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Damage Detection with Finite Element Model Updating using experimental modal data

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Summary

In this paper we present and experimentally validate the method of finite element model updating based on modal parameters with regard to the detection, localisation and determination of damage in a structure. The experimental modal data is extracted from measurements with the stochastic subspace system identification. The experimental setup consists of a cantilever beam excited by a static load and measured with four accelerometers. The structure is progessively damaged with a cutter.

The presented results are the present state of research in the project B4 in the Collaborative Research Center 398 "Lifetime oriented design concepts" funded by the German Science Foundation. The subject of the project is the diagnosis and localisation of discrete damage by vibration measurement with the aim of lifetime estimation.

Introduction

The monitoring of buildings, engines or general structures is still a wide field of research for engineers. One possibility to detect damage without damaging the structure is to measure and analyse the modal parameters. The subspace system identification is one method to extract the modal parameters from measurements that allows a direct interpretation in terms of mechanics. Similarly the finite element method is able to discretise a structure, to gather their mechanical properties and to simulate their dynamic behaviour. One enhancement of this method is the finite element model updating which is able to reconstruct system characteristics like modal parameters with balancing of unknown model parameters like elastic modulus or cross-sectional heights. One of the new aspects of the paper lies in the experimental validation of the finite element model updating applied on a progressively damaged structure in combination with an enhanced stochastic subspace system identification.

Subspace System Identification

The subspace system identification originally arised from system and control theory with the task to determine a subspace system that is able to reproduce a signal (measurement). The stochastic subspace system identification that we use is a special case of this method where the system response (output only) is included exclusively and the excitation (input) is unknown, see [1] and [2]. The theory supposes that the investigated system is linear and time-invariant for the considered period.

To derive a subspace in terms of mechanics we can use a second order linear ordinary differential equation (1) that describes the dynamic behaviour of the space-discrete

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mechanical structure and an output equation (2) that characterises the connection to the measurements.

$$P(t) = M\ddot{q}(t) + D\dot{q}(t) + Sq(t) \quad = \quad Gu(t) \tag{1}$$
$$Y(t) = C_d\, q(t) + C_v\dot{q}(t) + C_a\ddot{q}(t) + D'u(t) \tag{2}$$

In the above equations $P(t)$ is the excitation force vector that is factorised into the input location influence matrix G and the time-dependent input force vector $u(t)$; M, D and S are the system mass, damping and stiffness matrices; $Y(t)$ represent the measurements; C_d, C_v and C_a are the displacement, velocity and acceleration calibration matrices; D' is the direct transmission matrix. Using equation (1), the trivial statement $\dot{q}(t) = \dot{q}(t)$, the state-space vector $X(t) = [\, q(t) \;\; \dot{q}(t)\,]^T$ and adding the process noise $w(t)$, we obtain the state-space equation (3). Inserting equation (1) into equation (2), adding the measurement noise $v(t)$ and introducing the state-space vector $X(t)$, we obtain the extended output equation (4).

$$\dot{X}(t) = \begin{bmatrix} 0 & I \\ -M^{-1}S & -M^{-1}D \end{bmatrix} X(t) \quad + \quad \begin{bmatrix} 0 \\ M^{-1}G \end{bmatrix} u(t) + w(t) \tag{3}$$
$$Y(t) = \begin{bmatrix} C_d - C_aM^{-1}S & C_v - C_aM^{-1}D \end{bmatrix} X(t) \;+\; \begin{bmatrix} C_aM^{-1}G + D' \end{bmatrix} u(t) + v(t) \tag{4}$$

The first matrices of equations (3) and (4) are the continuous state-space matrix $\overline{A}$ and the input-output matrix C. They contain the basic information about the space-discrete mechanical structure and have to be identified.

The measurement, taken at discrete times, requires a discontinuous formulation of the subspace system. Gathering the measurement and the process noise by introducing the error matrix e_k and the product with the Kalman matrix K, we define the discontinuous state-space equation (5) and the discontinuous output equation (6) at the discrete time k.

$$x_{k+1} = Ax_k \;+\; Ke_k \tag{5}$$
$$y_k = Cx_k \;+\; e_k \tag{6}$$

In the above equations A and C are the discontinuous state-space matrix and the output matrix and can be identified with the stochastic subspace system identification as follows. At first, we sort the measurement outputs in so-called Hankel form and divide this matrix into a past reference part and a future part. Secondly, we use the projection to retain all information of the past that is substantial to predict the future. Next, we apply the singular value decomposition to estimate the maximum number n_2 of eigenvalues and eigenvectors that are excited by the input force to reduce the system and to calculate the extended observability matrix Γ_{n_2} that includes the discontinuous state-space matrix A and the output matrix C.

