Frictionless unilateral contact by Symmetric Boundary Element Method

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Summary

This paper proposes the analysis of the frictionless unilateral contact between two elastic bodies subjected to known external actions. The passage from the contact to the disjunction condition and vice-versa are analyzed through the introduction of a weighted value of the cohesion and of the distance in accordance with a strategy involving a symmetric boundary element formulation. The applications use the karnak.sGbem program made by some of the present authors.

Introduction

Elastic analysis of unilateral contact problems between elastic bodies and rigid obstacles or between (non-penetrable) elastic bodies has been of great interest in the last few decades. The pioneering work of Signorini and Fichera treated the theoretical aspects of this problem with mathematical rigour, whereas for practical purposes several numerical approaches dealing with the Finite Element Method have proved meaningful whether through iterative trial and error techniques or solving this problem directly as a linear complementary problem or as a mathematical programming problem. Subsequently these contact problems have been dealt with from a numerical point of view via Boundary Integral Equations mainly by the collocation approach through utilising a variation formulation or variation inequalities. Interest in the employment of the boundary element method arises either because the contact surface is on the body boundary, the natural site of the variables governing such a method, or as a consequence of the reduction in the problem dimension which for a plane body changes from two to one. The conventional direct BEM is characterised by lack of symmetry and sign definiteness of the matricial operators and exhibit drawbacks such as the absence of suitable variational principles for a consistent BE discretization and the lack of convergence criteria for step-by-step analysis.

The present paper aims to solve the unilateral contact problem between elastic bodies having different physical and geometrical characteristics by utilising the Symmetric Boundary Element Method (SBEM) through the trial and error iterative technique. The analysis will examine case of simple contact, i.e. slippingless and frictionless. The

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possibility of subdividing the body into boundary elements (B-elements) having physical and geometrical characteristics differing from one another permits one to apply such a method with a very large advantage especially when the contact problem exclusively regards one or more contact zones having limited dimension. Indeed, in this case the algebraic operators, referred to the B-elements not involved in the contact problem, remain the same, and only those operators of the B-elements including the contact zones and the load vector change. The paper will show the theoretical aspects of the problems of simple unilateral contact and will apply the formulation to some meaningful examples by using the Karnak-SBEM programme developed by some of the present authors. This programme, based on the symmetric formulation of the BEM, actually permits one in a substructuring process to evaluate the elastic response (displacements, stresses, tractions) of the structure subjected to external actions like body forces and imposed thermal-like strains, both in the domain, to imposed displacements on the constrained boundary and to forces on the free boundary.

Unilateral contact boundary conditions

Let us assign two elastic solids A and B coming into contact with one another, each having the boundary subdivided into a free portion, the remaining portion connected to another solid. Each body is subjected to forces $\bar{f}_2$ on Γ_2, to assigned displacements $\bar{u}_1$ on Γ_1 and body forces $\bar{b}$ and volumetric distorsions $\bar{\vartheta}$, both in Ω. Between two solids the tangential mutual force may take on very high values without the limit friction force being overcome: in this case it is not possible to have a slip between the mutual surfaces in contact.

Let the system formed by two solids in contact, referred to cartesian axes $\mathbf{x} \equiv (x_1, x_2)$, be subdivided into two bodies A and B both called bem-elements. Let the solids be embedded in their infinite domain, each having the same characteristics as A and B. In Figs. 1 b, c the domains Ω^A and Ω^B are shown and so are the boundaries Γ^A, Γ^B and Γ^{A+}, Γ^{B+}, the latter being the boundaries of the complementary domains $\Omega_\infty^A \setminus \Omega^A$ and $\Omega_\infty^B \setminus \Omega^B$. With reference to the local axes (**n**, **s**), defined on each boundary, the following vectors are considered:

$$\mathbf{u}^I = u_n^I \mathbf{n}^I + u_s^I \mathbf{s}^I \qquad \mathbf{t}^I = t_n^I \mathbf{n}^I + t_s^I \mathbf{s}^I \qquad \text{with } I = A, B \tag{1a, b}$$

where u_n, u_s and t_n ,t_s are the components of the displacements of Γ and of the tractions acting on Γ, in the **n** and **s** directions.

