Three Dimensional Coupled Analysis of Independently Modeled Substructures by Displacement Welding Technique

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Summary

In this paper, three dimensional displacement welding technique is proposed to carry out coupled analysis of the integrated whole model which is composed of independently modeled finite element substructures. In the proposed method, the incompatible displacement fields in the mismatching interfaces of independently modeled substructures are directly welded together by using a blended function made from moving least squares approximation. To investigate the validity of the proposed method, the patch test and convergence test are carried out. As a practical application, example of three dimensional coupled analysis of independently modeled substructures is presented.

Introduction

Large structural systems such as airplanes, spacecrafts, automobiles and others, are usually modeled in the form of substructures, and the substructures or subparts are modeled independently by several engineers. Therefore generally the independently modeled finite element substructures do not satisfy the nodal compatibility in the interfaces of each substructure, even though the nodal compatibility in the interface region is necessarily required for coupled finite element analysis of the whole integrated model. Due to the reason, considerable attention has been given to the coupled analysis of independently modeled finite element substructures during the past decade [1-3].

Of course, one easiest remedy may be the design of transition or interface elements to satisfy the nodal compatibility in the interface region of independently modeled substructures. However, this approach is somewhat cumbersome to apply and may need expensive human labor in practical situations because there are various cases and combinations to be considered. Therefore, different approaches have been explored to solve the trouble. Of these, one major approach is the introduction of Lagrange multiplier in the interface region to satisfy both of the displacement compatibility and force equilibrium in variational sense[3]. The approach is more flexible to handle the independently modeled finite element substructures than the transition element, and in mathematical sense the nodal compatibility is no longer required in the interface region.

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However this Lagrange multiplier approach necessarily requires the additional unknowns, and does not preserve the positive definiteness and banded structure of stiffness matrix of the whole coupled system. Also it frequently needs some additional interface meshes to appropriately enforce the constraint conditions of displacement compatibility.

Therefore, in this work a novel approach, where the incompatible displacement fields of independently modeled substructures are directly welded together, is proposed to efficiently carry out the coupled analysis of independently modeled finite element substructures without introducing any additional unknowns and interface mesh. Also, in the proposed method the positive definiteness and banded structures of the system stiffness matrix are no longer sacrificed because displacement fields of the independently modeled finite element substructures are directly welded into a compatible displacement field for whole coupled model. In displacement welding, moving least squares method [4-6], which does not require any fixed nodal connectivity, is utilized along with the original finite element shape functions.

Displacement Welding in Mismatching Interfaces

In this section, a novel displacement welding procedure is presented. Let us consider independently modeled substructures which do not satisfy the nodal compatibilities in the interfaces of each substructure as shown in Fig. 1.

If the substructures are assembled with no special treatment, then the displacement field at the interface of each substructure becomes discontinuous, and as a result coupled analysis of the whole model cannot be performed successfully. To solve the trouble, the following form of blended function, which was adopted to couple the meshless and finite element systems in the previous works [7, 8], is utilized at the interface region in this work.

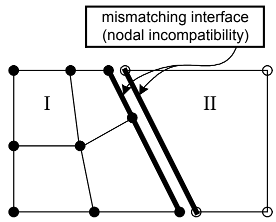


Fig. 1 Independently modeled substructures and mismatching interface

$$\Theta_i^{\mathrm{b}}(\mathbf{x}) = (1-\lambda(\mathbf{x}))N_i(\mathbf{x}) + \lambda(\mathbf{x})\psi_i(\mathbf{x}) \tag{1}$$

where $N_i(\mathbf{x})$ is the finite element shape function, and $\psi_i(\mathbf{x})$ is the moving least squares nodal shape function. It is noted that the function $(1-\lambda(\mathbf{x}))$ should be selected to be "0" at the mismatching interface in order to eliminate the discontinuity of finite element shape function at the interface. In the present work, the value of $\lambda(\mathbf{x})$ in each finite element region is obtained through the following relation.

$$\lambda(\mathbf{x}) = \sum_{i\in\Gamma_{\mathrm{int}}} N_i(\mathbf{x}) \tag{2}$$

where Γ_{int} denotes the mismatching interface. Additionally, one can show that the blended function (1) satisfies the consistency condition as follows.

