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Prediction of Welding Distortion Using Eigen Strain

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Summary

Since the inelastic strain or the inherent strain is produced only in the vicinity of the weld line, the welding distortion of large structure can be predicted by elastic FEM with introducing such inherent strain as the initial strain. In this report the general idea of the inherent strain or the eigen strain is discussed first. Then the prediction of welding distortion of curved structure is presented as an example. Further, a simple method using inverse analysis to determine the welding inherent deformation is proposed.

Introduction

Welding is a key technology for building metal structures such as ships, bridges and automobiles. However, it is impossible to avoid the welding distortion due to intrinsic nature of non-uniform heating and cooling of the welded area. Welding distortions degrades the performance of the product. Also, it is an obstacle for realizing automation and robotization in assembly process. In order to solve these problems, quantitative prediction and control of the welding deformations is necessary.

Since the phenomena involved in the welding is transient and nonlinear, thermal-elastic-plastic FEM must be employed. However, it requires unrealistically long computation time even for small test models. Alternative method is to employ the concept of eigen strain[1] or inherent strain. The idea of this method is that the welding distortion and the residual stress of a structure is produced by the inherent strain existing in the vicinity of the welding line. The local deformation due to the welding, such as the transverse shrinkage, the longitudinal shrinkage, the angular distortion (or the transverse bending) and the longitudinal bending can be regarded as the inherent deformations which are the integration of the inherent strains. It is known that the distribution of the inherent deformation is almost constant along the welding line and its value can be related to the heat input parameter Q/h^2 [2] as shown in Fig. 1. Thus, if the inherent deformations are known, the welding distortion of large structures can be predicted by elastic FEM with introducing the inherent deformations as the initial strains distributing in the elements along the welding line.

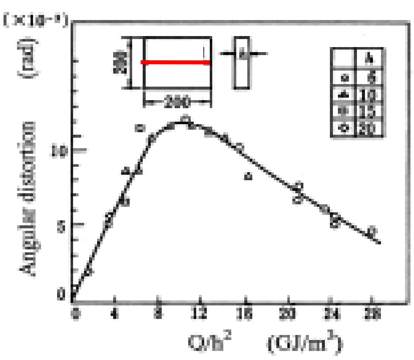

Fig. 1 Relation between angular distortion and heat input parameter Q/h^2.

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There are two alternative ways to estimate the inherent deformations. One is the experiment and the other is the thermal-elastic-plastic FEM. Though it requires time and cost, the experiment is simple when the weld length is long such as in the case of ships and bridges. When the weld length is long enough, the self-restraint is small and the inherent deformation can be directly measured. The latter is effective if all necessary information such as the temperature dependent material properties and the heat input distribution is known. In case of the assembly of automobile, short weld is often used. When the weld is short, the self-restraint is large and the inherent deformation terns into both the deformation and the residual stress which is invisible. Thus, the measurement becomes difficult.

In this report, the general idea of the inherent strain in welding is discussed and the prediction of the welding distortion of a curved structure is presented as an example. In this example, the influence of gap, which is also the inherent deformation in the general sense, is examined. Further, a simple inverse analysis to estimate the inherent deformation for short weld is proposed.

Inherent Deformation

To explain the fundamental concepts involved in problems associated with the welding residual stress and distortion, a simple model consists of three metal bars shown in Fig. 2 is often used. Three bars, namely (a), (b) and (c), are joined at both ends. One end of the three bars is fixed and the other end is free. The bar-(b) is assumed to be heated over the melting point partially or throughout its length. When the bar-(b) is cooled down to the room temperature after the thermal cycle, the length of the bar-(b) becomes shorter than the original length L by ΔL. In this case the deformation of the free end u and the tensile force F_b acting in the bar-(b) can be described in terms of ΔL, i.e.

$$u = \Delta L\, k/(k+k^*) = (1-\beta)\,\Delta L \qquad (1)$$

$$F_b = L\,k\,k^*/(k+k^*) = \beta k L \qquad (2)$$

where, k is the stiffness of the bar-(b) and k^* is the sum of the stiffnesses of bar-(a) and (c). $\beta=k^*/(k+k^*)$ is the restraining parameter which represents the restraint acting on the bar-(b). Since both the deformation and the stress are determined by the shrinkage ΔL, ΔL is called inherent deformation in this case.

If the restraint is small such as the case in which $k^*=0$ or $\beta=0$, the deformation u and the tensile force F_b become,

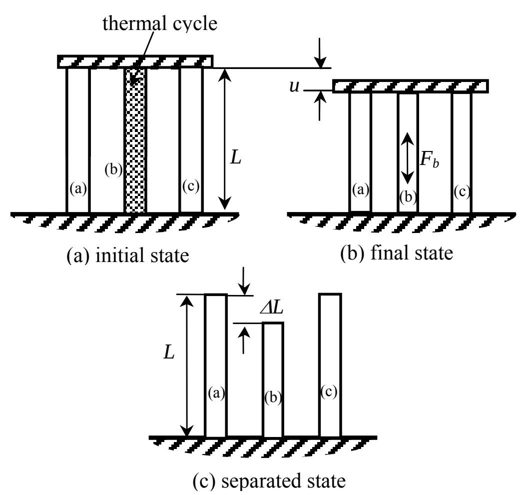


Fig. 2 Three bar models for welding deformation and residual stress.

