Characterization of Internal State Variable for fiber fracture in UD Composite

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Summary

The continuum damage mechanics is used to describe the constitutive behavior of UD laminate with damage. The progressive fiber fracture, as it is affected by interface strength, debonding growth and matrix cracking, is considered as a main damage mechanism causing the stiffness reduction. The internal state variable that accounts for fiber fracture is formulated within the used theory. The methodology is proposed to measure the values of the internal state variable experimentally. The FEM analysis and Monte Carlo simulations are used in order to predict the internal state variable according to the outlined theoretical formulation. The predicted values of internal state variable are compared with experimentally measured.

Introduction

Typical for reinforced composites are inelastic effects. The inelasticity is commonly associated with viscoelasticity, viscoplasticity and damage evolution. The damage evolution is a complicated phenomenon to account for and to characterize. The continuum damage mechanics is a promising approach to develop thermodynamically consistent formulation of constitutive law for media with damage. This would account for effects of temperature changes (non-isothermal conditions) and environmental (moisture) conditions. This is because the additional internal state variables (ISV) that would account for considered phenomenon can be included. The thermodynamically consistent damage dependent lamination theory, formulated by D.H. Allen [1,2], is used to describe inelastic response to the applied load for a long fiber composite laminates at reference conditions (isothermal environmental conditions). In this study, the internal state variable that accounts for damage of UD composite in fiber direction is formulated within the terms of the applied theory, and the function that describe the corresponding internal state variable is determined and analyzed theoretically and experimentally. The stiffness degradation is associated with corresponding damage evolution providing the tool for experimental determination of the ISV accounting for considered damage. FEM analysis and Monte Carlo simulations are used to calculate considered ISV using formulated micromechanics.

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General constitutive relationships for media with damage

The thermodynamically consistent constitutive relationships for media with cracks have been developed by [1,2] and will be used herein

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} + I^{\xi}_{ijkl}\alpha^{\xi}_{kl}, \tag{1}$$

where I^{ξ}_{ijkl} is damage dependent modulus, and $\xi = 1,...,N$ accounts for different damage modes. The locally averaged internal state variables associated with energy dissipation due to the cracking is defined as first proposed by [3] and utilized by [1,2] as

$$\alpha_{kl} = \frac{1}{V}\int_{S_2} u_k n_l dS_2 \tag{2}$$

where α_{kl} are components of internal state variable tensor, V is a local volume in which statistical homogeneity can be assumed, u_k and n_l are crack face displacement and normal respectively, S_2 is a crack surface area. (3)

Equations for the laminate with damage

The constitutive relationships for general laminate with damage can be obtained from (1) and written in the matrix form

$$\{N\} = [A]\{\varepsilon\} + [B]\{k\} + \{f^{(1)}\}, \tag{4}$$

where $[A]$ and $[B]$ are extensional stiffness matrix, and the coupling stiffness matrix of the laminate. The effect of the fiber fracture that takes place due to the loading in fiber direction is described by $\{f^{(1)}\} = \sum_{k=1}^{n}\left[I^{(1)}\right]_k (z_k - z_{k-1})\{\alpha^{(1)}\}_k$, where $\left[I^{(1)}\right]_k$ is a damage stiffness tensor of k^{th} layer, and $\{\alpha^{(1)}\}_k$ is internal state variable accounting for fiber fracture damage. Generally, $\{\alpha^{(1)}\}_k$ can be determined experimentally, or calculated using micromechanics approach. The measurements of stiffness degradation of UD laminate must be used in order to determine $\{\alpha^{(1)}\}_k$ experimentally.

Determination of damage tensor, $\{\alpha^{(1)}\}_k$

The progressive fiber fracture, combined with debonding and matrix cracking at higher applied strain levels, can be described by representative unit element as illustrated in Figure 1. The considered unit element can account for the debonding effects and matrix cracking effects on the crack opening displacement.

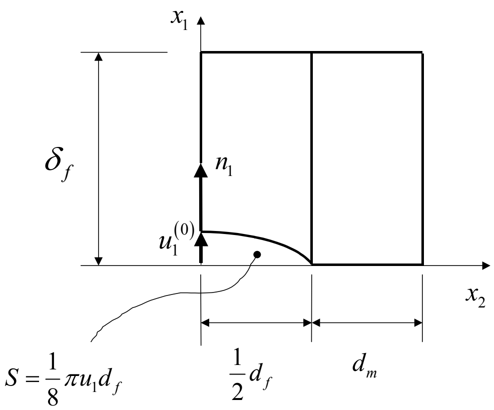

Figure 1. Representative volume element. δ_f is ineffective length, d_f is diameter of the fiber, d_m is dimensions of the surrounding matrix, and S is ¼ of the area of the ellipse under the crack profile line . The dimensions are calculated based on the fiber packing in hexagonal array.

