Wave scattering from a wavy interface between two anisotropic media

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Summary

The propagation of ultrasonic waves in a thick anisotropic plate with two layers is investigated in the two-dimensional case. The interface between the layers is assumed to be periodic. The surfaces of the plate are traction-free except where an ultrasonic probe is attached and emits waves into the plate. Two different methods have been used to solve the wave propagation problem. In order to obtain a solution which is essentially exact, the null field approach is used. For a slightly wavy surface the problem may be solved by an alternative method, where the exact boundary conditions on the interface are replaced by approximate boundary conditions on a flat reference surface. Some numerical results show the influence of the anisotropy and the wavy interface, as well as the difference between results obtained by the two methods.

Introduction

Ultrasonic nondestructive testing is a widely used method in the nuclear power industry. Even though the method has been in use for a long time and may be regarded as well-established, there is still a need for a good mathematical model of the testing situation. Such a model can be used for interpretation of test results, planning of tests, and for qualification of testing procedures.

One situation of importance in the nuclear power industry is that of a plate or pipe with a cladding, usually for corrosion protection. One common method is to apply an austenitic cladding on a ferritic base material by a welding process. From an ultrasonic testing point of view this leads to at least two difficulties. The austenitic cladding is anisotropic with the complications this lead to. Furthermore, the interface between the cladding and the base is usually corrugated, and this can lead to strong effects on the ultrasonic properties.

The purpose of this work is to model the propagation of ultrasound in a clad component taking both the anisotropy and the interface corrugation into account. The interface is taken to be sinusoidal, both because this is realistic and because it simplifies the analysis. Periodic interfaces are treated extensively in the literature, particularly in electromagnetism but also for elastic waves, see, *e.g.*, Fokkema [1], Boström [2], and Roberts et al. [3]. However, it seems that the combination of a periodic interface and an anisotropic medium has not been investigated previously.

To solve the wave propagation problem in the presence of a periodic interface two different methods are employed; the null field approach and an approximate

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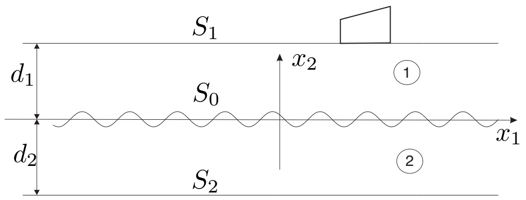


Figure 1: The 2D geometry with an ultrasonic transmitter on the free surface.

method based on a series expansion of the boundary conditions. The null field approach has previously been used by Boström [2] to treat elastic wave propagation and scattering problems for periodic surfaces.

Problem formulation

Consider a thick anisotropic plate with two layers separated by a periodic interface, $x_2 = s(x_1)$ as shown in Fig. 1. The x_1x_2 plane is a plane of elastic symmetry for both materials, which means that a two-dimensional wave propagation problem is possible. It should be noted, however, that the x_1 and x_2 axes do not need to be crystal axes of the materials. Only time harmonic conditions are considered.

The free surfaces of the plate are assumed to be traction-free except for the fact that an ultrasonic transmitter is placed on the upper surface. The transmitter is modelled by the traction exerted on the plate. The two layers are in perfect contact, so that the displacement u_j and the traction $t_j = \sigma_{ij}n_i$ are both continuous across the interface, *i.e.*,

$$u_j^1(x_1, s(x_1)) = u_j^2(x_1, s(x_1)), \tag{1}$$

$$t_j^1(x_1, s(x_1)) = t_j^2(x_1, s(x_1)). \tag{2}$$

Finally, to fully specify the wave propagation problem all waves must be outgoing at infinity, which means that the group velocity must be outgoing.