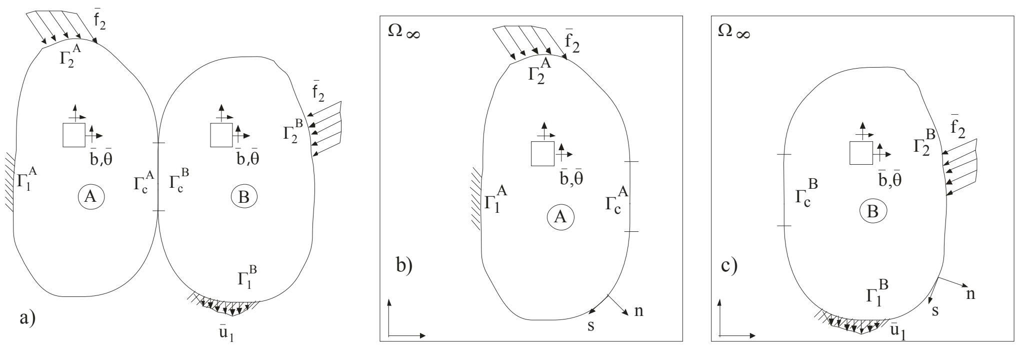


Fig.1: a) Elastic solids A and B in contact; b, c) Domains Ω_A and Ω_B embedded in their infinite domain.

If we denote by h the distance, measured along the normal $\mathbf{n}^A$, between two points on Γ_c^A and Γ_c^B which will touch and by c the value of the possible cohesion existing between the bodies in contact, the (linearized) unilateral contact boundary conditions read

$$\tilde{\mathbf{n}}^A(\boldsymbol{u}^A - \boldsymbol{u}^B) - h \le 0 \qquad \text{detachment conditions} \qquad (2a)$$

$$\tilde{\mathbf{n}}^A\mathbf{t}^A - c \le 0 \qquad \text{contact conditions} \qquad (2b)$$

$$[\tilde{\mathbf{n}}^A(\boldsymbol{u}^A - \boldsymbol{u}^B) - h\,][\tilde{\mathbf{n}}^A\boldsymbol{t}^A - c] = 0 \qquad \text{complementarity conditions} \qquad (2c)$$

where the symbol tilde means the transposition of the vector or matrix. In the latter equations $\mathbf{u}^A$ and $\mathbf{t}^A$ mean the displacements and the tractions on the boundary $\Gamma_2^A \cup \Gamma_c^A$ of Ω^A, and the same applies to $\mathbf{u}^B$ and $\mathbf{t}^B$.

These equations lead to the classic unilateral contact problem between the two bodies and are valid both for all the points on Γ_c with c>0 and h=0, and for those on Γ_2^A and Γ_2^B with c=0 and h>0.

In the case examined we assume

$$\mathbf{n}_s^A\ \mathbf{t}_c^A < A$$

the constant A being a high enough parameter, thus guaranteing the absence of slip between the two bodies.

Elastic model for each bem-element

If we assume a bem-element as a reference, we can find a relation connecting the unknown boundary quantities to the known quantities on the boundary and to the volume actions. This goal is reached when the complementary domain $\Omega_\infty \setminus \Omega$ of each bem-element is unstrained and unstressed. This phenomenon is guaranteed by imposing the Diriclet and Neumann conditions

$$\mathbf{u}_1^+ = \mathbf{0} \quad \text{on } \Gamma_1^+, \qquad \mathbf{t}_2^+ = \mathbf{0} \quad \text{on } \Gamma_2^+ \tag{3a,b}$$

and both the conditions on the contact boundary

$$\mathbf{u}_c^+ = \mathbf{0}, \qquad \mathbf{t}_c^+ = \mathbf{0} \quad \text{on } \Gamma_c^+ \tag{3c,d}$$

