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# Aeroelastic Modeling and Vibratory Load Analysis of Helicopters

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**Summary**

An aeroelastic model for helicopter rotors in forward flight is presented and applied to predict the vibratory loads transmitted to the hub. A nonlinear blade structural model is coupled with a free-wake, potential flow BEM solver and the periodic regime blade deformation is determined by the harmonic balance method. Numerical results examine the influence of the aerodynamic modeling on the prediction of the vibratory loads for the ECD BO-105 in a level flight configuration.

## Introduction

The aim of this paper is the presentation of an aeroelastic model for helicopter rotors in forward flight addressed to the evaluation of the vibrating hub loads arising during regime conditions.

The availability of a reliable tool for the prediction of vibrating hub loads is of primary interest in the helicopter design. Indeed, they are source of fuselage vibrations that have a significant impact on the fatigue-life of the structure (and hence on maintenance costs) and in turn produce acoustic disturbances inside the cabin that could cause unacceptable ride discomfort. Hub loads are the results of the rotor blade aeroelastic behavior and their evaluation requires the introduction of accurate structural and aerodynamic models. In particular, the structural model has to take into account both the strong coupling between bending and torsion degrees of freedom of the blade and the nonlinearities arising from the significant deflections that slender rotor blades usually undergo. The aerodynamic modeling has to be able to describe 3D, unsteady flows, with particular accuracy on the prediction of the wake effects that are of great importance in the aerodynamics of rotors in forward flight.

Here, the aeroelastic model is obtained by coupling the equations of the blade dynamics introduced by Hodges and Dowell [1], with the aerodynamic loads given by a free-wake boundary element method (BEM) for potential flows [2], [3]. The resulting integro–differential model is integrated through a Galërkin approach followed by a harmonic balance method for the definition of the periodic blade deformation arising in regime flight conditions. The numerical investigation concerns the prediction of vibratory hub loads on the ECD BO-105 in level flight conditions. Three models for the determination of the unsteady aerodynamic loads are used in order of assess the importance of the aerodynamic model used in the analysis, when the objective is the evaluation of the cabin vibrations induced by the main rotor. Specifically, the aeroelastic response is evaluated by using the BEM solver both under the assumption of prescribed wake shapes and using a free-wake procedure. These two solutions are compared also with that obtained through the quasi-steady aerodynamic model with inflow correction.

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### Blade structural dynamics model

In this work the blade structural dynamics is described through the nonlinear flap-lag-torsion equations of motion presented by Hodges and Dowell [1]. These are based on a beam-like model and are valid for straight, slender, homogeneous, isotropic, nonuniform, twisted blades, undergoing moderate diplacements (second order terms are retained in the equations). Eliminating the radial displacement from the set of equations by solving it in terms of local tension (*i.e.*, the blade is assumed to be inextensible for bending deflections and radial displacements are simply geometric consequences of transverse bending [4]), for no hinge offset and for mass and tensile axes coinciding with the elastic axis, the in-plane displacement, $v(x,t)$, and the out-of-plane displacement, $w(x,t)$, of the elastic axis along with the blade cross-section elastic torsion deflection, $\phi(x,t)$, are governed by the following set of three lead-lag, flap and torsion dimensionless integro–differential equations

$$[(\Lambda_2-\Lambda_{21}\sin^2\theta)v'']''+\frac{\Lambda_{21}}{2}[w''\sin(2\theta)]''+\Lambda_{21}[\phi w''\cos(2\theta)-\phi v''\sin(2\theta)]''$$
$$-[v'\int_x^1(x+2\dot{v})\mathrm{d}x]'-2\int_0^x(v'\dot{v}'+w'\dot{w}')\mathrm{d}x+\ddot{v}-v-2\beta_{pc}\dot{w}=\mathcal{L}_v \qquad (1)$$

$$[(\Lambda_1+\Lambda_{21}\sin^2\theta)w'']''+\frac{\Lambda_{21}}{2}[v''\sin(2\theta)]''+\Lambda_{21}[\phi v''\cos(2\theta)+\phi w''\sin(2\theta)]''$$
$$-[w'\int_x^1(x+2\dot{v})\mathrm{d}x]'+\ddot{w}+2\beta_{pc}\dot{v}+\beta_{pc}x=\mathcal{L}_w \qquad (2)$$

