solar powered aircraft, flying in the range 0-30.000m of altitude. The model includes an evaluation of the solar energy obtainable at any latitude and day of the year, the energy necessary for a 24 hour flight (i.e. for a long endurance flight) and allows one to carry out the different preliminary design aircraft. A preliminary analysis of the Helios aircraft, is used to check the reliability of the proposed model. As examples of application, a comparison is shown of three configurations, namely: a flying wing, a conventional aircraft and a multiplane aircraft. Finally, some comments on these configurations are reported.

Introduction.

The use of renewable energy and, in particular, of solar energy, is a challenge for the future. In the aerospace field, the main applications of the solar energy to the aircraft propulsion has been carried out in USA in the recent years. Flight tests of some technological demonstrators allowed the research community to evidence the open problems but, also, to understand the possibilities of applications of solar powered aircraft. A promising application in this respect is the HALE (High Altitude Long Endurance) aircraft. The requirements of HALE aircraft are to fly at high altitudes (20-30 km) for very long missions (some months), with a payload of about 100 Kg. At high altitudes, the solar propulsion is attractive because the solar irradiation is intense, the air pollution of the fuel engines is avoided and the knowledge and experience on the photovoltaic panels, gained during space missions, can be used. The long endurance flight is possible when some part of the total energy stored during the day is made available to fly during the night. The storage of energy is a critical aspect. It can be obtained through the use of conventional batteries or fuel cells; in the case of fuel cells, the energy is used for the water electrolysis, from which oxygen and hydrogen are extracted and, then, stored in tanks to be combined in the fuel cells.

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In this paper, a mathematical model for the preliminary design of a Solar Powered Aircraft is presented. The model is based on the energy balance for a cycle of 24 hours of a given day of the year, in level flight condition (loiter) at any altitude. The present model allows us to design different solar powered configurations. In this paper three of them are studied, namely: a flying wing, a conventional configuration and multiplane configuration. In order to check the reliability of the mathematical model, the Helios aircraft is simulated; the conclusion is that the results obtained are satisfactory, even though more information on the structural weight are needed. The main result obtained in this research is that a solar powered aircraft is optimum, when all the surfaces are both lifting and fully panelled with photovoltaic sheets.

Design methodology.

Let consider a panel oriented in a generic position on the earth (figure 1):

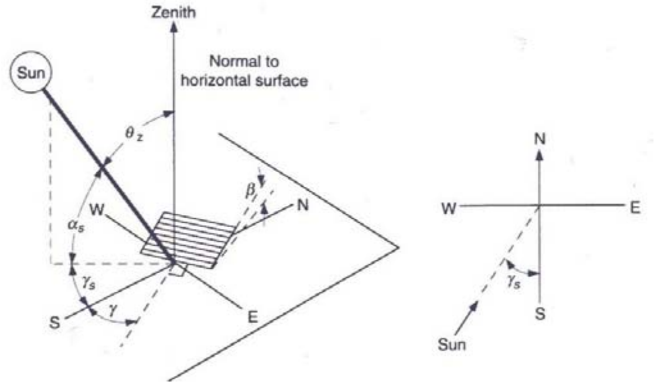

figure 1: Angle definitions (ref. [9])

let define the following angles:

φ: “latitude”, -90°≤φ≤90°, North positive.
δ: “declination”; -23.45°≤δ≤23.45°, North positive
β: “slope”, or angle between the surface in question and the horizon; 0°≤β≤180°.
γ: “surface azimuth angle”, the deviation of the projection on a horizontal plane of the normal to the surface from the local meridian; -180°≤γ≤180°, with zero at south, west positive;
ω: “hour angle”, due to earth rotation: $\omega = \frac{360}{24} t$ [deg h];

θ: "angle of incidence", the angle between the beam and the unit vector normal to the surface;
θ_z: "zenith angle", the angle between the vertical and the beam radiation;
α_s: "solar altitude angle", the angle between the horizon and the solar beam;
γ_s: "solar azimuth angle", the angle between South and the projection of beam radiation on the horizontal plane;
$AM = \frac{1}{\cos\theta_z}$: Air Mass, the ratio of the mass of atmosphere crossed by the beam radiation and the same mass with the sun in the Zenith.

