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Boundary element formulations applied to the analysis of anisotropic cracked plates under transient loads

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Summary

This paper presents a comparison of two techniques used with dual reciprocity boundary elements for the analysis of anisotropic cracked plates under transient loads: the subregion technique and the dual boundary element method. Characteristics of these techniques are discussed. A numerical example is analysed by both techniques. Although there is a good agreement between the techniques, special care should be taken in the positioning of internal nodes.

Introduction

In the recent years, important advances on boundary element techniques applied to anisotropic materials were published in the literature. For example, plane elasticity problems were analysed by Sollero and Aliabadi [1], Deb [2], and Albuquerque *et al.* [3] [4] [5] [6], out of plane elasticity problems Zhang [7], tri-dimensional problems by Kögl and Gaul [8].

For transient crack problems in anisotropic materials, three techniques have been applied for the crack modelling. The first one is the use of a fundamental solution that considers the existence of a crack on the domain. In this case, the crack doesn't need to be discretized as has been shown by Zhang [7] for anti-plane problems. Other possibility is the division of the domain of the problems into sub-regions so that each crack edge remains in different sub-regions. Details of this technique was presented by Albuquerque *et al.* [3] [4] for plane problems. The third possibility, named dual boundary element method, is to use different integral equations for each crack nodes avoiding the division of the domain into sub-regions.

In this paper, the last two techniques cited in the previous paragraph are compared for anisotropic problems under transient loads. It is shown that both of them present advantages and drawbacks. As will be seen, the dual boundary element method still demands some developments particularly to anisotropic transient problems.

Boundary element formulation for crack problems

The differential motion equation to a point in a linear elastic anisotropic and homogeneous body with an Ω domain enclosed by a boundary Γ can be written as

$$C_{ijkl}u_{k,jl} + \rho b_i - \rho \ddot{u}_i = 0 \qquad (1)$$

where C_{ijkl} is the elastic constant tensor, u_i is the displacement vector, ρ is the density, b_i is the body force vector, and double dot stands for the second time derivative.

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In the dual boundary element method two integral equations are used to model a crack problem in order to avoid an ill conditioned system of equations due the coincident nodes on the crack faces. Assuming continuity of the displacement at the boundary source points, the displacement integral equation is given by

$$c_{ij}u_j + ⨍_\Gamma T_{ij}u_j d\Gamma = \int_\Gamma U_{ij}t_j d\Gamma + \rho\ddot{\alpha}_l^m \left\{ c_{ij}\hat{u}_{lj}^m - \int_\Gamma \hat{t}_{lj}^m U_{ij} d\Gamma + \int_\Gamma T_{ij}\hat{u}_{lj}^m d\Gamma \right\} \tag{2}$$

where c_{ij} is a constant which depends on the position on the boundary; α_l^m is a vector of M determined coefficients; $m = 1, \ldots, M$; U_{ij} and T_{ij} are the displacement and traction fundamental solutions, respectively; $\hat{u}_{lj}$ and $\hat{t}_{lj}$ are the displacement and traction particular solution, respectively; $⨍$ stands for integration in the Cauchy sense.

Assuming continuity of strains at the boundary source points, the traction integral equation is given by

$$\begin{aligned} \frac{1}{2}t_j \quad &+ \quad n_i ⨎_\Gamma T_{kij}u_k d\Gamma - n_i ⨍_\Gamma U_{kij}T_k d\Gamma \\ &= \quad \rho\ddot{\alpha}_l^m \left\{ \frac{1}{2}\hat{t}_{lj}^m - n_i \int_\Gamma U_{kij}\hat{p}_{lk}^m d\Gamma + n_i \int_\Gamma T_{kij}\hat{u}_{lk}^m d\Gamma \right\} \end{aligned} \tag{3}$$

where U_{ijk} and T_{ijk} are linear combination of U_{kj} and T_{kj}, respectively, n_i is the normal vector and the symbol $⨎$ stands for integration in the Hadamard sense.