When we introduce the Somigliana Identities of the displacements and of the tractions, the following boundary integral equations can be obtained:

$$\mathbf{u}_1^+ \equiv \int_{\Gamma_1} \mathbf{G}_{uu}\mathbf{f}_1 + \int_{\Gamma_2} \mathbf{G}_{ut}(-\mathbf{u}_2) + \int_{\Gamma_c} \mathbf{G}_{uu}\mathbf{f}_c + \int_{\Gamma_c} \mathbf{G}_{ut}(-\mathbf{u}_c) + \int_{\Omega} \mathbf{G}_{uu}\overline{\mathbf{b}} + \int_{\Omega} \mathbf{G}_{u\sigma}\overline{\vartheta} = \mathbf{0} \tag{4a}$$

$$\mathbf{t}_2^+ \equiv \int_{\Gamma_1} \mathbf{G}_{tu}\mathbf{f}_1 + \int_{\Gamma_2} \mathbf{G}_{tt}(-\mathbf{u}_2) + \int_{\Gamma_c} \mathbf{G}_{tu}\mathbf{f}_c + \int_{\Gamma_c} \mathbf{G}_{tt}(-\mathbf{u}_c) + \int_{\Omega} \mathbf{G}_{tu}\overline{\mathbf{b}} + \int_{\Omega} \mathbf{G}_{t\sigma}\overline{\vartheta} = \mathbf{0} \tag{4b}$$

$$\begin{aligned}\mathbf{u}_c^+ \equiv & \int_{\Gamma_1} \mathbf{G}_{uu}\mathbf{f}_1 + \int_{\Gamma_2} \mathbf{G}_{ut}(-\mathbf{u}_2) + \int_{\Gamma_c} \mathbf{G}_{uu}\mathbf{f}_c + \oint_{\Gamma_c} \mathbf{G}_{ut}(-\mathbf{u}_c) - \frac{1}{2}\mathbf{u}_c \\ & + \int_{\Omega} \mathbf{G}_{uu}\overline{\mathbf{b}} + \int_{\Omega} \mathbf{G}_{u\sigma}\overline{\vartheta} = \mathbf{0}\end{aligned} \tag{4c}$$

$$\begin{aligned}\mathbf{t}_c^+ \equiv & \int_{\Gamma_1} \mathbf{G}_{tu}\mathbf{f}_1 + \int_{\Gamma_2} \mathbf{G}_{tt}(-\mathbf{u}_2) + \oint_{\Gamma_c} \mathbf{G}_{tu}\mathbf{f}_c - \frac{1}{2}\mathbf{f}_c + \int_{\Gamma_c} \mathbf{G}_{tt}(-\mathbf{u}_c) \\ & + \int_{\Omega} \mathbf{G}_{tu}\overline{\mathbf{b}} + \int_{\Omega} \mathbf{G}_{t\sigma}\overline{\vartheta} = \mathbf{0}\end{aligned} \tag{4d}$$

where the small circle appearing in the integrals defines these integrals as the Cauchy Principal Value (CPV), whereas the terms ½ $\mathbf{u}_c$ and ½ $\mathbf{f}_c$ are the free terms.

If in the latter two equations the terms $\mathbf{u}_c$ and $\mathbf{f}_c$ are added on both sides of the equations, one obtains:

$$\begin{aligned}\mathbf{u}_c &= \int_{\Gamma_1}\mathbf{G}_{uu}\mathbf{f}_1 + \int_{\Gamma_2}\mathbf{G}_{ut}(-\mathbf{u}_2) + \int_{\Gamma_c}\mathbf{G}_{uu}\mathbf{f}_c + \oint_{\Gamma_c}\mathbf{G}_{ut}(-\mathbf{u}_c) + \frac{1}{2}\mathbf{u}_c \\ &\quad + \int_{\Omega}\mathbf{G}_{uu}\overline{\mathbf{b}} + \int_{\Omega}\mathbf{G}_{u\sigma}\overline{\vartheta}\end{aligned} \tag{5a}$$