Useful equations relating the angle of incidence, θ, to the other angles are (ref. [9]):

$$\begin{aligned}\cos\theta &= \sin\delta\sin\varphi\cos\beta - \sin\delta\cos\varphi\sin\beta\cos\gamma + \cos\delta\cos\varphi\cos\beta\cos\omega + \\ &\cos\delta\sin\varphi\sin\beta\cos\gamma\cos\omega + \cos\delta\sin\beta\sin\gamma\sin\omega\end{aligned} \tag{1}$$

and

$$\cos\theta = \cos\theta_z\cos\beta + \sin\theta_z\sin\beta\cos(\gamma_s - \gamma) \tag{2}$$

The total solar energy for unit area on panel as in figure 1, can be expressed as:

$$E_{Sun} = \int_T I_{SC}\left(1 + 0.033\cos\frac{360n}{365}\right)\cos\theta_z(t)\bar{\tau}_b^{AM}(t)dt \tag{3}$$

where:

$I_{SC}=1367\ W/m^2$ is the *solar constant*, it is the solar power for unit area just outside the Earth atmosphere;
n is the *n-th* day of the year;
$\bar{\tau}_b$ is the averaged transmittance; i.e. it represents a correction for I_{SC} when a panel is located inside the atmosphere; it depends on the atmosphere composition hence on the altitude.

The solar energy depends on the hour of the day, the day of the year, the latitude, the position of the photovoltaic panels and it changes as the altitude does. This energy (available only in the morning, $T=T_d$) is reduced because of the plant efficiencies (let η_p, η_m, η_{sc}, the propeller, the engines and the solar cells efficiencies, respectively) and must be equal to the sum of the energy necessary for flight and the energy to be stored in batteries to accomplish the nocturnal mission; then the following balances must hold:

$$\begin{cases} \hat{\eta} E_{sun} = E_{flight}\big|_d + E_{StorIN} & \text{diurnal balance} \\ E_{StorOUT} = E_{flight}\big|_n & \text{nocturnal balance} \end{cases} \tag{4}$$

where $\hat{\eta} = \eta_P \eta_m \eta_{SC}$.

The energy necessary for flight, in the minimum power conditions can be expressed as:

$$E_{flight}\big|_{P\min} = \int_T P_{flight}\big|_{\min} dt = \int_T \left[\frac{2^{5/2}}{3^{3/4}} \sqrt{\frac{1}{\rho}} \left(\frac{W}{S} \right)^{3/2} S C_{D0}^{1/4} K^{3/4} + W V_{zP\min} \right] dt \tag{5}$$

Equation (5) is general at all; suitable hypotheses can be introduced in order to obtain some results for a preliminary design:

1. Leveled flight at constant height (loiter motion) at: $\rho = cost$ and $V_{zPmin} = 0$.
2. Constant weight (W=cost) or: the fuel cell system is a closed loop.
3. 24 hours time interval: $T_d = 24 - T_n$.
4. Aerodynamics. Let use the flat plate analogy to predict the parasite drag coefficient C_{D0}: $C_{D0} = C_f \frac{S_b^{TOT}}{S}$, $C_{D0} = C_f \frac{S_b^{TOT}}{S}$, where C_f is the flat plate friction drag coefficient, it is a function of Reynold's number.
5. Aerodynamics coefficients (C_f, e, AR) are constant in the period considered.
6. The percentage of adjunctive paneled surfaces respect to the wing reference area is constant in the daytime and in the nighttime.

From these hypotheses we obtain the maximum allowable wing loading, such that the energetic balance on the 24 hours is satisfied:

$$\left(\frac{W}{S_{wing}} \right) = \left[\int_{T_d} \eta I_n dt \right]^{2/3} \left\{ \frac{2^{5/2}}{3^{3/4}} \sqrt{\frac{1}{\rho}} \left(C_f (\mathrm{Re}) \right)^{1/4} \frac{1}{(\pi e AR)^{3/4}} \frac{24}{(1+\gamma)^{N/2}} \frac{\alpha_1(\gamma)}{\alpha(\gamma)} \right\}^{-2/3} \tag{6}$$

where:

$$\begin{aligned} \alpha(\gamma) &= \sum_{i \neq c} \alpha_i \beta_i \delta_i + \alpha_c \gamma \delta_c \\ \alpha_1(\gamma) &= \sum_{i \neq c} \alpha_i \beta_i + \alpha_c \gamma \end{aligned} \tag{7}$$

and α_i, β_i, are geometric parameters referred to the *i-th* aircraft's component, while γ_i and δ_i are geometric parameters referred to panelled surfaces; a simple aircraft's architecture can be modeled by them; the *c* subscript is referred to the adjunctive panelled surfaces.