$$-\frac{\mu^2K}{2}[\phi'(1-x^2)]'-\kappa\phi''+\mu^2\ddot{\phi}+\phi(\mu_2^2-\mu_1^2)\cos(2\theta)-\mu^2K[\theta'\int_x^1(x+2\dot{v})\mathrm{d}x]'+\mu^2\ddot{\theta}$$
$$+2\dot{\theta}[\mu_2^2(v'\sin^2\theta-w'\sin\theta\cos\theta)+\mu_1^2(v'\cos^2\theta+w'\sin\theta\cos\theta)]+(\mu_2^2-\mu_1^2)\frac{\sin(2\theta)}{2}$$
$$+\Lambda_{21}[(w''^2-v''^2)\sin\theta\cos\theta+v''w''\cos(2\theta)]=\mathcal{M}_\phi \qquad (3)$$

where $x$ is the spanwise position, $\theta(x,t)$ is the pitch angle distribution, $\Lambda_1$ and $\Lambda_2$ are the dimensionless flap and lag bending stiffnesses ($\Lambda_{21}=\Lambda_2-\Lambda_1$), $\kappa$ is the dimensionless torsion rigidity $\mu_1$, $\mu_2$, $\mu$ are the dimensionless mass radii of gyration, $K$ is the square of the ratio between the blade cross-section polar radius of gyration and the blade cross-section mass radius of gyration, whereas $\beta_{pc}$ is the precone angle. In addition, $\mathcal{L}_v$ and $\mathcal{L}_w$ are, respectively, in-plane and out-of-plane dimensionless aerodynamic forces per unit length, whereas $\mathcal{M}_\phi$ is the dimensionless aerodynamic pitching moment per unit length (see Ref. [4] for details on the definition of the parameters appearing in Eqs. 1, 2 and 3).

### Free-wake BEM rotor aerodynamics

The aerodynamic loads forcing the blade dynamics equations are evaluated by a boundary integral approach for potential, incompressible flows. It is based on the formulation introduced in Ref. [2], and on the free-wake algorithms presented in Ref. [5] for hovering rotors and in Ref. [3] for rotors in forward flight.

Introducing the velocity potential, $\varphi$, such that $\mathbf{v} = \nabla\varphi$, the conservation of mass yields the Laplace equation, $\nabla^2\varphi = 0$, with solution that may be given in terms of the following boundary integral representation

$$\varphi(\mathbf{x},t) = \int_{\mathcal{S}_B} \left( \frac{\partial\varphi}{\partial n} G - \varphi \frac{\partial G}{\partial n} \right) \mathrm{d}\mathcal{S}(\mathbf{y}) - \int_{\mathcal{S}_W} \Delta\varphi \frac{\partial G}{\partial n} \mathrm{d}\mathcal{S}(\mathbf{y}) \tag{4}$$

In Eq. 4, $G = -1/4\pi\|\mathbf{x}-\mathbf{y}\|$ is the fundamental solution of the Laplace equation, $\mathcal{S}_B$ and $\mathcal{S}_W$ denote body and wake surfaces, respectively, whereas $\Delta\varphi(\mathbf{x}_W,t) = \Delta\varphi(\mathbf{x}_{TE},t-\tau)$, with $\tau$ denoting the time taken by a wake material point to be convected from the trailing edge to $\mathbf{x}_W$. In addition, $\partial\varphi/\partial n = \mathbf{v}_B \cdot \mathbf{n}$, for the boundary condition of body surface impermeability. For $\mathbf{x}$ approaching $\mathcal{S}_B$, Eq. 4 represents a compatibility condition between $\varphi$, $\partial\varphi/\partial n$ on the body and $\Delta\varphi$ on the wake. Since $\partial\varphi/\partial n$ is known from the boundary condition, Eq. 4 yields an integral equation which may be used to obtain the values of $\varphi$ on $\mathcal{S}_B$. Finally, once the potential on the body surface has been evaluated, the Bernoulli theorem yields the pressure distribution and hence the aerodynamic forces acting on it.

In the case of free-wake analysis, the shape of $\mathcal{S}_W$ is obtained as part of the solution. Indeed, once $\varphi$ on the surface is known, from $\mathbf{v} = \nabla\varphi$ the velocity is evaluated in the field and, in particular, at the wake points. Then, at each step of the time-marching procedure these are moved accordingly and the shape of $\mathcal{S}_W$ is continuously renewed.