$$\mathbf{f}_c = \int_{\Gamma_1}\mathbf{G}_{tu}\mathbf{f}_1 + \int_{\Gamma_2}\mathbf{G}_{tt}(-\mathbf{u}_2) + \oint_{\Gamma_c}\mathbf{G}_{tu}\mathbf{f}_c + \frac{1}{2}\mathbf{f}_c + \int_{\Gamma_c}\mathbf{G}_{tt}(-\mathbf{u}_c) + \int_{\Omega}\mathbf{G}_{tu}\overline{\mathbf{b}} + \int_{\Omega}\mathbf{G}_{t\sigma}\overline{\vartheta} \tag{5b}$$

where we note that the equations are not identically null as in eqs. (4c,d), but equal to $\mathbf{u}_c$ and $\mathbf{f}_c$ and where the sign of the free terms changes.

The same eqs. (4a,b) and (5a,b) can be written symbolically in the following way:

$$\mathbf{u}_1^+ \equiv \mathbf{u}_1^+[\mathbf{f}_1, -\mathbf{u}_2, \mathbf{f}_c, -\mathbf{u}_c] + \hat{\mathbf{u}}_1^+[\overline{\mathbf{b}}, \overline{\vartheta}] = \mathbf{0} \tag{6a}$$

$$\mathbf{t}_2^+ \equiv \mathbf{t}_2^+[\mathbf{f}_1, -\mathbf{u}_2, \mathbf{f}_c, -\mathbf{u}_c] + \hat{\mathbf{t}}_2^+[\overline{\mathbf{b}}, \overline{\vartheta}] = \mathbf{0} \tag{6b}$$

$$\mathbf{u}_c = \mathbf{u}_c[\mathbf{f}_1, -\mathbf{u}_2, \mathbf{f}_c, -\mathbf{u}_c] + \hat{\mathbf{u}}_c[\overline{\mathbf{b}}, \overline{\vartheta}] \tag{6c}$$

$$\mathbf{f}_c = \mathbf{f}_c[\mathbf{f}_1, -\mathbf{u}_2, \mathbf{f}_c, -\mathbf{u}_c] + \hat{\mathbf{f}}_c[\overline{\mathbf{b}}, \overline{\vartheta}] \tag{6d}$$

Let us introduce the boundary discretization into the boundary elements by making the following modelling:

$$\mathbf{f}_1 = \Psi_f\mathbf{F}_1, \qquad \mathbf{u}_2 = \Psi_u\mathbf{U}_2, \qquad \mathbf{f}_c = \Psi_f\mathbf{F}_c, \qquad \mathbf{u}_c = \Psi_u\mathbf{U}_c \tag{7a,d}$$

and let us perform the weighting of the quantities defining the boundaries through the same shape functions as those of the response modelling, but employed in an energetically dual way, so obtaining the following block system

$$\begin{Bmatrix} \mathbf{W}_1^+ = \mathbf{0} \\ \mathbf{P}_2^+ = \mathbf{0} \\ \hdashline \mathbf{W}_c \\ \mathbf{P}_c \end{Bmatrix} = \left[\begin{array}{cc:cc} \mathbf{A}_{u1,u1} & \mathbf{A}_{u1,f2} & \mathbf{A}_{u1,uc} & \mathbf{A}_{u1,fc} \\ \mathbf{A}_{f2,u1} & \mathbf{A}_{f2,f2} & \mathbf{A}_{f2,uc} & \mathbf{A}_{f2,fc} \\ \hdashline \mathbf{A}_{uc,u1} & \mathbf{A}_{uc,f2} & \mathbf{A}_{uc,uc} & \overline{\mathbf{A}}_{uc,fc} \\ \mathbf{A}_{fc,u1} & \mathbf{A}_{fc,f2} & \overline{\mathbf{A}}_{fc,uc} & \mathbf{A}_{fc,fc} \end{array}\right] \begin{Bmatrix} \mathbf{F}_1 \\ -\mathbf{U}_2 \\ \hdashline \mathbf{F}_c \\ -\mathbf{U}_c \end{Bmatrix} + \begin{Bmatrix} \hat{\mathbf{W}}_1^+ \\ \hat{\mathbf{P}}_2^+ \\ \hdashline \hat{\mathbf{W}}_c \\ \hat{\mathbf{P}}_c \end{Bmatrix} \tag{8}$$

where the terms $\overline{\mathbf{A}}_{uc,fc}$ and $\overline{\mathbf{A}}_{fc,uc}$ include the CPV integrals and the corresponding free terms.