0th order consistency of blended function

$$\begin{aligned}&\sum_i \Theta_i^b(\mathbf{x}) = \sum_i (1-\lambda(\mathbf{x}))N_i(\mathbf{x}) + \sum_i \lambda(\mathbf{x})\psi_i(\mathbf{x})\\&= (1-\lambda(\mathbf{x}))\sum_i N_i(\mathbf{x}) + \lambda(\mathbf{x})\sum_i \psi_i(\mathbf{x}) = (1-\lambda(\mathbf{x})) + \lambda(\mathbf{x}) = 1\end{aligned} \tag{3}$$

1st order consistency of blended function

$$\begin{aligned}&\sum_i \Theta_i^b(\mathbf{x})x_i = \sum_i (1-\lambda(\mathbf{x}))N_i(\mathbf{x})x_i + \sum_i \lambda(\mathbf{x})\psi_i(\mathbf{x})x_i\\&= (1-\lambda(\mathbf{x}))\sum_i N_i(\mathbf{x})x_i + \lambda(\mathbf{x})\sum_i \psi_i(\mathbf{x})x_i = (1-\lambda(\mathbf{x}))x + \lambda(\mathbf{x})x = x\end{aligned} \tag{4}$$

$$\begin{aligned}&\sum_i \Theta_i^b(\mathbf{x})y_i = \sum_i (1-\lambda(\mathbf{x}))N_i(\mathbf{x})y_i + \sum_i \lambda(\mathbf{x})\psi_i(\mathbf{x})y_i\\&= (1-\lambda(\mathbf{x}))\sum_i N_i(\mathbf{x})y_i + \lambda(\mathbf{x})\sum_i \psi_i(\mathbf{x})y_i = (1-\lambda(\mathbf{x}))y + \lambda(\mathbf{x})y = y\end{aligned} \tag{5}$$

It is noted that the consistencies of finite element shape function $N_i(\mathbf{x})$ and moving least squares nodal shape function $\psi_i(\mathbf{x})$ are used for the above derivation. And the newly introduced blended function is used to weld the displacement field in the interface region.

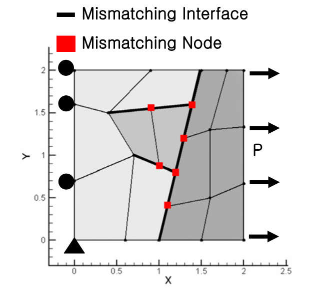
$$\mathbf{u}(\mathbf{x}) \cong \begin{Bmatrix} \sum_i \Theta_i^b(\mathbf{x})u_i \\ \sum_i \Theta_i^b(\mathbf{x})v_i \end{Bmatrix} \tag{6}$$

Since the blended function is rational function[5,9], numerical integration should be carried out carefully. In this work, a higher order Gaussian quadrature rule will be adopted to integrate the rational function appropriately. Again it is noted that the proposed welding procedure does not require any additional unknown and interface mesh, and preserves the positive definiteness and banded structure of the system stiffness matrix unlike the Lagrange multiplier approaches, since the incompatible displacement fields in the interface region of independently modeled substructures are directly transformed into a compatible displacement field over the whole model through the newly defined blended function.

Numerical Examples

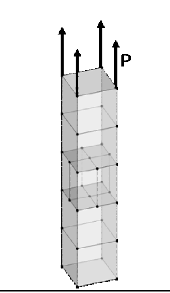
In this section, patch test and convergence test are carried out to verify the validity of the proposed method, and a practical example in three dimension is worked out for future practical application.

At first, patch tests are performed in two and three dimensions, and the results are presented in Fig. 2 and Fig. 3. From the results of Fig. 2, it is observed that constant strain field is reproduced in two dimension if one uses sufficient Gaussian integration order. Likewise, it is confirmed from the results of Fig. 3 that three dimension patch test is also passed successfully. In Fig. 3, Gaussian integration order of 16 is adopted for sufficiently accurate numerical integration.



