Prediction mode numerical simulation for non-straight propagation crack in elasto-plastic material

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Summary

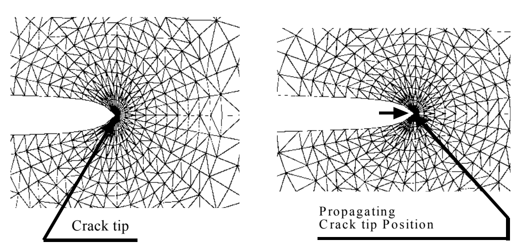
In this study, first, using local symmetry criterion, the fracture path prediction simulation for mixed-mode loaded elasto-plastic material was carried out. The computed fracture path agreed excellently with experimentally observed one. Then, using maximum hoop stress criterion and maximum hoop strain criterion the fracture path prediction simulations for material including voids were carried out. The influence of voids on crack tip was investigated.

Introduction

The prediction of fracture path is extremely important for prevention the complete destruction of large constructions. The elasto-plastic deformations are often accompanied the materials fracture. In this study, it was conducted using the moving finite element method on Delaunay automatic triangulation [1,2]. First, using local symmetry criterion(T_2^{*0} =0 criterion) [3]., the fracture path prediction simulation for mixed-mode loaded elasto-plastic material was carried out. The computed fracture path agreed excellently with experimentally observed one. Then, using maximum hoop stress criterion ($\sigma_{\theta\theta}$ max)[4] and maximum hoop strain criterion ($\varepsilon_{\theta\theta}$ max) the fracture path prediction simulations for material including voids were carried out. The influence of voids on crack tip was investigated.

Variational principle for a propagating crack in a nonlinear material

Let us consider a propagating crack in an elasto- plastic material. Here we use the incremental infinitesimal deformation theory. In the moving finite element procedure, the mesh pattern near the propagating crack tip translates in each time step as illustrated in Fig.1. After the mesh translation, the moving finite element method requires the mapping of solution fields in the previous mesh onto those in the present mesh. To satisfy the governing equations and the boundary conditions in the present



(a) Previous mesh pattern (b) Present mesh pattern

Fig.1 Moving finite element procedures for a propagating crack

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mesh at time t=t+Δt, a new incremental variational principle[5] is derived as follows:

$$\int_V \left(\Delta\sigma_{ij}\delta\varepsilon_{ij} + \rho\ddot{u}_i\delta u_i\right)dV - \int_V \Delta\bar{f}_i\delta u_i dV - \int_{S_t} \Delta t_i \delta u_i =$$

$$+\int_V \bar{f}_i^{0(t)}\delta u_i dV + \int_{S_t} \bar{t}_i^{0(t)}\delta u_i dS - \int_V \left(\sigma_{ij}^{0(t)}\delta\varepsilon_{ij} + \rho\ddot{u}_i^{0(t)}\delta u_i\right)dV \quad (1)$$

where the superscript 0(t) denotes a quantity of the previous step (t_0) in the present mesh pattern at time t. The new crack surfaces $\Delta\Gamma^{+}_{\square}$ and$\Delta\Gamma^{-}_{\square}$ are created by the crack propagation during time increment Δt. The finite element solution field obtained from the new variational principle satisfies the following equilibrium equation and the mechanical boundary condition including the crack surfaces at time $t=t_0+\Delta t$:

$$\Delta\sigma_{ij,j} + \Delta\bar{f}_i - \rho\Delta\ddot{u}_i = -\left(\sigma_{ij,i}^{0(t)} + f_i^{0(t)} - \rho\ddot{u}_i^{0(t)}\right) \qquad \text{in V} \quad (2)$$

$$\Delta\sigma_{ij}n_j = \bar{t}_i^{0(t)} + \Delta\bar{t}_i - \sigma_{ij}^{0(t)}n_j \qquad \text{in S} \quad (3)$$

The finite element method was developed based on Eq. (1)

