Stress Gradient Effects in the Failure Criteria for Textile Composites

Ryan L. Karkkainen* and Bhavani V. Sankar†

Summary

A micromechanical analysis of the representative volume element (RVE) of a plain weave textile composite has been performed using the finite element method. Stress gradient effects are investigated, and it is assumed that the stress state is not uniform across the RVE. The stress state is defined in terms of laminate theory load matrices [N], [M], i.e. applied loads and applied moments. Structural stiffness matrices analogous to the [A], [B], [D] matrices are defined. These are computed directly from the micromechanical models, rather than making estimations based upon the homogeneous Young's modulus and plate thickness. Failure envelopes for a plain-weave textile composite have been constructed using microstresses from finite element analysis of the RVE. Transverse failure of the fiber tow was the dominant mode of initial failure. The DMM failure envelope compared closely in form to the Tsai-Wu failure theory, but was more conservative in some areas.

Introduction

Conventional micromechanical models for textile composites assume that the state of stress is uniform over a distance comparable to the dimensions of the representative volume element (RVE). However, due to complexity of the weave geometry, the size of the RVE in textile composites can be large comparable to structural dimensions. In such cases, severe non-uniformities in the stress state will exist, and conventional models may fail. Such stress gradients also exist when the load is applied over a very small region, as in static contact or foreign object impact loading, and when there are stress concentration effects such as open holes in a structure. Although micromechanical models have been successfully employed in predicting thermo-elastic constants of fiber-reinforced composite materials, their use for strength prediction in multiaxial loading conditions is not practical, as computational analysis must be performed in each loading case. Thus phenomenological failure criteria are still the predominant choice for design in industry. There are three major types of engineering failure criteria for unidirectional composite materials: maximum stress criterion, maximum strain criterion, and quadratic interaction criterion (such as the Tsai-Hill and Tsai-Wu failure theories) [1]. These may also be

* Graduate Research Assistant
† Professor, Mechanical and Aerospace Engineering Department, 231 MAE-A Building / P.O. Box 116250, University of Florida, Gainesville, Florida, 32611-6250, USA

employed in the micromechanical analysis of textile composites, to build a macroscopic failure envelope upon which a textile composite failure criterion may be developed.

Most of the modeling work done thus far have focused on predicting thermo-mechanical properties [2-4]. To facilitate the use of textile composites in lightweight structures, it is required to have a lucid understanding of failure mechanisms, and design engineers must have an accurate and practical model for prediction of failure strength. Most of the current analytical and numerical methodologies developed to characterize textile composites [5-7] assume that the textile is a homogeneous material at the macroscopic scale. A previous study by the authors [8] extended a method, know as the Direct Micromechanics Method [9], to develop failure envelopes for a plain-weave textile composite under plane stress in terms of applied macroscopic stresses. In the current paper, micromechanical finite element analysis is performed to determine the constitutive relations and failure envelope for a plain-weave graphite/epoxy textile composite.

Methods

In the current study, stress gradient effects are investigated, and it is assumed that the stress state is not uniform across the RVE. This represents an extension of the micromechanical models used to predict the strength of textile composites [8-10]. The stress state is defined in terms of the well-known laminate theory load matrices [*N*] and [*M*], i.e. applied force resultants and applied moment resultants. Furthermore, structural stiffness coefficients analogous to the [*A*], [*B*], [*D*] matrices are defined. In this approach, these structural stiffness coefficients are computed directly from the micromechanical models, rather than making estimations based upon the homogeneous Young's modulus and plate thickness. Conventional models essentially neglect the presence of [*M*] terms that result from non-uniformity or gradients in applied force resultants. The additional analysis of the [*M*] term includes information about the distribution, or gradient, of a non-uniform load. These additions can greatly increase the ability of a failure model to accurately predict failure for load cases in which such effects may well be predominant, such as in thin plates, concentrated loading, or impact loading.

A typical weave architecture has been selected and this RVE is detailed in Table 1 and Figure 1. Total fiber volume fraction, given these dimensions, will be 25% (incorporating the fact that the resin-impregnated yarn itself has a fiber volume fraction of 65%).