The general definition of internal state variable α_{kl}, expression (2), can be formulated for considered damage as

$$\alpha_{kl}^{(1)} = \frac{1}{V}\sum_{m=1}^{M}\int_{S_2^{(0)}} u_k n_l ds_2^{(0)} , \tag{5}$$

where $S_2^{(0)}$ is crack surface area of single crack. If the crack, see Figure 1, does not change the orientation the normal to crack surface have only one component, $n_1 = 1$, $n_2 = n_3 = 0$. It leads to the $\alpha_{kl}^{(1)} = \left\{\alpha_{11}^{(1)} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \alpha_{31}^{(1)} \quad 0 \quad \alpha_{21}^{(1)}\right\}^T$, where the $\alpha_{11}^{(1)}$ accounts for crack opening in Mode-I, and according to (5), can be expressed in form of crack opening displacement

$$\alpha_{11}^{(1)} = \frac{1}{V}\sum_{m=1}^{M}\int_{S_2^{(0)}} u_1 dS_2^{(0)} = \frac{1}{V}\sum_{m=1}^{M}\int_{x_2} u_1 dx_2 . \tag{6}$$

Considering the finite size of the representative volume, $V = dx_1 dx_2 dx_3$, and two dimensional case with unit thickness, $dx_3 = 1$

$$\alpha_{11}^{(1)} = \frac{m}{k\delta_f d_f \dfrac{1}{\sqrt{V_f}}}\int_{x_2} u_1 dx_2 , \tag{7}$$

where $dx_1 = \delta_f$ is ineffective length, and $dx_2 = kd_f \frac{1}{\sqrt{V_f}}$, d_f is diameter of the fiber, V_f is fiber volume fraction, and $k = 0.8244$ is a generally proportionality coefficient. Both, dx_2 and k, are calculated on bases of the hexagonal fiber packing. The integral part in the (7) give the area under the crack surface profile, $S = \int_{x_2} u_1 dx_2 = \frac{1}{8}\pi u_1^{(0)} d_f$, where the, $u_1^{(0)}$ is the crack opening displacement at $x_2 = 0$, see Figure 1. Than the (7) can be rewritten as

$$\alpha_{11}^{(1)} = \frac{1}{8}\frac{\pi u_1^{(0)} m\sqrt{V_f}}{k\delta_f},\ k = 0.8244\,, \tag{8}$$

where $u_1^{(0)}$ is generally function of the applied strain, geometry and stiffness ration of fiber and matrix. The number of crack, m are counted per ineffective length, δ_f, and naturally will have limits $0 \le m \le 1$. The maximum crack density can be calculated by using fact that one crack per ineffective length is possible, $\rho = \frac{m}{\delta_f} = \rho_{\max}\big|_{m=1} = \frac{1}{\delta_f}$.

Both, ineffective length, δ_f and number of cracks per unit fiber length as a function of applied strain, can be obtained experimentally from Single Fiber Fragmentation Test (SFFT). However, The Weibull statistics for the fiber can be used to generate the crack density function, and in this case, FEM can be used to determine ineffective length. Also analytical expressions could be used to calculate δ_f. This approach will lead to overestimated $\alpha_{11}^{(1)}$ for large applied strain levels. It is because practically the crack density in composite would be expected somewhat less than theoretical, $\rho_{\max} = 1/\delta_f$. The stochastic methods, or Monte-Carlo simulations could give more realistic crack density function for the composite.

Experimental considerations

Let's consider the unidirectional laminate with fiber orientation in loading direction, subjected to static loading, $\{N\} = \{N_x \quad 0 \quad 0\}^T$. There is only one type of damage expected, and it can be described with the internal state variable $\alpha^{(1)}$. Further, the constitutive relationships for the considered laminate can be written in a compact matrix form as $\{N\} = [A]\{\varepsilon\} + [I^{(1)}]\{\alpha^{(1)}\}h$. Considering loading conditions, $\{N\} = \{N_1 \quad 0 \quad 0\}^T$, and using $-C_{ijkl} = I_{ijkl}$ [1,2], and $A_{ij} = C_{ij}h$, it can be reduced to, $N_1 = A_{11}\varepsilon_1 + A_{12}\varepsilon_2 + C_{11}\alpha_{11}^{(1)}h$. Further, introducing the definition of the stiffness, $E_1(\varepsilon) \equiv \frac{1}{h}\frac{\partial N_1}{\partial \varepsilon_1}$, and using the expression for N_1 in it, the relationship for $\alpha_{11}^{(1)}$ as function of measured stiffness for particular applied strain level is obtained,

$$\alpha_{11}^{(1)}(\varepsilon_1)=\frac{h\int_{\varepsilon_1} E_1(\varepsilon_1)\,d\varepsilon_1+\frac{A_{12}A_{21}}{A_{22}}\varepsilon_1-A_{11}\varepsilon_1}{\left(\frac{A_{12}A_{21}}{A_{22}}-A_{11}\right)}. \tag{9}$$

Results and discussion

The stiffness degradation of UD laminate, $[0_4]_T$, is measured experimentally using quasi-static tensile loading-unloading test. Reference material, and the same preconditioned (exposed to salt water for 6 month) have been tested. The whole loading-unloading loop of the i^{th} loading-unloading cycle is used to calculate the Young's modulus, $E_1(\varepsilon_1)$ for corresponding applied strain. It was observed, that the stiffness degradation measured is consistent for all individual specimens.