## Harmonic balance solution for blade aeroelastic equations

Equations 1, 2 and 3 governing the blade structural dynamics coupled with the aerodynamic forcing terms given by the BEM aerodynamic solution yield the aeroelastic integro–differential model to be integrated. Here, considering a hingeless rotor, the space integration is performed through the Galërkin method using the non-rotating modes of the blade as shape functions. The resulting aeroelastic system consists of a set of nonlinear ordinary differential equations of the type

$$\mathbf{M}(t)\,\ddot{\mathbf{q}} + \mathbf{C}(t)\,\dot{\mathbf{q}} + \mathbf{K}(t)\,\mathbf{q} = \mathbf{f}_{str}^{nl}(t,\mathbf{q}) + \mathbf{f}_{aer}(t,\mathbf{q}) \tag{5}$$

where $\mathbf{q}$ denotes the vector of the Lagrangean coordinates (modal amplitudes), whereas $\mathbf{M},\mathbf{C}$, and $\mathbf{K}$ are linear, time-periodic, mass, damping, and stiffness structural matrices (note that these matrices are time-variant because of the cyclic pitch). Nonlinear structural contributions are collected in the forcing vector $\mathbf{f}_{str}^{nl}(t,\mathbf{q})$, whereas vector $\mathbf{f}_{aer}(t,\mathbf{q})$ collects the aerodynamic loads. Specifically, for the pressure $p$ given by the BEM solution, the generalized forces related to lead-lag, flap, and torsion equations are, respectively, given by

$$f_{aer_j}^{v} = -\int_{\mathcal{S}} p\, n_y \Psi_j \,\mathrm{d}\mathcal{S}; \quad f_{aer_j}^{w} = -\int_{\mathcal{S}} p\, n_z \Psi_j \,\mathrm{d}\mathcal{S}; \quad f_{aer_j}^{\phi} = -\int_{\mathcal{S}} p\,(y\,n_z + z\,n_y)\,\Theta_j \,\mathrm{d}\mathcal{S}$$

where $\Psi_j$ are the bending shape functions and $\Theta_j$ are the torsion shape functions used in the analysis ($y$ is the chordwise coordinate, $z$ is the coordinate parallel to the shaft and $n_y$ and $n_z$ denote components of the unit vector orthogonal to the blade surface).

Since the aim of the work is the prediction of the vibrating hub loads during regime forward flight conditions, the aeroelastic system in Eq. 5 is integrated using the harmonic balance approach. For a periodic forcing term, $\mathbf{f} = \mathbf{f}_{str}^{nl} + \mathbf{f}_{aer}$, it consists of assuming periodic blade deformations, $\mathbf{q}$, given as the solution of the algebraic equations arising from equating the harmonic components of the LHS of Eq. 5 with the harmonic components of its RHS. Specifically, expressing $\mathbf{q}$ and $\mathbf{f}$ in terms of the following Fourier series

$$\mathbf{q}(t)=\mathbf{q}_0+\sum_{n=1}^{N}[\mathbf{q}_n^c\cos(\Omega_n t)+\mathbf{q}_n^s\sin(\Omega_n t)];\;\; \mathbf{f}(t)=\mathbf{f}_0+\sum_{n=1}^{N}[\mathbf{f}_n^c\cos(\Omega_n t)+\mathbf{f}_n^s\sin(\Omega_n t)]$$

where $\Omega_n = n\Omega$, with $\Omega$ representing the rotational speed of the rotor (fundamental frequency of periodicity), and combining with Eq. 5 yields the following periodic aeroelastic solution in terms of the sine and cosine harmonic components

$$\hat{\mathbf{q}} = \left[\hat{\mathbf{M}}+\hat{\mathbf{C}}+\hat{\mathbf{K}}\right]^{-1}\hat{\mathbf{f}} \tag{6}$$