In order to obtain the solution of an elastodynamic problem with complex geometry, boundary Γ is divided here into boundary elements with tractions and displacements interpolated by quadratic shape functions. The final system of equation can be written as:

$$\begin{aligned} \mathbf{H}^d\mathbf{u} \quad &= \quad \mathbf{G}^d\mathbf{t} + \rho\left[\mathbf{H}^d\hat{\mathbf{U}} - \mathbf{G}^d\hat{\mathbf{T}}\right]\mathbf{E}\ddot{\mathbf{u}}, \\ \mathbf{H}^t\mathbf{u} \quad &= \quad \mathbf{G}^t\mathbf{t} + \rho\left[\mathbf{H}^t\hat{\mathbf{U}} - \mathbf{G}^t\hat{\mathbf{T}}\right]\mathbf{E}\ddot{\mathbf{u}}, \end{aligned} \tag{4}$$

where subscripts d and t stand for terms of displacement and traction integral equations, respectively. Matrices **H** and **G** contain the integrals of the product of anisotropic fundamental solutions, shape functions and Jacobian of transformation. Vectors **t** and **u** contain boundary traction and displacement components, respectively. $\hat{\mathbf{U}}$ and $\hat{\mathbf{T}}$ are matrices which contain particular solutions for displacement and traction, respectively. Matrix **E** is given by:

$$\mathbf{E} = \mathbf{F}^{-1}, \tag{5}$$

where **F** is the matrix of approximation functions.

As the inverse of **F** matrix is necessary in the formulation, nodes with the same coordinates cannot be used in the assembly of this matrix in order to avoid a singular matrix. So, a traditional approach when using dual reciprocity together with dual boundary element method is not to use the crack nodes as dual reciprocity nodes. Owing to this, there is the necessity of using a shadow of nodes around the crack to replace the absence of crack nodes in the approximation of inertia terms.

In a compact form, equation (4) can be written as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{H}\mathbf{u} = \mathbf{G}\mathbf{t}, \tag{6}$$

where

$$\mathbf{M} = \rho \left[\mathbf{G}\hat{\mathbf{T}} - \mathbf{H}\hat{\mathbf{U}} \right] \mathbf{E}. \tag{7}$$

Using a time step integration, the acceleration $\ddot{u}$ is written in terms of displacements u and then, applying the boundary conditions, the matrix equation (4) is rearranged. Finally, displacements and tractions are computed for each time step.

In the sub-region technique, the domain is divided into sub-domains so that different crack edges belong to different sub-domains. In this case, only the displacement equation (2) is used and the **F** matrix are computed using nodes of all crack edges since they are integrated following different paths. As the crack nodes are taking into account in approximation of inertial terms, uniformly distributed internal nodes can be used. Besides, as continuous elements are used in the crack discretization, singular elements, as for example the traction singular quarter point element (Martinez and Dominguez [9]), can be applied at the crack tip in order to increase the accuracy of the results. From authors experience, none of the proposed special elements for crack problems provide an improvement in the accuracy of dual boundary for anisotropic transient problems.

Numerical example

Consider a plate with a slanted edge crack (see Figure 1) under a uniform tensile stress applied as a step load at time $\tau_o = 0$. A state of plane stress is assumed. The dimensions of the plate are $h = 44$ mm, $w = 32$ mm, $c = 6$ mm, $a = 22.63$ mm, and $\alpha = 45^o$.