The measured stiffness degradation is normalized, E_i/E_0 and described by a second order polynomial, see Figure 2.

Further the obtained function for E_i/E_0 can be used into (9) in order to calculate $\alpha_{11}^{(1)}(\varepsilon_1)$. The experimentally determined $\alpha_{11}^{(1)}(\varepsilon_1)$ reflects the total effect of the multiple fracture mechanism, and it would be difficult to separate them. The micromechanics approach is used (8) to calculate $\alpha_{11}^{(1)}(\varepsilon_1)$ accounting for fiber cracking and crack density assumptions. The FEM is used for calculating the crack opening displacement as function of applied strain, $u_1^{(0)}$ in (8). The plane strain conditions with unit thickness have been considered. The debonding crack and the crack growth into the matrix, are not allowed in this study. It is still to be analyzed whether the same ISV should be used, or a new one should be formulated in order to account for both. The obtained value of $\delta_f \approx 125(\mu m)$ is rather close to what can be calculated according to Rosen. The Monte Carlo simulations, utilizing weakest link assumptions and stress global shearing mechanism, are used to produce the crack density function. The experimentally determined and theoretically calculated values of $\alpha_{11}^{(1)}(\varepsilon_1)$ are given in Figure 2. The theoretically predicted $\alpha_{11}^{(1)}(\varepsilon_1)$ are lover than experimental up till applied strain $\varepsilon_1 \approx 1.25(\%)$. It is due to the fact that the debonding is not considered. Also predictions dive higher values for higher applied strain levels. It is due to use of weakest link and global stress shearing assumptions as well as using the method of chain of bundles in Monte Carlo simulations. It leads to artificially increased number of cracks. The local stress shearing mechanism gives more realistic description of mechanism, and will be implemented in future. As well as the use of conditions of formation clusters of multiple fiber breaks would reduce the predicted number of cracks.

In order to validate the outlined approach and obtained $\alpha_{11}^{(1)}(\varepsilon_1)$, the strain-stress curve of independent tensile test of UD laminates, $[0_4]_T$, can be predicted by using the obtained function of $\alpha_{11}^{(1)}(\varepsilon_1)$ into the constitutive equations for laminate, (4), and predictions are in rather good agreement with experimental data.

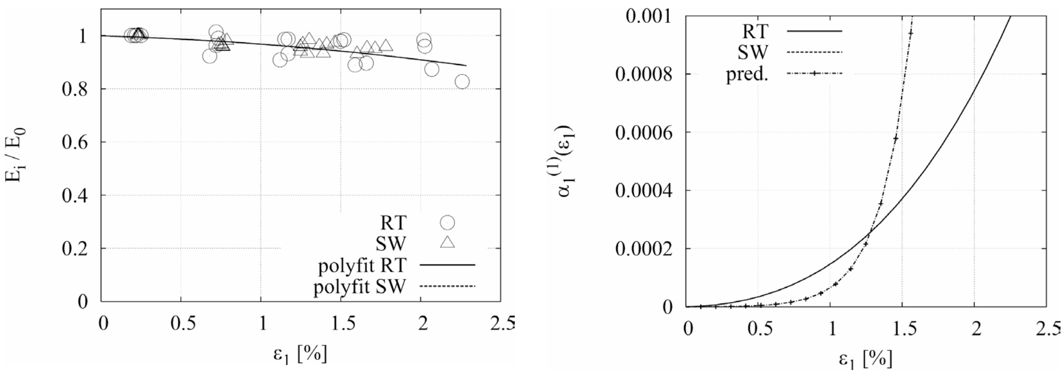

Figure 2: Stiffness degradation of UD composite, and Calculated ISV $\alpha_{11}^{(1)}(\varepsilon_1)$*. The second order polynomial,* $y = a_2 x^2 + a_1 x + a_0$ *is used for approximation. The calculated constants are,* $a_0 = 1.0$, $a_2 = -1.76$ *and* $a_2 = -136.9$.

Conclusions

The ISV that characterizes the fiber fracture is formulated within the used continuum damage mechanics approach. The methodology is proposed how to measure the considered ISV experimentally. The FEM analysis and Monte Carlo simulations are used to calculate the ISV using proposed micromechanics formulation. Theoretically calculated and experimentally measured values of the ISV are compared, and both are used within the constitutive expressions for laminate to predict strain-stress behavior.

Acknowledgment

Part of the work is done within frames of the EU 5th frame program, OPTIMAT BLADES, Contract Nr. ENK 6-CT-2001-00552. It is also a subject of the Nordic Co-operation Network, supported by Nordic Council of Ministers. Contract j.nr. 2003-8003-23-Modris Megnis, DK.

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