where $\hat{\mathbf{q}}^T = \{\mathbf{q}_0\; \mathbf{q}_1^c\; \mathbf{q}_1^s\; \mathbf{q}_2^c\; \mathbf{q}_2^s\; \cdots\}$, $\hat{\mathbf{f}}^T = \{\mathbf{f}_0\; \mathbf{f}_1^c\; \mathbf{f}_1^s\; \mathbf{f}_2^c\; \mathbf{f}_2^s\; \cdots\}$, and matrices $\hat{\mathbf{M}}, \hat{\mathbf{C}}$ and $\hat{\mathbf{K}}$ are given by the combination of the $\mathbf{q}$–harmonics with those of the $\mathbf{M}, \mathbf{C}$, and $\mathbf{K}$ matrices. If $\mathbf{M}, \mathbf{C}$, and $\mathbf{K}$ were constant, in Eq. 6 one would have

$$\hat{\mathbf{M}}+\hat{\mathbf{C}}+\hat{\mathbf{K}} = \begin{bmatrix} \mathbf{K} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & -\Omega_1^2\mathbf{M}+\mathbf{K} & \Omega_1\mathbf{C} & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & -\Omega_1\mathbf{C} & -\Omega_1^2\mathbf{M}+\mathbf{K} & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\Omega_2^2\mathbf{M}+\mathbf{K} & \Omega_2\mathbf{C} & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\Omega_2\mathbf{C} & -\Omega_2^2\mathbf{M}+\mathbf{K} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \end{bmatrix}$$

and the equations for the $\mathbf{q}$–harmonics would be uncoupled. On the contrary, in the problem under examination the structural matrices are periodic and hence, once expressed in terms of the Fourier series and combined with the harmonics of $\mathbf{q}$, they yield fully-populated $\hat{\mathbf{M}}, \hat{\mathbf{C}}$ and $\hat{\mathbf{K}}$ matrices and the $\mathbf{q}$–harmonic equations in Eq. 6 are coupled each other. This is shown, for instance, by the form of the stiffness matrix that reads

$$\hat{\mathbf{K}} = \begin{bmatrix} \mathbf{K}_0 & \frac{1}{2}\mathbf{K}_1^c & \frac{1}{2}\mathbf{K}_1^s & \frac{1}{2}\mathbf{K}_2^c & \frac{1}{2}\mathbf{K}_2^s & \cdots \\ \mathbf{K}_1^c & \mathbf{K}_0+\frac{1}{2}\mathbf{K}_2^c & \frac{1}{2}\mathbf{K}_2^s & \frac{1}{2}\mathbf{K}_1^c & \frac{1}{2}\mathbf{K}_1^s & \cdots \\ \mathbf{K}_1^s & \frac{1}{2}\mathbf{K}_2^s & \mathbf{K}_0-\frac{1}{2}\mathbf{K}_2^c & -\frac{1}{2}\mathbf{K}_1^s & \frac{1}{2}\mathbf{K}_1^c & \cdots \\ \mathbf{K}_2^c & \frac{1}{2}\mathbf{K}_1^c & -\frac{1}{2}\mathbf{K}_1^s & \mathbf{K}_0+\frac{1}{2}\mathbf{K}_4^c & \frac{1}{2}\mathbf{K}_4^s & \cdots \\ \mathbf{K}_2^s & \frac{1}{2}\mathbf{K}_1^s & \frac{1}{2}\mathbf{K}_1^c & \frac{1}{2}\mathbf{K}_4^s & \mathbf{K}_0-\frac{1}{2}\mathbf{K}_4^c & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \end{bmatrix}$$

(similar expressions for $\hat{\mathbf{M}}$ and $\hat{\mathbf{C}}$ are used) The solution of Eq. 6 is obtained iteratively. Starting from an initial-guess periodic blade deformation, first the corresponding periodic aerodynamic forces are calculated and then their harmonics are used in Eq. 6 to obtain a new blade deformation. In turn, this yields a new set of periodic aerodynamic forces and the procedure is applied until two subsequent iterations give the same blade deformation.

## Numerical results

The main rotor of the ECD BO–105 is considered as a test case, in order to investigate about the impact of the aerodynamic model on the prediction of the vibrating hub loads. This rotor has four blades with radius $R = 4.91$m, constant chord $c = 0.39$m, and a twist angle of $-8^\circ$. The configuration analyzed is a level flight with advance ratio $\mu = 0.3$ and rotational speed $\Omega = 40.4$rad/s.