The global efficiency $\eta = \hat{\eta}\eta^*$, (with $\eta^* = \left[1 + \frac{T_n}{24}\left(\frac{1}{\eta_{Stor}} - 1\right)\right]^{-1}$, η_{Stor}: storage system efficiency; T_n: nighttime period), is time dependent, and $N=0$ if the adjunctive surfaces are no-lifting, and $N=1$ if the adjunctive surfaces are lifting. The weight in equation (6) is the sum of the weights of the different aircraft's components; since some of these are dependent of the parameter in the right side of equ. (6) it can be solved only iteratively and only if the weight equations are known.

Empty weight prediction model.

For this class of aircraft, we can identify the following main weights:

Structural weight W_{af};
Solar cells weight W_{sc};
Fuel cells/batteries weight W_{fc};
Reactant weight (only for the fuel cells system) W_r;
Storage weight (only for the fuel cells system) W_{ta};
Propeller and electric motors weight W_{prop};
Payload and avionics weight W_{pay}.

The component W_{pay}.is known; the other weight components are unknown and they depend on the solution adopted. At this stage of the project, we can only try to give a statistical evaluation of the main weight components.

Structural weight model.

For this class of aircraft, it is very difficult to define a good structural weight model because there are not available data or only a few number of flying aircrafts have been fabricated. Among these, there are the NASA prototypes Pathfinder, Centurion, Helios.

In this paper, two equations have been adopted; the first one, from reference [1], is obtained by statistical data for sailplanes with two bottom fins, whilst the second one it is obtained by interpolating NASA prototype data.

The first weight model should be preferred if the vehicle is piloted by humans.

For the second model, we get the following equation:

$$W_{af} = 0.693 S^{0.656} AR^{0.2335} \qquad [\text{kg}] \tag{8}$$

This last one should be preferred for UAVs vehicles.

The above models cannot give good weight estimations when applied to a biplane configuration. Preliminary studies indicate that the solution adopted for a biplane is stiffer than the other examined configurations and the weight is assumed as 80% of the flying wing weight.

Photovoltaic panels weight estimation.

In this paper, we suppose to use Si-cells with a weight of 0.3651 kg/m^2 [2] and with an 19% efficiency. Hence the solar cells weight can be expressed as:

$$W_{sc} = 0.3651 S_c \tag{9}$$

Storage system weight estimation.

The fuel cells available to-day have a maximum specific power of 650W/kg; while the hydrogen storage system has a specific energy in the range 1.5 – 2 kWh/kg.

Specific power values for the reactants are the following [1]:
reactants specific power: 2.6 kW/kg
container specific power: 3.4 kW/kg

If we use batteries, they have a specific energy between 35 Wh/kg (PbA) and 100 Wh/kg (Li-Ion).

Electric motors weight estimation.

The data are obtained from the actual brushless electric motors, a linear approximation of the weight function is:

$$W_{prop} = 0.0045 P_{prop} \qquad (\text{W in [kg], P in Watts}) \tag{10}$$

A validation of the present model.

In order to validate the proposed mathematical model for the solar powered aircraft design, the same tool is applied to analyse an existing solution. A certain amount of data are available from the literature about the Helios aircraft, designed and manufactured by NASA-Dryden Flight Research Center in collaboration with AeroVironment Inc. (USA).

A comparison between the results obtained and the parameters relevant to Helios is shown in with the following design data:

$h=30000$ m
$\varphi=14°$ N
date: September the 30th
$W_{pay}=330$ kg.