The material is orthotropic, with the following properties: Young's moduli $E_1 = 82.4$ GPa and $E_2 = 164.8$ GPa, shear modulus $G_{12} = 29.4$ GPa, Poison's ratio $\nu = 0.4006$, and density $\rho = 2450$ Kg/m^3. The time step used was $\Delta\tau = 0.4$ μs. The analysis with dual boundary element method was carried out with 30 boundary elements and 42 internal points (Figure 2) while with sub-region technique it was using 48 boundary elements and 20 internal points (Figure 3). Figures 4 and 5 show the normalised dynamic stress intensity

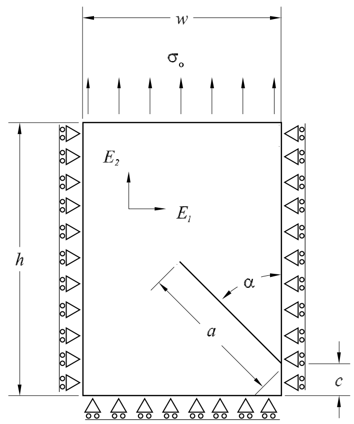

Figure 1: Plate with a slanted edge crack.

factors obtained by dual boundary element method for mode I and mode II, respectively, with the results obtained by sub-regions and traction singular quarter point elements. It can be seen that there is good agreement between the formulations.

Conclusions

This paper presents a comparison between the dual boundary element method and sub-region technique in the computation of dynamic stress intensity factors for transient anisotropic plane problems. It was shown that, at the moment, due to the use of dual reciprocity formulation to approximate the inertial terms, the sub-region technique is more suitable, provided that the crack edge nodes can be used as dual reciprocity nodes. However, the authors has no doubt that dual boundary element method is more robust and efficient than sub region technique for the most part of boundary element formulations, particularly when crack propagation is a concern. The application of the dual reciprocity together with dual boundary is not straightforward. The coincidence of crack nodes causes a singular **F** matrix. A standard procedure to overcome this difficulty is not to use crack nodes as dual reciprocity nodes. Instead internal nodes concentrated near the crack are applied. So, the dual reciprocity with single region formulation is sensitive to the position of internal points. Consequently, the sub region technique still remains with some advantages over single region in dual reciprocity formulations.

Acknowledgment

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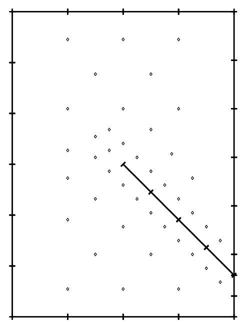

Figure 2: Dual boundary element mesh.

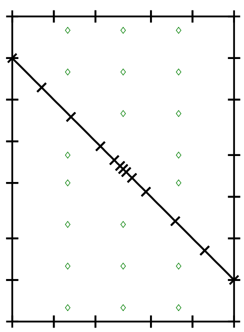

Figure 3: Sub-region boundary element mesh.

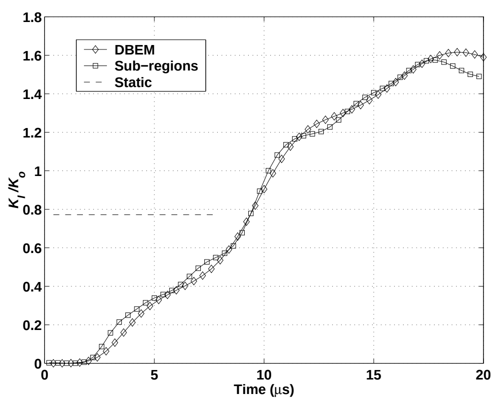

Figure 4: Normalised mode *I* dynamic stress intensity factors.

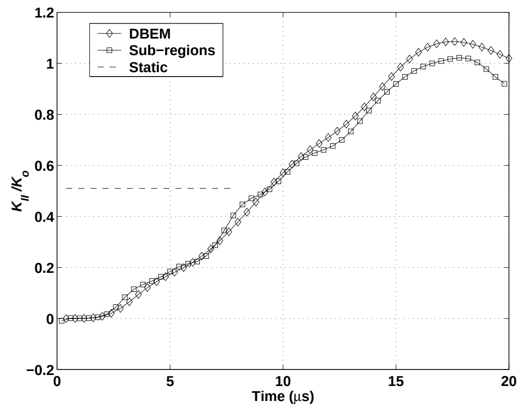

Figure 5: Normalised mode *II* dynamic stress intensity factors.

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