The result of the above sequence is the discontinuous state-space matrix A which has to be shifted into continuous domain and which still includes an arbitrary state-space transformation. The knowledge of the state-space transformation matrix allows us finally to calculate the undamped modal parameters of the submatrix $M^{-1}S$. The fact that the finite element model updating can be performed on the basis of the undamped modal data has the important advantage that we do not need to define any assumptions about the damping of the system.

Finite Element Model Updating

So far we have concentrated on one category of finite element model updating algorithms, the iterative ones [3] which have some important advantages. First of all, the positive definiteness of the (updated) mass and stiffness matrices is retained and the connectivity of the structure is conserved which means that the updated matrices can be still derived from a finite element model. The possibility to include selected fragments of the mode shapes and to weight the measured data, the analytical data and the model parameters are further important advantages.

Assembling the measured eigenvalues λ_M and the mode shapes Φ_M we receive a measurement modal vector (7). Considering a finite element model depending on the model parameters θ (elastic modulus and/or cross-sectional height and/or ...), we calculate the analytical modal data and assemble an equivalent analytical modal vector (8). The index i denotes an iteration index.

$$Z_M^T = (\lambda_{M1}, \Phi_{M1}^T, \lambda_{M2}, \Phi_{M2}^T, \cdots, \lambda_{Mn}, \Phi_{Mn}^T) \tag{7}$$

$$Z_{F,i}^T = (\lambda_{F1,i}, \Phi_{F1,i}^T, \lambda_{F2,i}, \Phi_{F2,i}^T, \cdots, \lambda_{Fn,i}, \Phi_{Fn,i}^T) \tag{8}$$

Note that we work here only with the real modal data. Therefore, only the mass and the stiffness matrix of the finite element model have to be taken into account (the computation of the damping matrix is avoided). Pre-requisite for the updating algorithm is a correct mode pairing, e. g. with the modal assurance criterion (9) and mode scaling, e. g. with the modal scaling factor (10).

$$MAC_{jk} = \frac{\left|\Phi_{Mj}^T \Phi_{Fk}\right|^2}{\left(\Phi_{Fk}^T \Phi_{Fk}\right)\left(\Phi_{Mj}^T \Phi_{Mj}\right)} \tag{9}$$

$$MSF_j = \frac{\Phi_{Fj}^T \Phi_{Mj}}{\Phi_{Mj}^T \Phi_{Mj}} \tag{10}$$

The aim of the finite element model updating is to determine the model parameters θ in such a way that the difference $\Delta Z = Z_M - Z_{F,i}$ between the measurement modal vector Z_M and the analytical modal vector $Z_{F,i}$ is minimised. If we introduce the parameter difference $\Delta\theta$ between the iteration steps i and $i+1$ and the sensitivity matrix S_u, we can define the analytical modal error ε (11) of the analytical modal vector $Z_{F,i}$ with respect to the model parameters θ.

$$\varepsilon = \Delta Z - S_u \Delta\theta \tag{11}$$

The sensitivity matrix S_u describes the change of the analytical modal data included in Z with respect to an incremental modification of the model parameters θ evaluated in the iteration step i. Using the analytical error ε, we can define the extended weighted penalty function $j(\Delta\theta)$ (12) which is a non-linear function of the model parameters θ. Inserting the modal difference ΔZ, the parameter difference $\Delta\theta$ and the analytical modal error ε into the extended weighted penalty function $j(\Delta\theta)$, we receive an equation that has to be minimised. Solving the first derivation of this equation to the new parameters θ_{i+1} serves the updating algorithm (13).

$$j(\Delta\theta) = \varepsilon^T W_{\varepsilon\varepsilon}\varepsilon + \Delta\theta^T W_{\theta\theta}\Delta\theta \tag{12}$$

$$\theta_{i+1} = \theta_i + \left[S_u^T W_{\varepsilon\varepsilon} S_u + W_{\theta\theta}\right]^{-1} S_u^T W_{\varepsilon\varepsilon}\left(Z_M - Z_{F,i}\right) \tag{13}$$