Exact solution, the null field approach

When employing the null field approach the starting point is an integral representation instead of the differential equation, *i.e.*,

$$(-1)^i \int_{S_i - S_0} \left[\Sigma^i_{mjl}(\mathbf{x};\mathbf{x}')u_j(\mathbf{x}) - G^i_{jl}(\mathbf{x};\mathbf{x}')\sigma_{mj}(\mathbf{x})\right] n_m \mathrm{d}S = \begin{cases} u_l(\mathbf{x}'), & \text{inside material } i, \\ 0, & \text{outside material } i, \end{cases} \tag{3}$$

where $G^i_{jj'}(\mathbf{x};\mathbf{x}')$ is the Green's tensor of material i, and $\Sigma^i_{mjl}(\mathbf{x};\mathbf{x}')$ is the Green's stress triadic. The Green's tensor in material i is chosen as the half-space Green's tensor which satisfies traction-free boundary conditions on S_i. Using also the traction-free boundary conditions on S_i for the fields, the integrals over S_1 and S_2 disappear in Eq. (3) except that the integral over S_1 yields the incoming field u_j^{in}, which thus appears as a term of its own. The half-space ($x_2 < d_1$ and $x_2 > -d_2$) Green's tensors for materials 1 and 2, respectively, can be expressed as the free space Green's tensor, and a second term that is added to satisfy the condition of vanishing traction on the free surface. Both tensors can be expanded as Fourier transforms in x_1. The explicit expressions are given by Boström et al. [5].

To proceed the total field in material 1 is split into an incident and a scattered part, $u_l^1 = u_l^{\text{in}} + u_l^{\text{sc1}}$ The displacement fields, u_l^{in}, u_l^{sc1} and u_l^2 can also be expanded in Fourier transforms in x_1. Inserting into Eq. (3) yields four equations with u_l^{sc1}, u_l^2, and the displacement and traction on the interface as unknowns. Expanding the fields at the interface in a suitable way, taking the inverse Fourier transform and exploiting the periodicity of the surface leads to a set of simultaneous equations for the unknown Fourier coefficients. The elements of the coefficient matrix are all integrals over one period of the interface. In the case of a sinusoidal surface,

$$s(x_1) = b \sin \frac{2\pi x_1}{a}, \tag{4}$$

it is even possible to carry out the integrations analytically. It is then straightforward to determine the scattered fields.

Approximate solution

The exact boundary conditions on the surface $x_2 = s(x_1)$, see Eqs. (1)–(2), can be replaced by approximate conditions on the surface $x_2 = 0$. If the displacement and the traction are expanded in Taylor series, second order terms and higher can be neglected, provided that the slope of the interface is small, *i.e.*, $|s'(x_1)| \ll 1$, and that the height $|s(x_1)|$ is small compared to the wavelength. The result is

$$u_j^1 + s(x_1)\frac{\partial u_j^1}{\partial x_2} = u_j^2 + s(x_1)\frac{\partial u_j^2}{\partial x_2}, \tag{5}$$

$$-s'(x_1)\sigma_{1j}^1 + \sigma_{2j}^1 + s(x_1)\frac{\partial \sigma_{2j}^1}{\partial x_2} = -s'(x_1)\sigma_{1j}^2 + \sigma_{2j}^2 + s(x_1)\frac{\partial \sigma_{2j}^2}{\partial x_2}. \tag{6}$$

Expanding the displacement fields as Fourier transforms in x_1, inserting into Eqs. (5)–(6), specializing to the case of a sinusoidal surface and taking the inverse Fourier transform with respect to x_1 yields after some lengthy but straightforward algebra a set of simultaneous equations for the Fourier coefficients. No explicit expressions are given here, but it should be mentioned that the coefficient matrix is a band matrix, which makes the problem well suited for numerical computation. Once the Fourier coefficients have been solved for, the displacement field can easily be calculated in the same way as for the exact method.

Numerical results and conclusions

In this section a few numerical results are given that illustrate the effects of the anisotropy and, in particular, the corrugated interface. Only parameters related to the corrugation, *i.e.*, a and b, are varied. The upper part of the plate is chosen as an isotropic steel, while the weld material in the lower part is taken as transversely isotropic. The crystal axes are rotated 50° counterclockwise from the x_1-axis.

To excite ultrasonic waves an angled 1 MHz SV probe is applied to the upper surface at $x_1 = 0$. The absolute value of the real part of the displacement vector is plotted in the relevant part of the plate. A linear grey scale is used with black as the largest amplitude and white as practically zero amplitude. The scale is the same in all the plots.