The T* Integral

The global components the T* integral [6] T_k^* can be expressed as

$$T_k^* = \int_{\Gamma_\varepsilon} \left[(W+K)n_k - t_i u_{i,k}\right]dS$$

$$= \int_{\Gamma+\Gamma_c} \left[(W+K)n_k - t_i u_{i,k}\right]dS + \int_{V_\Gamma - V_\varepsilon} \left[\sigma_{ij}\varepsilon_{ij,k} - W_{,k} + \rho\ddot{u}_i u_{i,k} - \rho\dot{u}_i\dot{u}_{i,k}\right]dV \quad (4)$$

where W and K are the stress working and kinetic energy densities, respectively. Integral paths are defined as follows: Γ is a far-field contour that encloses the crack tip and envelops a volume V_Γ; and Γε is a near-field contour arbitrarily close to the propagating crack tip and envelops a small volume Vε ; and Γc is the crack surface enclosed by Γ as illustrated in Fig.2. Physically, the near-tip region Vε can be considered as the process zone in which micro-processes associated with fracture occur. The crack-axis components of the T* Integral can T_l^{*0} be easily obtained by the coordinate transformation: $T_l^{*0} = \alpha_{lk}(\theta_0)T_k^*$ where θ is the crack direction. The tangential crack –axis component T_1^{*0} has the meaning of the energy flow rate to the process zone for a propagating crack in an elasto-plastic material under a steady-

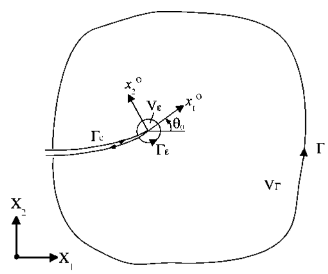


Fig.2 T* integral path

state condition. The vertical crack-axis component T_2^{*0} can be considered as a measure of mixed-mode state of the crack tip [7].

The crack tip which approaches the voids make it very difficult to accurately evaluate fracture parameters such as the T* Integral. To overcome this difficulty, T* Integral evaluation method using continuous function s is derived as follows:

$$T_k^* = \int_{\Gamma+\Gamma_c} \left[(W+K)n_k - t_i u_{i,k}\right] s\,dS + \int_{V_\Gamma - V_\varepsilon} \left[\sigma_{ij}\varepsilon_{ij,k} s - W_{,k} s + \left(\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}\right) s\right] dV \quad (5)$$

where s is a continuous function defined in V_Γ. In this technique, continuous function s is defined on each point in the material, and the T^* Integral is evaluated by the product of this s and calculation term. Equation (5) agrees with the conventional evaluation of T^* Integral , when all s is made to be 1 in V. And, it agrees with the equivalent domain integral method [8], when s is made to be s=0 on Γ and s=1 in the V_Γ inside. In equation (5), when the voids are included in region V_Γ, we set the s function to be s=1 for all of points in the domain except the voids where the value is set s=0, and vice versa. Hence, it is possible to easily obtain the T^* Integral for crack tip approaching the voids.

Fracture-Path Prediction Procedure

Many criteria have been proposed for crack propagation direction prediction. These criteria can be classified into (a) explicit prediction theory or (b) implicit prediction theory. In this study, two propagation-direction criteria were employed: (i) the maximum hoop stress criterion ($\sigma_{\theta\theta}$ max) [4] and (ii) the local symmetry criterion (T_2^{*0}=0) [3]. The maximum hoop stress criterion falls into the explicit prediction theory, which predicts the propagation direction by satisfying the postulated criterion based on a physical quantity for the current crack tip. The crack grows in the predicted direction by a small crack-length increment.

On the contrary, the local symmetry criterion falls into the implicit prediction theory, which searches the propagation direction that satisfies the postulated criterion, based on a physical quantity evaluated after the crack has been grown a small crack-length increment. Using this theory, an iteration procedure is generally needed to find the propagation direction that exactly satisfies the postulated criterion. Furthermore, it is known that the implicit prediction theories are usually more accurate.