$u = \Delta L$ (3) $F_b = 0$ (4)

This means that the mismatch or the inherent deformation ΔL directly appears in the form of deformation.

To the contrary, if the restraint is extremely large ($k^* = \infty$ or β=1), the mismatch ΔL directly appears in force F_b not in deformation u, i.e.

$u = 0$ (5) $F_b = k\,\Delta L$ (6)

Inherent Strain

The transient or the residual deformation in welding is produced by various strain components of different origins. In general, apparent strain ε, which corresponds to the deformation, is regarded as the sum of the thermal strain ε^T, elastic strain ε^e, plastic strain ε^p, strain due to the phase transformation ε^t and the creep strain ε^{cr}, i.e.

$$\varepsilon = \varepsilon^T + \varepsilon^e + \varepsilon^p + \varepsilon^t + \varepsilon^{cr} \quad (7)$$

The thermal strain becomes zero when the temperature returns to the original temperature after the complete thermal cycle. Thus, if we are interested in the state after the complete thermal cycle Eq. (7) becomes.

$$\varepsilon = \varepsilon^e + \varepsilon^p + \varepsilon^t + \varepsilon^{cr} \quad (8)$$

The strain components in the above equation can be separated into the elastic strain ε^e and other strain components ε^*. The elastic strain is produced by stress and reversible. On the other hand, all components of ε^*, namely plastic strain, creep strain and that produced by phase transformation, are irreversible. Because of this nature, it is called as inherent strain or eigen strain.

The same three bar model can be used for more general discussion. Let's assume that the apparent strain in the three bars can be decomposed into the elastic strains $\varepsilon^e{}_a$, $\varepsilon^e{}_b$, $\varepsilon^e{}_c$ and the inherent strains $\varepsilon^*{}_a$, $\varepsilon^*{}_b$, $\varepsilon^*{}_c$. Then the equations governing the mechanical behavior of the model are given as follows.

(a) strain-displacement relation (3 equations)

$u/L = \varepsilon^*{}_a + \varepsilon^e{}_a$ (9) $u/L = \varepsilon^*{}_b + \varepsilon^e{}_b$ (10)

$u/L = \varepsilon^*{}_c + \varepsilon^e{}_c$ (11)

(b) stress-strain relation (3 equations)

$\sigma_a = E\,\varepsilon^e{}_a = E(u/L - \varepsilon^*{}_a)$ (12) $\sigma_b = E\,\varepsilon^e{}_b = E(u/L - \varepsilon^*{}_b)$ (13)

$\sigma_c = E\,\varepsilon^e{}_c = E(u/L - \varepsilon^*{}_c)$ (14)

(c) equilibrium equation (1 equation)

$$A(\sigma_a+\sigma_b+\sigma_c)=0 \tag{15}$$

where, L: length of bars A: cross-sectional area of bars
E: Young's modulus u: displacement
σ_a , σ_b , σ_c : stresses of bars

In this problem, 10 variables, namely u, ε^e_a, ε^e_b, ε^e_c, ε^*_a, ε^*_b, ε^*_c, σ_a, σ_b and σ_c , are involved. Among them, ε^e_a, ε^e_b, ε^e_c and u can be measured. On the other hand, we have 3+3+1=7 equations relating these variables. This means that 6 variables must be determined by 7 equations.

When we are interested in welding residual stress, we can measure elastic strain by various methods such as the strain gage or the X-ray diffraction. But the welding deformation can not be measured because the initial configuration before welding is not available in the real structures. Therefore, let's assume that the elastic strains ε^e_a , ε^e_b and ε^e_c are measurable and known. By eliminating displacement u and stresses σ_a , σ_b and σ_c from Eqs. (9)-(15), the following equations are derived.

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{Bmatrix} \varepsilon^*_a \\ \varepsilon^*_b \\ \varepsilon^*_c \end{Bmatrix} = \begin{Bmatrix} \varepsilon^e_a \\ \varepsilon^e_b \\ \varepsilon^e_c \end{Bmatrix} \tag{16}$$

It can be readily shown that the determinant of the coefficient matrix is zero. This means that the inherent strains ε^*_a , ε^*_b and ε^*_c can not be uniquely determined. Undetermined part of the strain corresponds to the solution of the following homogeneous equation.

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{Bmatrix} \varepsilon^*_a \\ \varepsilon^*_b \\ \varepsilon^*_c \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{17}$$

The solution of the above equation is given as,

$$\varepsilon^*_a = \varepsilon^*_b = \varepsilon^*_c = \lambda = u/L \tag{18}$$

where, λ is an arbitrary constant. Physically this implies that the inherent strain satisfying the compatibility produces deformation but no stress. This argument is important in measuring residual stress based on the inherent strain[3].