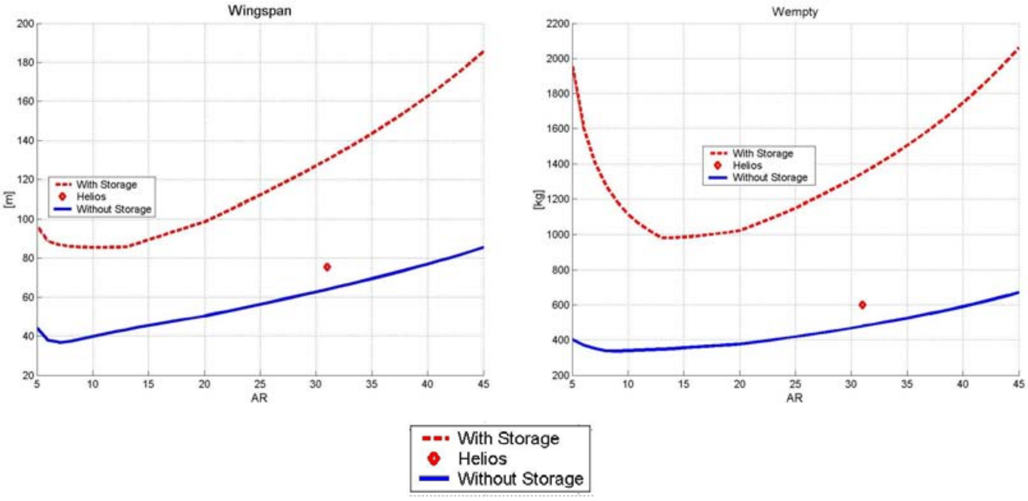

figure 2: Wingspan and empty weight vs. AR plots

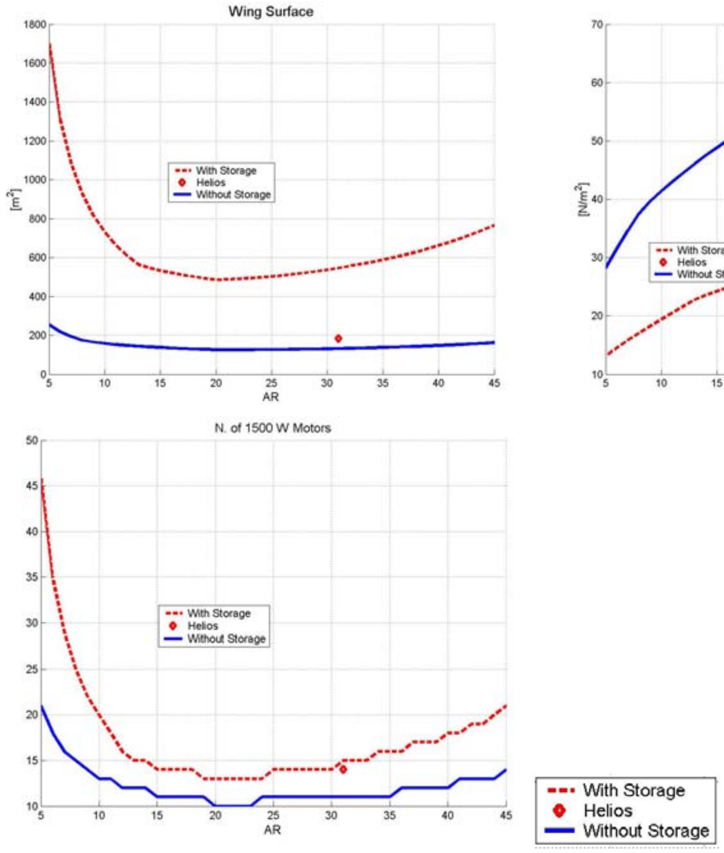

figure 3: Wing surface, wing load and number of electric motors

The model gives reasonable predictions of the parameters examined, compared with the actual data of Helios; the worst prediction is the weight estimation, as could be expected given the lack of information on weight. The predictions depend on the Aspect Ratios, AR; in particular, there exists the range 20-25 of AR, in which the number of the electric motors is minimum (i.e. the energy lost is minimum).

Comparison among candidate configurations for HALE aircraft.

In the next figures, a comparison among the three different architectures is shown.