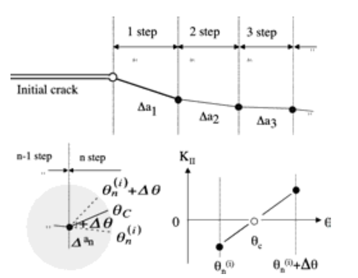


Fig.3 Fracture-path prediction procedure

The maximum hoop stress criterion is based on the hypothesis that crack grows from the tip in the radial direction along which the hoop stress is maximum. The local symmetry criterion supposes that crack grows in the direction in which the T_2^{*0} is equal to zero after the crack growth by a small increment.

Fig.3 schematically outlines the numerical procedures for the path-prediction mode of the mixed-phase simulation for the local symmetry criterion. At each time step, the crack is advanced by a small increment according to the experimental history (the crack-length versus time curve).

The fracture path is predicted as follows. At a generic time step, n, taken as the first trial, the crack is advanced in the tangential direction (θ_n) from the crack tip of step n-1. If the chosen propagation-direction criterion, for example the local symmetry (T_2^{*0} =0) criterion, is satisfied at the attempted crack-tip location, the crack is advanced in the direction θ_n. If the T_2^{*0} value is negative, the crack is tentatively advanced in the direction $\theta_n+\Delta\theta$, as the next trial. A check is then made to see if the criterion is satisfied. If the criterion is not satisfied, and the sign of the T_2^{*0} value at the present trial is different from that at the previous trial, the fracture direction θ_c that exactly satisfies the employed propagation-direction criterion is determined from the T_2^{*0} versus θ curve, as shown in Fig.3. If the sign of the T_2^{*0} value is positive, a negative $\Delta\theta$ is set to seek the direction θ_c of T_2^{*0} =0. These procedures are repeated until the criterion is satisfied.

Numerical result

The mesh pattern of the through wall crack problem is shown in Fig. 4. The specimen material is A5052P, the material properties of A5052P are shown in Table1. Equivalent stress and equivalent plastic strain relation is express by strain hardening equation as shown in Eq (6). The numerical simulation carried out mixed-phase prediction mode [9]. In these numerical simulations, the quasi-static experimental history (the crack-length versus displacement of load points) and propagation-direction criterion (the local symmetry criterion, T_2^{*0} =0) are used to estimate fracture behavior.

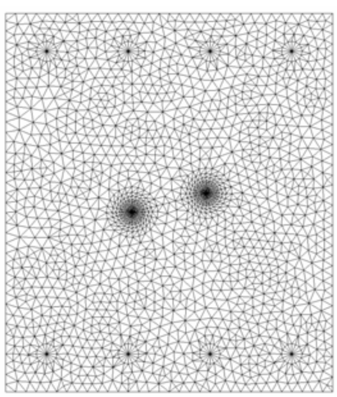


Fig.4 Mesh pattern

Table.1 Elasto-plastic properties

Young's modulus	E	69.6GPa
Strain hardness exponent	n	11
Yield stress	σys	145.75MPa
Referencial stress	E'	300MPa
Poisson rate	ν	0.33

$$\bar{\varepsilon} = \left\{ \left(\frac{\bar{\sigma}}{E'} \right)^n - \left(\frac{\sigma_{ys}}{E'} \right)^n \right\} \qquad (6)$$

In the void problem, location of cracks and defects in Fig.5 is assumed. One unit cell was made to be an object of the calculation. In the numerical model for the void problem, it assumed that shape of defect is circular and four voids were placed in the unit cell. The simulation model is shown in Fig.6. In this study, the maximum hoop stress criterion($\sigma_{\theta\theta}$Max) [4] and the maximum hoop strain criterion ($\varepsilon_{\theta\theta}$Max) are emploed as propagation-direction criteria. Uniform displacement are applied on top surface and bottom surface of the unit cell. It is assumed that crack growth length per each step is 0.0075 mm, and the incremental displacement Δu is 0.0001667 mm.