Eq. (8) can be expressed in extended form in the following way:

$$\mathbf{0} = \mathbf{A}\mathbf{X} + \mathbf{A}_c\mathbf{X}_c + \hat{\mathbf{U}} \tag{9a}$$

$$\mathbf{Z}_c = \mathbf{A}_c^{T}\mathbf{X} + \mathbf{A}_{cc}\mathbf{X}_c + \hat{\mathbf{U}}_c \tag{9b}$$

where the vectors $\mathbf{X}$ and $\mathbf{X}_c$ collect the boundary quantities ($\mathbf{F}_1$ and $\mathbf{U}_2$) and the contact zone quantities ($\mathbf{F}_c$ and $\mathbf{U}_c$) respectively, whereas $\mathbf{Z}_c$ collects the weighted quantities (displacement $\mathbf{W}_c$ and traction $\mathbf{P}_c$) on the same contact zone.

By performing a variable condensation through the replacement of the $\mathbf{X}$ vector extracted from eq. (9a) into eq. (9b), we obtain the following equation:

$$\mathbf{Z}_c = (\mathbf{A}_{cc} - \mathbf{A}_c^{T}\mathbf{A}^{-1}\mathbf{A}_c)\mathbf{X}_c + (\hat{\mathbf{U}}_c - \mathbf{A}_c^{T}\mathbf{A}^{-1}\hat{\mathbf{U}}_c) \tag{10}$$

which can be expressed in extended form as follows:

$$\mathbf{W}_c = \mathbf{D}_{uc,uc}\mathbf{F}_c + \mathbf{D}_{uc,fc}(-\mathbf{U}_c) + \hat{\mathbf{W}}_c \tag{11a}$$

$$\mathbf{P}_c = \mathbf{D}_{fc,uc}\mathbf{F}_c + \mathbf{D}_{fc,fc}(-\mathbf{U}_c) + \hat{\mathbf{P}}_c \tag{11b}$$

with obvious symbol meaning.

The latter equations relate the quantities defined on the single boundary Γ_c (displacements and tractions, both weighted) to the forces acting on the nodes and to the displacements of the same nodes and to the load terms, also.

Elastic analysis

For every bem-element, equations like eqs.(11) can be written, i.e.

$$\begin{cases} \mathbf{W}_c^A = \mathbf{D}_{uc,uc}^A \, \mathbf{F}_c^A + \mathbf{D}_{uc,fc}^A \, (-\mathbf{U}_c^A) + \hat{\mathbf{W}}_c^A \\ \mathbf{P}_c^A = \mathbf{D}_{fc,uc}^A \, \mathbf{F}_c^A + \mathbf{D}_{fc,fc}^A \, (-\mathbf{U}_c^A) + \hat{\mathbf{P}}_c^A \end{cases} \quad \text{for body A} \tag{12a,b}$$

$$\begin{cases} \mathbf{W}_c^B = \mathbf{D}_{uc,uc}^B \, \mathbf{F}_c^B + \mathbf{D}_{uc,fc}^B \, (-\mathbf{U}_c^B) + \hat{\mathbf{W}}_c^B \\ \mathbf{P}_c^B = \mathbf{D}_{fc,uc}^B \, \mathbf{F}_c^B + \mathbf{D}_{fc,fc}^B \, (-\mathbf{U}_c^B) + \hat{\mathbf{P}}_c^B \end{cases} \quad \text{for body B} \tag{13a,b}$$

By imposing the regularity conditions in the nodes of the contact zone, i.e.

$$\mathbf{U}_c = \mathbf{U}_c^A = \mathbf{U}_c^B \qquad \mathbf{F}_c = \mathbf{F}_c^A = -\mathbf{F}_c^B \tag{14a,b}$$

and the weighted regularity conditions along contact sides, i.e.