To exclude the compatible mode of inherent strain, one component, for example ε^*_c , is assumed to be zero, thus,

$$\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \begin{Bmatrix} \varepsilon^*_a \\ \varepsilon^*_{ab} \end{Bmatrix} = \begin{Bmatrix} \varepsilon^e_a \\ \varepsilon^e_{ab} \end{Bmatrix} \tag{19}$$

The solution for this equation is shown to be,

$$\left[\varepsilon^*_a,\ \varepsilon^*_b,\ \varepsilon^*_c\ \right] = \left[-(2\varepsilon^e_a+\varepsilon^e_b)/3,\ -(\varepsilon^e_a+2\varepsilon^e_b)/3,\ 0\right] \tag{20}$$

The inherent strain estimated following the above procedure is called effective inherent strain which means that it reproduces the residual stress exactly but not the deformation.

Similarly, if the displacement u is measured and known, the following relation is derived from the equilibrium equation by eliminating the elastic strains and the stresses.

$$u/L = (\varepsilon^*_a + \varepsilon^*_b + \varepsilon^*_c)/3 \tag{21}$$

This relation tells us that the inherent strains, which satisfy the following relation, produce stress but no deformation.

$$\varepsilon^*_a + \varepsilon^*_b + \varepsilon^*_c = 0 \tag{22}$$

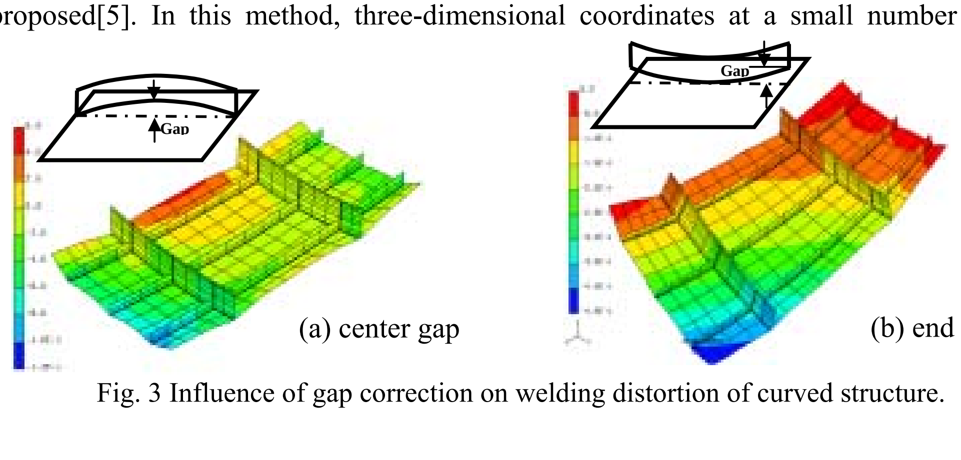
In other words, the inherent strain field, which corresponds to the equilibrated stress field, produces stress but no deformation. This is important in the engineering problems such as plate forming.

Welding Distortion of Curved Structure

The distortions during the assembly by welding are very difficult to predict based on experience, especially, when the structures have asymmetric curved geometry such as ships and automobiles. I such cases, both the local shrinkage and the correction of gap and misalignment between parts are influential. We proposed an elastic FEM which takes into account these through the concept of inherent deformation and the interface element. Figure 3 shows the influence of the gap correction before welding on the final distortion[4]. When the structure is asymmetric, significant distortion of twisting mode is produced by welding. The size and the mode of the gap are also influential to the distortion as shown in Fig.3.

Estimation of Inherent Deformation Using Inverse Analysis

When the weld is partial and its length is short, the inherent deformation can not be measured directly. To solve this problem, nonlinear inverse analysis using FEM is proposed[5]. In this method, three-dimensional coordinates at a small number of the



(a) center gap (b) end gap

Fig. 3 Influence of gap correction on welding distortion of curved structure.

selected points on the specimen are measured before and after welding. From these coordinates, the deformations $F^m{}_j$, such as the shrinkages between two points and the deflection relative to the reference triangle defined in the specimen are computed. In this way, the effect of the rigid body deformation can be removed. On the other hand, if the distribution of the inherent deformation is described using parameters a_i, the relation between the parameter a_i and the deformation $F_j(a_i)$ can be computed using elastic shell FEM. When the plate is thin, the functions $F_j(a_i)$ are nonlinear. The parameters of inherent deformation a_i which produce the measured deformations $F^m{}_j$ are determined through iterative inverse analysis which is based on the following equation.

$$F_j(a_i + \Delta a_i) \approx F_j(a_i) + (\partial F_j / \partial a_i)\Delta a_i = F^m{}_j \tag{23}$$

Conclusions

(1) Compatible inherent strain field produces deformation but no stress.

(2) Inherent strain field corresponding to self-equilibrated stress field produces stress but no deformation.

(3) Both the local shrinkage due to welding and the gap existing before the welding are the inherent strain in general sense and they produce distortion.

(4) To estimate the inherent deformation of the short weld, iterative inverse analysis is proposed.

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