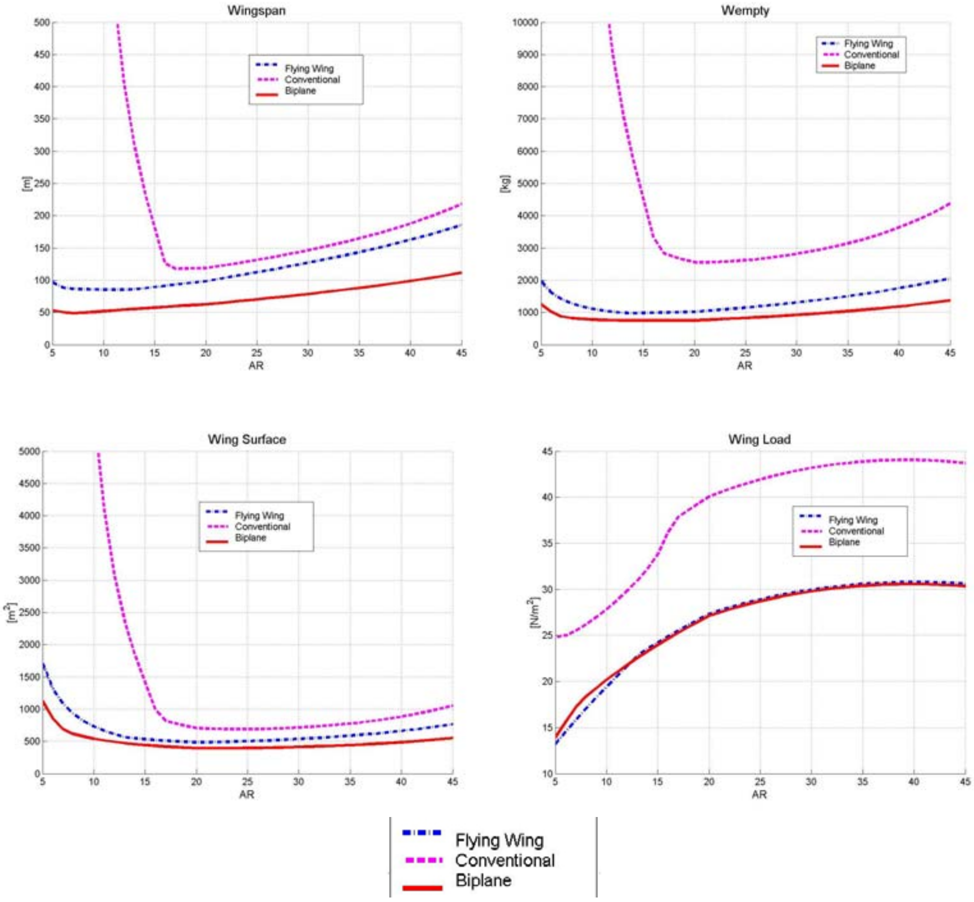

figure 4: Wingspan, empty weight, wing surface and wing load

Even if the fuselage is panelled, the previous plots indicate that the conventional architecture is the most expensive from an energetic point of view in comparison to the other architectures.

The biplane configuration is the less expensive, more it has the shortest wingspan. In an high aspect ratio wing, a short wingspan is desirable for aeroelastic reasons as [3] shows.

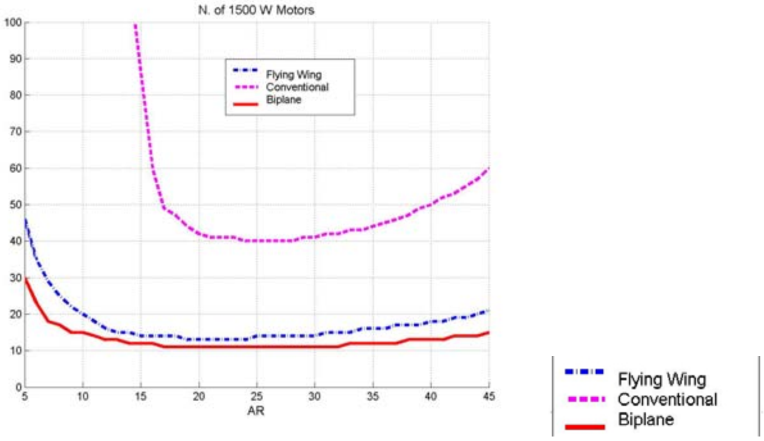

figure 5: Number of electric motors

Conclusions.