One problem that arises is that the fields calculated by the null field method are only valid above the top and below the bottom of the corrugations. For small corrugations, however, it is possible to use the expansion all the way down to the interface as will be seen in the following. This is in accordance with the Rayleigh hypothesis, see, *e.g.*, van den Berg and Fokkema [4]. For larger corrugations the region inside the corrugations is left blank when the corrugations are not small. The approximate solution, on the other hand, should be valid all the way down to the flat surface as long as the approximations are justified.

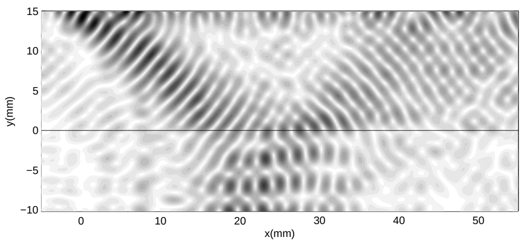
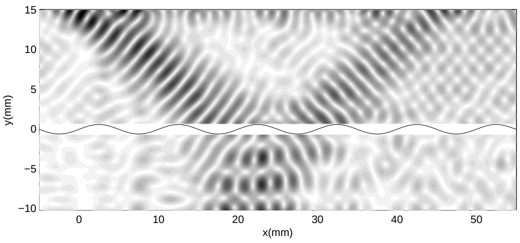


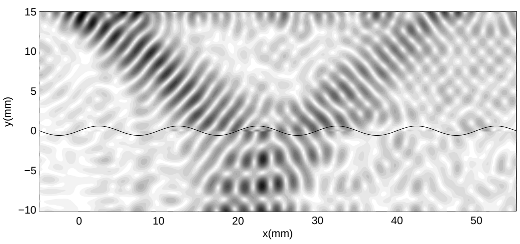
Figure 2: The field in the plate with a flat interface.

Figure 2 shows the field in the plate for a flat interface. The direct field from the probe is a shear wave at the angle 45°. The Rayleigh wave along the surface is also clearly seen. In the lower anisotropic part the wave is more or less propagating vertically, giving a standing wave due to the total reflection from the lower plate surface. The reflected field in the upper part is at 45° as expected. Both methods give, of course, identical results.

Figure 3 shows the field for $a = 10$ mm and $b = 0.6$ mm calculated using both methods. The field is plotted also inside the corrugations. According to the Rayleigh hypothesis the solution should be valid everywhere for $b/a < 0.072$ for the sinusoidal profile. Obviously, the reflected beam is more concentrated than in the case of a flat interface. It is also seen that the results obtained by the approximate



(a) Exact solution



(b) Approximate solution

Figure 3: The field in the plate with a corrugated interface with $a = 10$ mm, $b = 0.6$ mm.

method cannot easily be distinguished from the exact solution.

Figure 4 shows the same case as Fig. 3 except for the fact that $a = 5$ mm and $b = 1.2$ mm. In particular, the field distribution in the lower anisotropic medium is completely altered. It is also obvious that the approximate method no longer give reliable results.

The conclusion is that the effects of the wavy interface can be quite important, especially with shorter periods and higher amplitudes. The approximate method is considerably more efficient from a computational point of view, but the numerical results are only valid for moderately large corrugations. However, a more thorough investigation is needed before any definite conclusions can be drawn about the range of applicability.

Acknowledgment

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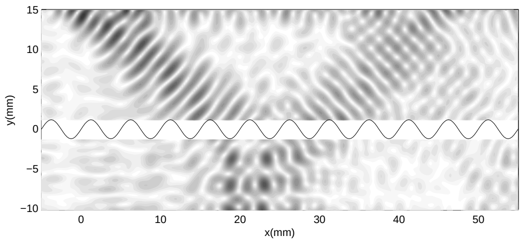

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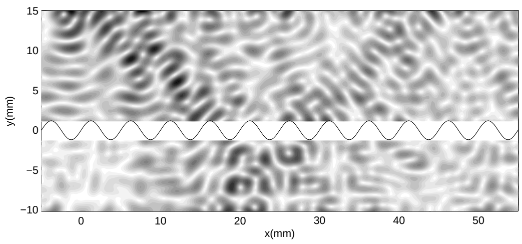

(b) Approximate solution

Figure 4: The field in the plate with a corrugated interface with $a = 5$ mm, $b = 1.2$ mm.

Reference

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