Table 1: RVE Dimensions

Dimension	***a* , *b***	***c***	***p***	***t***	***w***
Length (mm)	1.68	0.254	0.84	0.066	0.70

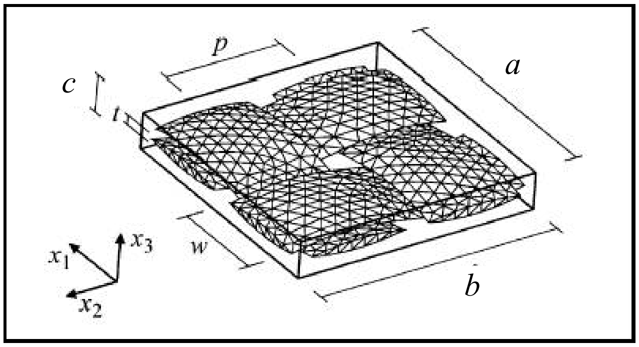

Figure 1: RVE Geometry

To ensure continuity of macrostresses and compatibility of displacements across an RVE, periodic traction and displacement boundary conditions must be employed. Any macroscopically homogeneous deformation can be represented as a prescription of relative displacements on points on opposite faces of the RVE, as in Table2. In addition, traction boundary conditions which enforce equal and opposite traction forces on opposite lateral faces of the RVE are required.

In the Direct Micromechanics Method (DMM), the RVE is subjected to macroscopic force and moment resultants, which are related to macroscopic strain and curvature according to:

$$\begin{Bmatrix} [N] \\ [M] \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} [\varepsilon] \\ [\kappa] \end{Bmatrix} \tag{1}$$

Thus the constitutive matrices must be evaluated to determine this correlation. Once this has been determined, a macroscopic deformation can be applied using an FEM code. In this way, the FEM results for stress in each element yield the microstresses resulting from an applied force or moment resultant.

The RVE is subjected to independent macroscopic unit deformations in order to evaluate the stiffness matrices of Eq 1. In each of the six cases shown in Table 2 below, a single unit strain or a single unit curvature is applied, and all other terms are set to zero, and the appropriate periodic boundary conditions are applied. The four-node linear tetragonal elements in the commercial ABAQUS™ FE software package were used to model the yarn and matrix for all cases. The FEM results for each element yield the microstresses resulting from an applied macro level strain and curvature. The corresponding macro level force and moment resultant in each case can be computed by averaging the microstresses over the entire volume of the RVE:

Table 2: Periodic Displacement Boundary Conditions

		u(a,y)-u(0,y)	*v(a,y)-v(0,y)*	*w(a,y)-w(0,y)*	*u(x,b)-u(x,0)*	*v(x,b)-v(x,0)*	*w(x,b)-w(x,0)*
1.	$\varepsilon_x^M = 1$	*a*	*0*	*0*	*0*	*0*	*0*
2.	$\varepsilon_y^M = 1$	*0*	*0*	*0*	*0*	*b*	*0*
3.	$\gamma_{xy}^M = 1$	*0*	*a/2*	*0*	*b/2*	*0*	*0*
4.	$\kappa_x^M = 1$	*az*	*0*	$-a^2/2$	*0*	*0*	*0*
5.	$\kappa_y^M = 1$	*0*	*0*	*0*	*0*	*bz*	$-b^2/2$
6.	$\kappa_{xy}^M = 1$	*0*	*az/2*	*-ay/2*	*bz/2*	*0*	*-bx/2*

$$N_{ij} = \left(1/ab\right)\sum \sigma_{ij}^e V^e \tag{2}$$

$$M_{ij} = \left(1/ab\right)\sum z\sigma_{ij}^e V^e \tag{3}$$

Thus the six load cases can be evaluated to completely describe the six columns of the [*A*], [*B*], [*D*] matrices. This information having been determined, one is then able to evaluate the microstress field resulting from any general loading case.

The method described above can be used to predict failure strength by comparing the computed microstresses in each element against failure criteria for the constituent yarn and matrix of the textile composite. The microstress state for a general applied force or moment resultant is obtained by superposing multiples of the results from the unit macrostrain analysis.

Failure is checked on an element-by-element basis, and the failure criterion of each element can be selected appropriately based upon whether it is a yarn or matrix element. Initial failure is defined when one of the yarn or matrix elements has failed. This is analogous to a first-ply failure of a laminated composite. For matrix elements, which are isotropic, the maximum principal stress criterion is used to evaluate element failure. The yarn is in essence a unidirectional composite at the micro level, thus the Tsai-Wu quadratic failure criterion is used to determine its failure. The current study considers *x-y* plane stress analysis in terms of *N* and *M*.