The following conclusions can be accomplished:

1. the model gives good results when compared to existing solutions as NASA solar powered aircrafts (Helios), but more investigations must be done especially for the component's weight equations;
2. the power required for flight at high altitudes and for long endurance is about 20-25 kW;
3. when applied at different architectures, the model predicts that a new aircraft configuration must be considered instead of conventional one;
4. a biplane configuration with reinforcement bulkheads has been considered (figure 6). This solution is stiffer than the flying wing and its structural weight can be considered as the 80% of the flying wing weight. This weight reduction, together with an aerodynamic efficiency improvement, they permit to achieve a better solution than the flying wing. Particularly, both the number of electric motors required for flight is less than the flying wing (-21%) and the range where this number is minimum is greater (*AR*=[17-33] instead of [18-24]). More, the wing span is reduced about the 40% compared with the flying wing: this result could avoid the typical aeroelastic problems of high aspect ratio wings;
5. further improvements in the storage system equipment are needed, especially in terms of specific power or specific energy and in terms of plant simplicity.

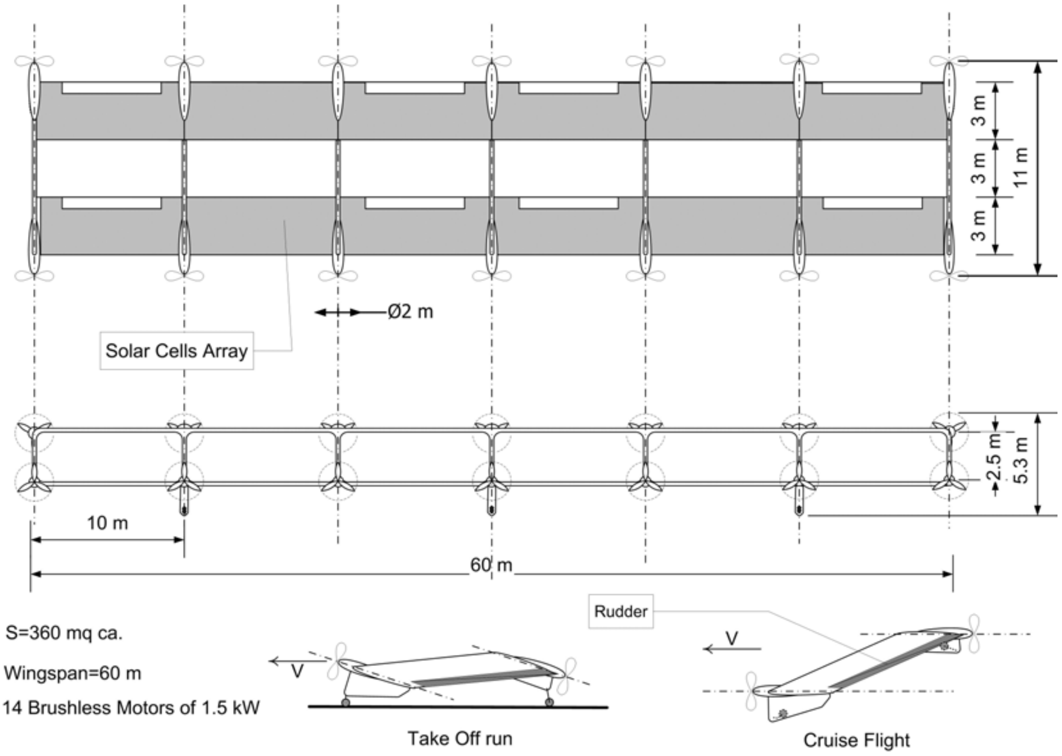

figure 6: Biplane Configuration for a Solar Powered Aircraft

The previous drawing is referred to one of the previous solutions for *AR=20*, flying at *30000 m*, the 30th of September, at *φ=14 °N*, carrying a payload of *330 kg*, with a fuel cells storage system and structural weight about *100 kg*.

Reference.

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