The modal weighting matrix $W_{\varepsilon\varepsilon}$ in equation (13) considers that (I) the measured mode shapes are less reliable than the measured eigenvalues and (II) that the measured modal data of the lower frequencies are more accurate than the ones of the higher frequencies. If the sensitivity matrix S_u indicates that some model parameters θ have little or same influence on the analytical modal data, the model weighting matrix $W_{\theta\theta}$ can be applied. Methods concerning the choice of weighting matrices belong to the so-called regularisation techniques and are treated e. g. in [4].

Experimental validation

The experimental setup is a clamped cantelever beam with a length of 1.6 m and a rectangular cross-section (40x15 mm) made of steel. The used measurement technology (Hottinger Baldwin) consists of four accelerometers (B12), an amplifier (Spider 8) and the software (catman 3.1). The system is excited by a static displacement and the accelerations are measured at four positions (Figure 1).

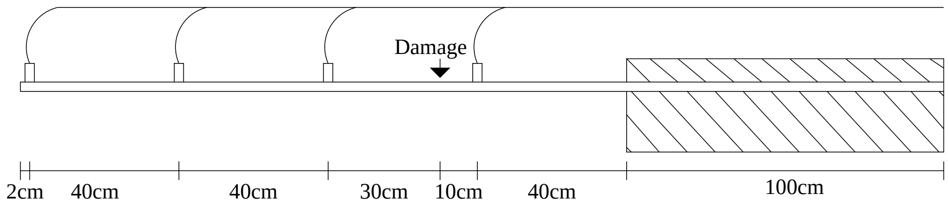

Figure 1: Laboratory dimensions

Using the stochatic subspace system identification, we obtain the continuous transformed state-space matrix $\overline{A}$ that provides the undamped experimental modal parameters. Similarly we use a space-discrete finite element model with 16 elements, 33 degrees-of-freedom to calculate the analytical modal parameters. The updating algorithm is employed to balance the model parameters like the rotational spring stiffness of the clamping, elastic modulus and cross-sectional heights. In this way the difference between the measured modal vector and the corresponding analytical result is minimised (see Table 1).

Table 1: Initial model updating

Experimental					Model Updating			
λ_{M1}	=	823.3371	-	823.4005	$\lambda_{F1,i}$	=	821.1100	(0.3%)
$\phi_{M1}(1)$	=	1.0000	-	1.0000	$\phi_{F1,i}(1)$	=	1.0000	(0.0%)
$\phi_{M1}(2)$	=	0.6438	-	0.6442	$\phi_{F1,i}(2)$	=	0.6596	(2.4%)
$\phi_{M1}(3)$	=	0.3426	-	0.3428	$\phi_{F1,i}(3)$	=	0.3452	(0.7%)
$\phi_{M1}(4)$	=	0.1075	-	0.1076	$\phi_{F1,i}(4)$	=	0.1027	(4.7%)
λ_{M2}	=	32277.20	-	32279.56	$\lambda_{F2,i}$	=	31819.40	(1.4%)
$\phi_{M2}(1)$	=	1.0000	-	1.0000	$\phi_{F2,i}(1)$	=	1.0000	(0.0%)
$\phi_{M2}(2)$	=	-0.1774	-	-0.1775	$\phi_{F2,i}(2)$	=	-0.1649	(7.6%)
$\phi_{M2}(3)$	=	-0.7688	-	-0.7691	$\phi_{F2,i}(3)$	=	-0.7201	(6.8%)
$\phi_{M2}(4)$	=	-0.4812	-	-0.4816	$\phi_{F2,i}(4)$	=	-0.4419	(9.0%)

Additionally, the structure will be damaged locally and progressively between the first and second accelerometer by using a cutter (2 mm) (Figure 1). The change of the identified first and second eigenvalue with increasing damage is displayed in Figure 2.