Results

The fiber tow was assumed to have material properties of a unidirectional composite, in this case AS/3501 graphite-epoxy. The constitutive matrices relating macroscopic loads to strains and curvatures were found to be:

$$[A] = \begin{bmatrix} 4.14 & 0.52 & 0 \\ 0.52 & 4.14 & 0 \\ 0 & 0 & 0.18 \end{bmatrix} \times 10^6 (Pa - m) \qquad [D] = \begin{bmatrix} 7.70 & 2.53 & 0 \\ 2.53 & 7.70 & 0 \\ 0 & 0 & 1.35 \end{bmatrix} \times 10^{-3} (Pa - m^3)$$

The character of the constitutive matrices is analogous to an orthotropic stiffness matrix with identical elastic constants in the material principal directions. These have been calculated directly from the micromechanics model without any homogenizing assumptions. By comparison to the direct micromechanics results of the DMM, calculation of flexural stiffness via homogenization schemes are assumptions can misrepresent flexural stiffness values D_{11}, D_{12}, and D_{66} by as much as factors of 2.9, 1.1, and 0.7 respectively.

Figure 2 shows the failure envelope for the plain weave graphite/3501 textile composite using the DMM, and a comparison to a Tsai-Wu failure ellipse. In-plane biaxial loading is considered. DMM failure envelopes are shown with no applied moment, and with an applied moment M_x equal to half the critical value that would cause failure if it were the only applied load. There is no Tsai-Wu failure envelope to include applied moment resultants, as the theory is not developed to include such load types.

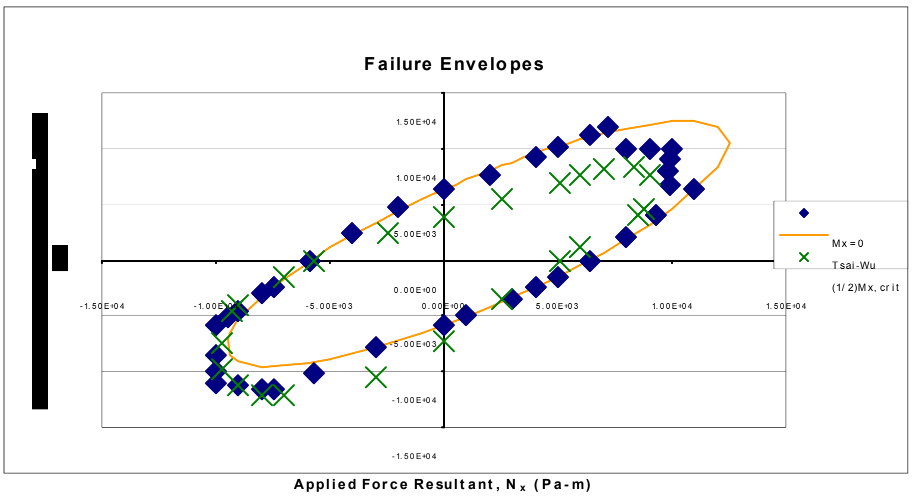

Figure 2: Comparison of DMM Failure Envelopes to a Tsai-Wu Failure Ellipse

Although the strength is less than the pure tow strength, the woven tow is not completely aligned in the loading directions. Some is curved into the thickness direction, providing through-thickness reinforcement. Furthermore, after initial transverse failure of the fiber tow (indicative of the introduction of intra-tow microcracking), the structure will still maintain load-bearing capacity, though stress concentrations will begin to build up and part integrity will be degraded.

Without applied moment, the DMM failure envelope follows closely with the form of a Tsai-Wu failure ellipse. Generally, the initial failure mode is transverse failure of the fiber tows. However, at the extremes of the major axis of the failure envelope, the initial failure mode transitions to failure of the matrix material. Thus the DMM failure envelope is cut short at the ends (compared to the failure envelope that would exist if matrix failure were not considered) and is squared-off in these regions in the form of the maximum failure stress criterion.

An applied moment in the x-direction has the effect of shrinking the failure envelope in regions where tensile applied loads dominate. However, when only compressive loads are applied, an applied moment can actually increase the in-plane load capacity by offsetting some of the compressive stress with bending-induced tension. As with the case of pure in-plane loading, the failure envelope at the outer corner of quadrant III is dominated by matrix failure. The effects of applied moment on the failure envelope of the plain weave textile represents the importance of the consideration of stress gradients, or load non-uniformities. The appreciable difference that arises suggests that such consideration could be critical to the successful design or optimization of a textile structural component.

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