$$\mathbf{W}_c^A = \mathbf{W}_c^B \qquad \mathbf{P}_c^A = -\mathbf{P}_c^B \tag{15a,b}$$

we reach the elastic solution $\mathbf{X}_c$ trough the following solving equation

$$\mathbf{K}\,\mathbf{X}_c + \hat{\mathbf{L}} = \mathbf{0} \tag{16}$$

where one sets:

$$\mathbf{K} = \left|\begin{array}{c:c} \mathbf{D}_{uc,uc}^A + \mathbf{D}_{uc,uc}^B & \mathbf{D}_{uc,fc}^A - \mathbf{D}_{uc,fc}^B \\ \hdashline \mathbf{D}_{fc,uc}^A - \mathbf{D}_{fc,uc}^B & \mathbf{D}_{fc,fc}^A + \mathbf{D}_{fc,fc}^B \end{array}\right| \qquad \mathbf{X}_c = \left|\begin{array}{c} \mathbf{F}_c \\ \hdashline -\mathbf{U}_c \end{array}\right| \qquad \hat{\mathbf{L}} = \left|\begin{array}{c} \hat{\mathbf{W}}_c^A - \hat{\mathbf{W}}_c^B \\ \hdashline \hat{\mathbf{P}}_c^A + \hat{\mathbf{P}}_c^B \end{array}\right| \tag{17a-c}$$

The remaining boundary quantities of bodies A and B, containing the reactive forces of the constrained nodes and the displacements of the free nodes, i.e. $\mathbf{X}^A$ and $\mathbf{X}^B$, are obtainable by using equations like eqs.(9a), written for each of the two bodies.

Unilateral contact analysis

Once the solution vector is computed for each body in terms of nodal quantities on all the boundary, then the displacement and traction distributions along the boundary sides concerning the unilateral contact phenomenon are easily obtainable through the use of the S.I.:

$$\mathbf{u}=\int_{\Gamma_1}\mathbf{G}_{\mathrm{uu}}\Psi_{\mathrm{f}}\mathbf{F}_1+\int_{\Gamma_2}\mathbf{G}_{\mathrm{uf}}\Psi_{\mathrm{u}}(-\mathbf{U}_2)+\int_{\Gamma_1}\mathbf{G}_{\mathrm{uu}}\Psi_{\mathrm{f}}\mathbf{F}_{\mathrm{c}}+\int_{\Gamma_{\mathrm{c}}}\mathbf{G}_{\mathrm{uf}}\Psi_{\mathrm{u}}(-\mathbf{U}_{\mathrm{c}})+\hat{\mathbf{u}}\quad \text{on } \Gamma_{\mathrm{c}}\cup\Gamma_2 \tag{18a}$$

$$\mathbf{t}=\int_{\Gamma_1}\mathbf{G}_{\mathrm{fu}}\Psi_{\mathrm{f}}\mathbf{F}_1+\int_{\Gamma_2}\mathbf{G}_{\mathrm{ff}}\Psi_{\mathrm{u}}(-\mathbf{U}_2)+\int_{\Gamma_1}\mathbf{G}_{\mathrm{fu}}\Psi_{\mathrm{f}}\mathbf{F}_{\mathrm{c}}+\int_{\Gamma_{\mathrm{c}}}\mathbf{G}_{\mathrm{ff}}\Psi_{\mathrm{u}}(-\mathbf{U}_{\mathrm{c}})+\hat{\mathbf{t}}\quad \text{on } \Gamma_{\mathrm{c}}\cup\Gamma_2 \tag{18b}$$

where the CPV integrals as well as the free terms are to be introduced when necessary. If, for example, we want to evaluate the displacement $\mathbf{u}_2$ on Γ_2, the second integral of eq.(18a) is considered as the CPV integral and the free term must be introduced.