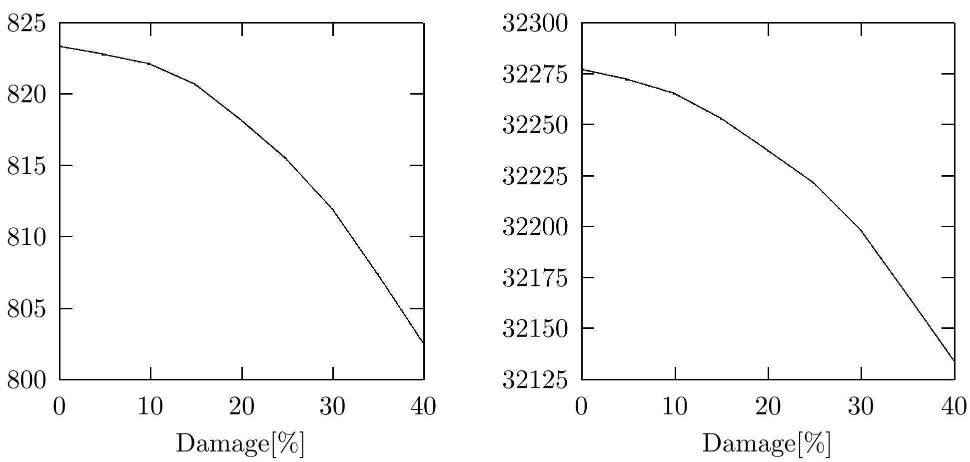

Figure 2: First and second eigenvalue with increasing damage

Finally we use the iterative finite element model updating again to detect and to localise the damage. In Figure 3 it is shown that the algorithm detects the damaged area more precisely, if one increases the number of elements. Beyond a certain discretisation level the localisation cannot be improved anymore.

Conclusion

The stochastic subspace system identification is able to extract the change of modal parameters with increasing damage with sufficient accuracy. The reconstruction of the initial experimental modal parameters includes a certain error. Regardless, the combination of both methods enables us to locate damage in the structure with relatively high accuracy. The focus of our present research work lies on the quantitative assessment of damage and the prediction of lifetime.

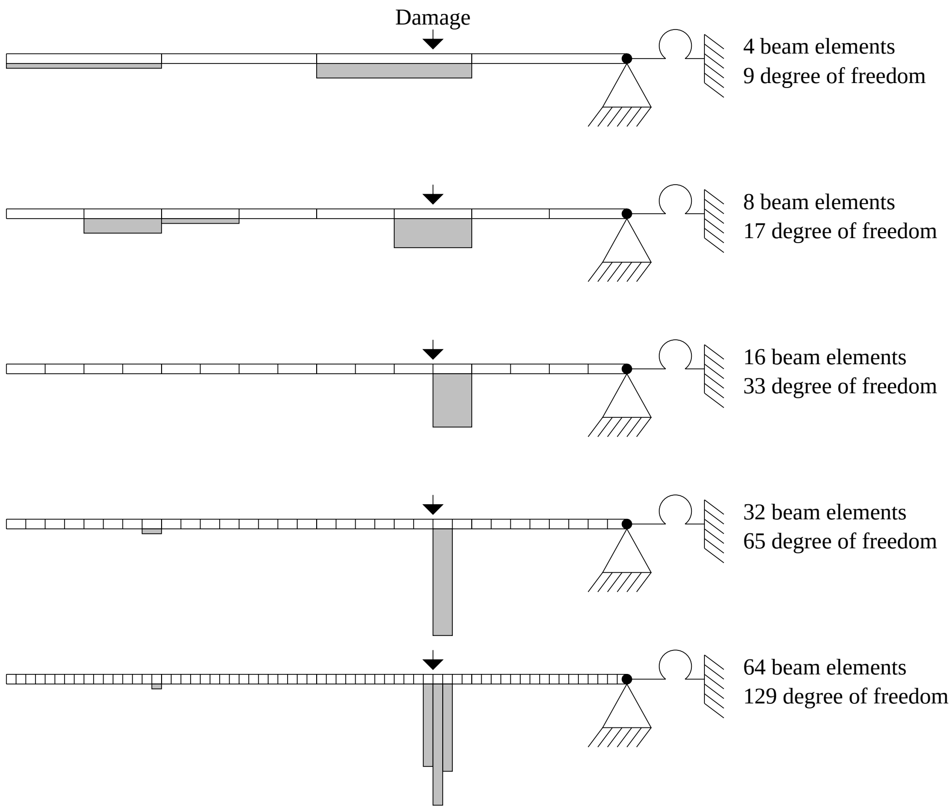

Figure 3: Damage localisation

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