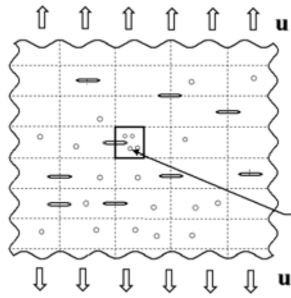


Fig.5 Cracks and defects in the material

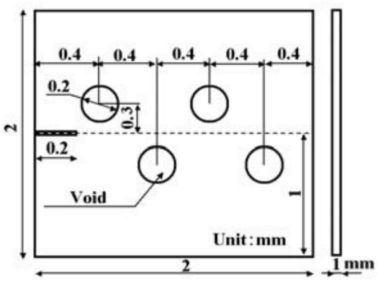


Fig.6 Simulation model

In the through wall crack problem, Fig.7 shows path independence of T^* Integral [6]. We can see that we were able to obtain T^* Integral with good path independence. Fig.8 shows the hysteresis of the T^* Integral. The result of using the local symmetry criterion show that the T^* Integral value is almost 0. Fig.9 shows crack tip coordinate histories. It is found that both experimental result and numerical result shows similar fracture path.

In the void problem, Fig.10 and Fig.11 show the fracture paths, which are predicted in the numerical simulations for the elastic material and the elasto-plastic material. Under the same boundary condition, the curvature of path crack of elasto-plastic fracture is larger than the one of elastic fracture. In the elasto-plastic material, higher stress concentrations have been occurred in the local area between void and crack tip. These stress concentration affect the hoop stress and strain distributions near the crack tip.

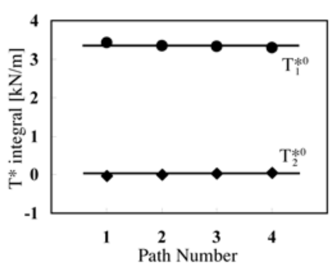


Fig.7 Path independence

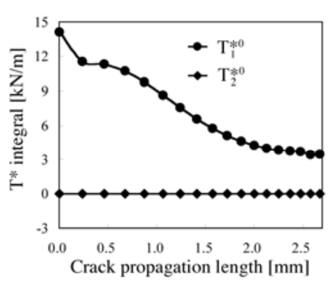


Fig.8 History of T^* Integral

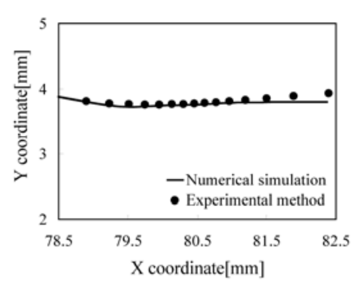
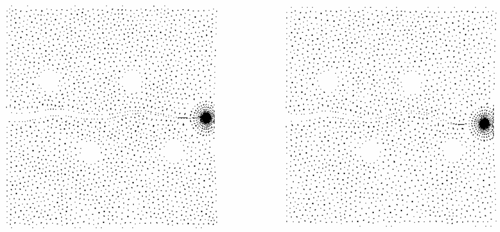
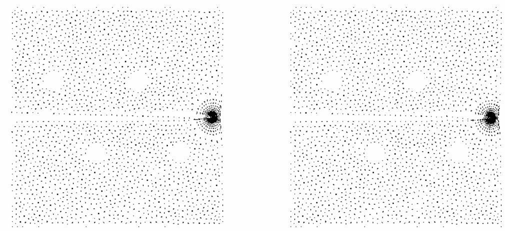


Fig.9 History of crack tip coordinate



(a) $\sigma_{\theta\theta}$Max criterion (b) $\varepsilon_{\theta\theta}$Max criterion
Fig.10 Elasto-plastic material



(a) $\sigma_{\theta\theta}$Max criterion (b) $\varepsilon_{\theta\theta}$Max criterion
Fig.11 Elastic material

Conclusion

In this study, the numerical path-prediction simulations of elasto-plastic fracture phenomena based on moving finite element method were carried out. The T* integral values, calculated in the analysis, are excellently path independent. The propagation direction of crack tip is considered to be controlled by T2*=0 criterion. The simulated results agreed excellently with the experimental ones. In the void problem the curvature of crack path is much larger in case of elasto-plastic fracture, than in elastic fracture.

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