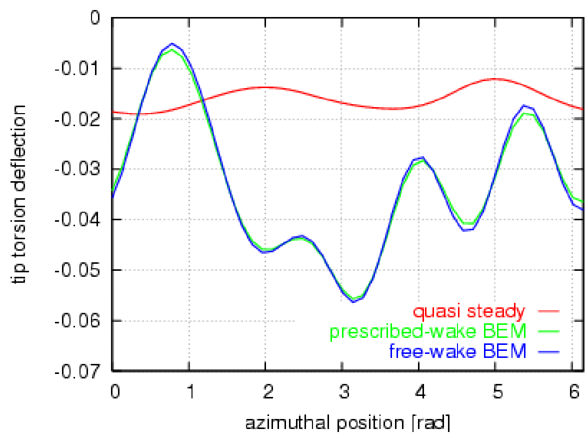


Figure 1: Torsion deflection at blade tip.

The solutions obtained considering three different aerodynamic models in the aeroelastic equations are examined. These are the free-wake BEM model and the prescribed-wake BEM model presented above, with the addition of the widely-used quasi-steady model (with induced velocity from Glauert's model). Figure 1 shows the comparison in terms of the torsion deflection, whereas Fig. 2 compares the $4/rev$ hub loads (forces and moments in a fuselage–fixed frame of reference) computed with the different aerodynamic models (note that the hub is forced by load harmonics that are multiple of the number of the rotor blades). In both comparisons the results from the BEM model differ considerably from those given by the quasi-steady model, whereas the inclusion of wake distorsion produces limited effects in terms of blade deflection. However, the $4/rev$ hub loads are more significantly affected by the wake shape and this is because of the fact that they are given by a combination of the higher harmonics of the blade deflections ($3/rev, 4/rev, 5/rev$).

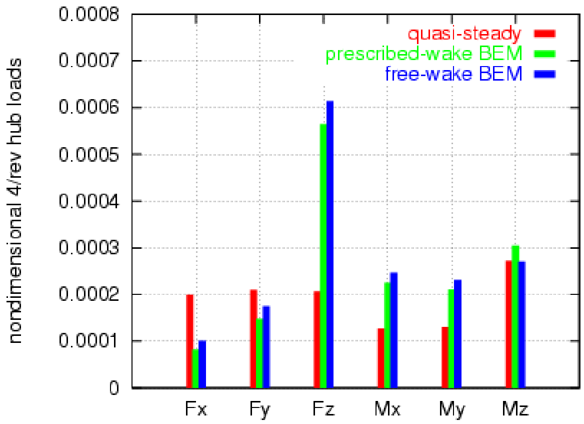


Figure 2: Fuselage–frame $4/rev$ hub loads.

## Conclusions

For helicopter rotors in forward flight, a nonlinear structural model has been coupled with a free-wake BEM aerodynamic solver and a harmonic balance approach is used to integrate the resulting aeroelastic equations. The numerical results given by the aeroelastic model based on BEM aerodynamics differ considerably from those obtained through the widely-used quasi-steady aerodynamic model and this confirms the necessity of using an accurate aerodynamic model for rotor aeroelasticity analysis. In addition, the results have shown that a free-wake algorithm has to be used for an accurate prediction of the high-frequency loads transmitted from the rotor to the hub.

## Reference

1. Hodges, D.H., and Dowell, E.H. (1974): “Nonlinear Equation for the Elastic Bending and Torsion of Twisted nonuniform Rotor Blades”, *NASA TN D-7818*.
2. Morino, L. (1974) “A General Theory of Unsteady Compressible Potential Aerodynamics,” *NASA CR–2464*.
3. Gennaretti, M., Corsetti, E., and Morino, L. (1998) “Coupled Free-Wake-Aerodynamics/Blade-Dynamics Analyses of Rotors in Forward Flight,” AIAA Paper 98-2241, 4th AIAA/CEAS Aeroacoustic Conference, Toulouse, France.
4. Hodges, D.H., and Ormiston R.A. (1976): “Stability of Elastic Bending and Torsion of Uniform Cantilever Rotor Blades in Hover with Variable Structural Coupling”, *NASA TN D-8192*.
5. Morino, L., Kaprielian, Z., Jr., and Sipcic, S.R. (1985) “Free Wake Analysis of Helicopter Rotors,” *Vertica*, Vol. 9, No. 2, pp. 127-140.