Displacement	*u*	*v*
Exact	0.6	-0.18
2nd order integ.	0.61289027104	-0.1816811702
4th order integ.	0.60000209594	-0.1800195193
8th order integ.	0.59999398587	-0.1799923851
16th order integ.	0.59999512494	-0.1799946816
32 th order integ.	0.59999515627	-0.1799946985

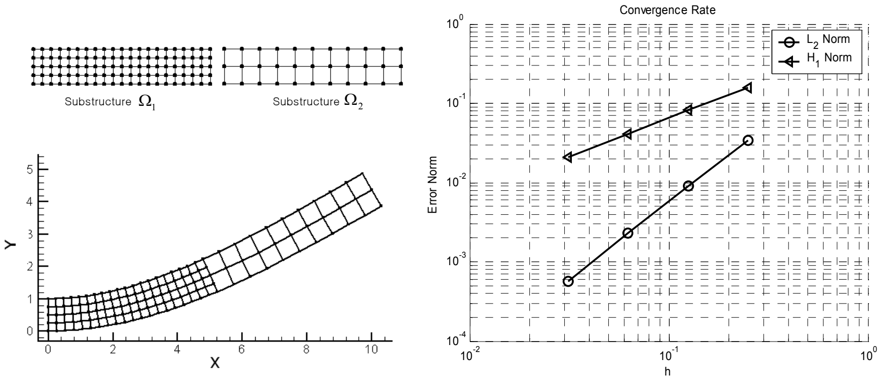
Fig. 2 Patch test model in two dimension and the test results



z-dir. coord.	2	4	6	8	10
z-dir. disp. (Exact)	2	4	6	8	10
z-dir. disp. (Num.) using 16th order integ.	2	4	6	8	10

Fig. 3 Patch test model in three dimension and the test results

Secondly, convergence test is carried out for the beam type problem of Fig. 4(a) to investigate the performance of the proposed method, and the results are presented in Fig. 4(b). From the results, it is identified that the rate of convergence in coupled analysis by using the proposed method is nearly same as that of the finite element method. Finally, as shown in Fig. 5, coupled analysis in three dimension is worked out for the structural system composed of three independently modeled substructures in order to verify the usefulness of the proposed method in practical design and analysis procedures. The analysis result clearly shows the practical usefulness of the proposed displacement welding technique.



(a) (b)

Fig. 4 Convergence in coupled analysis of beam type problem

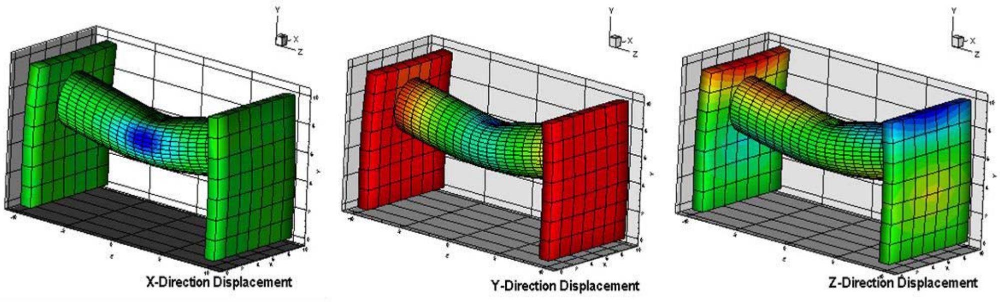


Fig. 5 Three dimension coupled analysis of structural system composed of three independently modeled substructures

Conclusions

In this work, a novel displacement welding technique is proposed for coupled analysis of independently modeled finite element substructures. In this method, the discontinuity of displacement field at the mismatching interface is eliminated by a new blended function, while preserving the consistency of the displacement field. The blended function is constructed by combining the moving least squares nodal shape function and the original finite element shape function. And the meshless character of the moving least squares nodal shape function makes it possible to weld the incompatible displacement fields easily without any remeshing job or special treatment. Moreover, the proposed welding technique does not introduce any additional unknown, and preserves the favorable positive-definiteness and banded structure of the system stiffness matrix unlike the Lagrange multiplier approaches.

Patch and convergence tests are carried out, and practical example is worked out to identify the validity and usefulness of the proposed method. From the numerical tests, it is identified that the acceptable results can be obtained both in two and three dimensional cases by using the proposed welding technique. Based on the observation, it is confirmed that the proposed technique can be efficiently utilized for coupled analysis of independently modeled finite element substructures in practical 3D design and analysis procedures.

Acknowledgements

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