In order to reach the unilateral contact conditions in the ambit of the symmetric BEM eqs.(2) must be rewritten, but evaluated in weighted form on Γ_{c} and Γ_2. Explicitly the following conditions must be valid:

$$\tilde{\boldsymbol{n}}^{\mathrm{A}}(\boldsymbol{W}^{\mathrm{A}}-\boldsymbol{W}^{\mathrm{B}})-\boldsymbol{H}\leq\boldsymbol{0} \tag{19a}$$

$$\tilde{\mathbf{n}}^{\mathrm{A}}\mathbf{P}^{\mathrm{A}}-\mathbf{C}\leq\mathbf{0} \tag{19b}$$

$$[\tilde{\boldsymbol{n}}^{\mathrm{A}}(\boldsymbol{W}^{\mathrm{A}}-\boldsymbol{W}^{\mathrm{B}})-\boldsymbol{H}\leq 0][\tilde{\boldsymbol{n}}^{\mathrm{A}}\boldsymbol{P}^{\mathrm{A}}-\boldsymbol{C}]=0 \tag{19c}$$

where $\mathbf{n}^{\mathrm{A}}$ is the external unit vector from the generic boundary element of the body A and where the weighting of the displacements and of the tractions was made. In the latter equations the terms

$$\mathbf{H}=\int_{\Gamma_{\mathrm{c}}\cup\Gamma_2}\Psi_{\mathrm{f}}\ \mathrm{h}\qquad\qquad \mathbf{C}=\int_{\Gamma_{\mathrm{c}}\cup\Gamma_2}\Psi_{\mathrm{u}}\ \mathrm{c} \tag{20a,b}$$

represent the weighted gap vectors between $\boldsymbol{\Gamma}_2^{\mathrm{A}}$ and $\boldsymbol{\Gamma}_2^{\mathrm{B}}$ and the weighted value of the hypothesized cohesion. The following weighting positions are valid:

$$\mathbf{W}^{\mathrm{A}}=\int_{\Gamma_{\mathrm{c}}\cup\Gamma_2}\Psi_{\mathrm{f}}\ \mathbf{u}^{\mathrm{A}}\qquad \mathbf{W}^{\mathrm{B}}=\int_{\Gamma_{\mathrm{c}}\cup\Gamma_2}\Psi_{\mathrm{f}}\ \mathbf{u}^{\mathrm{B}}\qquad \mathbf{P}^{\mathrm{A}}=\int_{\Gamma_{\mathrm{c}}\cup\Gamma_2}\Psi_{\mathrm{u}}\ \mathbf{t}^{\mathrm{a}} \tag{20c-e}$$

The contact conditions between the bodies A and B are evaluated through a step by step process: indeed, when the mechanical and thermic distorsion load changes in time, the orthogonalities (19) are to be verified.

Two cases can occur:

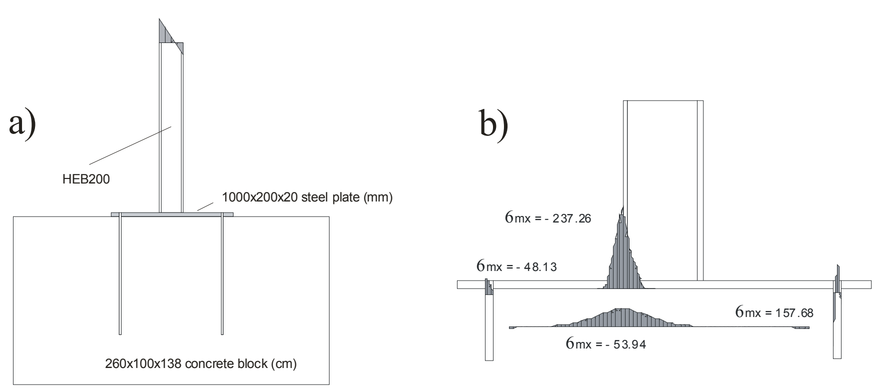
– On the boundary Γ_2 (**C**=**0**) eq.(19b) is verified as an equality. The analysis involves the inequality (19a) to be violated; as a consequence the interface sides are joined and (19b) will be verified as an inequality.

– On the boundary Γ_c (**H** = **0**) eq.(19a) is verified as an equality. The analysis involves the inequality (19b) to be violated; as a consequence the interface sides must be separated and (19a) will be verified as an inequality.

Example

Let a steel I-pillar, welded to a deliberately thin steel plate, be constrained to a concrete block through two bars embedded in the concrete. The plate is in touch with the higher zone of the concrete block, also. The tip of the pillar is subjected to the eccentric force (M, N). If we assume as external actions M = daN·m 10000, N = daN 100000, (e = M/N = cm 0.1), it is possible to analyze the contact zone between two bodies by verifying the disjunction zone.

We verified that the contact zone is limited to the bars embedded in the concrete and to the zone near to the pillar extremity on the left. A strong compression is found in this zone (σ_y = daN/cmq 237), whereas the maximum disjunction on the right of the contact zone takes on the value d = cm 0.0314. In Fig. 2b the vertical stress σ_y was evaluated in accordance with prefixed lines both inside the concrete and the steel bars, where the maximum stress values are shown. Two mappings of the compression and traction stress of the concrete are shown in Fig. 2d,e.



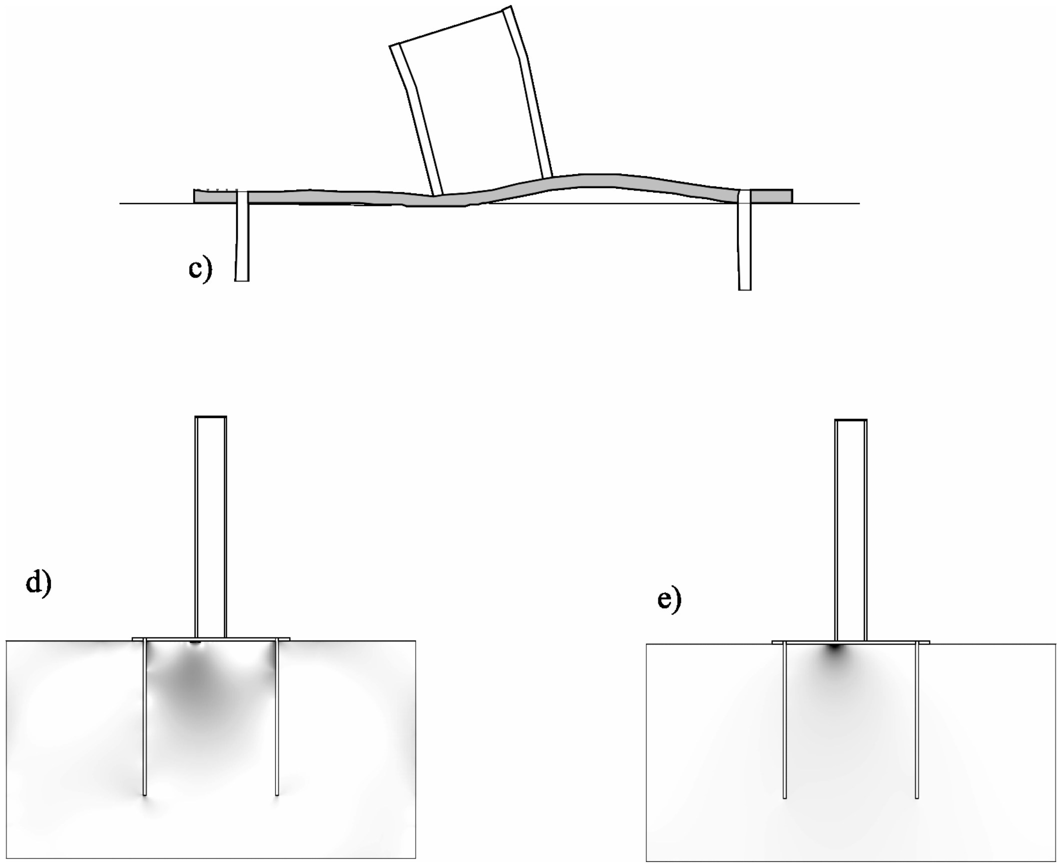


Fig.2: a) Steel I-pillar and concrete block, b) Vertical stress σ_y at prefixed lines, c) Particular regarding the contact and disjunction zones, d,e) Mappings of the compression and tractions stress in the concrete.

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