Validation of various linear spring stiffness definitions for a simple physical model for vibrational analysis of cracked beam elements subjected to uniform transverse loads

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Summary

As an alternative to the huge finite-element model solution, simple physical models must be implemented in some cases to adequately describe the behaviour of the structure. Such models are usually required in engineering problems where only a limited amount of data is available. A typical example of such problems are inverse problems where the simplified model has to be capable of intensive modification of crack location because a detailed discretisation of the crack and its surrounding with an appropriate detailed mesh of finite elements has severe practical limitations. The paper discuses the results obtained with a simplified computational model, where the complete crack is replaced by linear springs implementing various available definitions of rotational spring that represents crack.

Introduction

As cracks may change significantly the behaviour of the whole structure, the development of reliable computational models of the mechanical behaviour is especially important. A detailed discretisation of the crack and its surrounding can be achieved with an appropriate mesh of finite elements. However, such an approach is not recommended in inverse problems where when searching for a potential crack a model suitable for intensive crack's location and position modification is required. In such cases simplified models are implemented. One of the simplest and popular models for inverse identification of cracks by measurements of vibrational parameters of the structure is the model where the crack is introduced as a rotational linear spring connecting the uncracked parts of the structure that are modelled as elastic elements. In the case of a single-sided crack also a centric tensile axial load causes transverse displacements that do not appear when the crack is symmetrically double-sided. By introducing an additional virtual bending moment at the crack location and with a new appropriate definition of the rotational spring also this phenomenon can be efficiently modelled. The presented study is limited to transverse displacement computation due to uniform transverse load.

Simplified model for transverse displacements due to uniform transverse load

The analysis is based on the governing differential equation of the elastic curve of displacement of a straight beam that can be found in almost all references considering

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structural analysis (as for example Bathe [1]). The transverse displacements v are a function of the coordinate x, transverse uniform load p_y and flexural rigidity EI which is the product of Young's modulus E with the moment of inertia I of the cross section. The solution of governing differential equation is a polynomial of third degree with four unknown coefficients. To model beams with transverse cracks various approaches can be used. From the inverse identification point of view huge meshes of finite elements are not convenient and thus simple models, as for example the model where the crack itself is replaced by a rotational spring (Figure 1), seem to be more promising. The crack is defined by its location (i.e. the distance L_1 from the left end) and depth d.

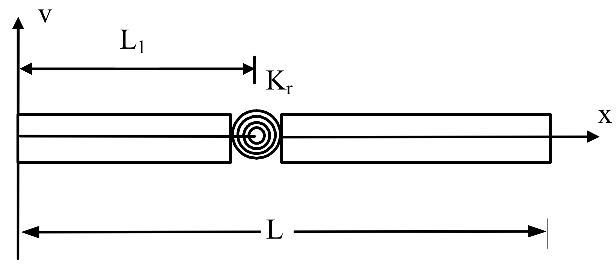

Figure 1: Computational model for transverse displacements due to uniform transverse load

Since the crack separates the beam into two elastic parts two governing differential equations for the parts on the left and right side of the crack remain to be solved. Four of eight unknown coefficients can be determined from the actual boundary conditions, and the remaining four from the continuity conditions at the crack location: the equality of displacement, the condition of a discrete increase of rotations due to the crack, the equality of bending moments, and the equality of shear forces. In the references several definitions of the rotational stiffness K_r for cracked rectangular cross section can be found. The stiffness of the string depends on the height of the uncracked cross section h, the relative depth of the crack $\delta = d/h$ (where d is the depth of the crack), and the product of Young's modulus with the moment of inertia of the uncracked cross section. The earliest definition for rotational stiffness was introduced by Okamura et. al [2]. The remaining definitions were presented by Ostachowicz and Krawczuk [3], Dimarogonas and Papetis (in Liang et al. [4]), Krawczuk and Ostachowicz [5], Sundermayer and Weaver [6], and Hasan [7]. All existing functions share the same mathematical formulation and differ only in the coefficients used. A comparison of the definitions shows that they all exhibit considerable agreement, especially for values of δ smaller than 0.5 (Skrinar [8]).

Implementation of rotational springs into simplified model and comparison of results

In order to numerically validate the definitions presented, the transverse displacements on a number of structures were computed with a commercial finite element program COSMOS/M. In the computational model 4000 2D 8 noded quadrilateral elements were implemented with 12300 nodal points. The discrete crack approach was utilised for the crack description.

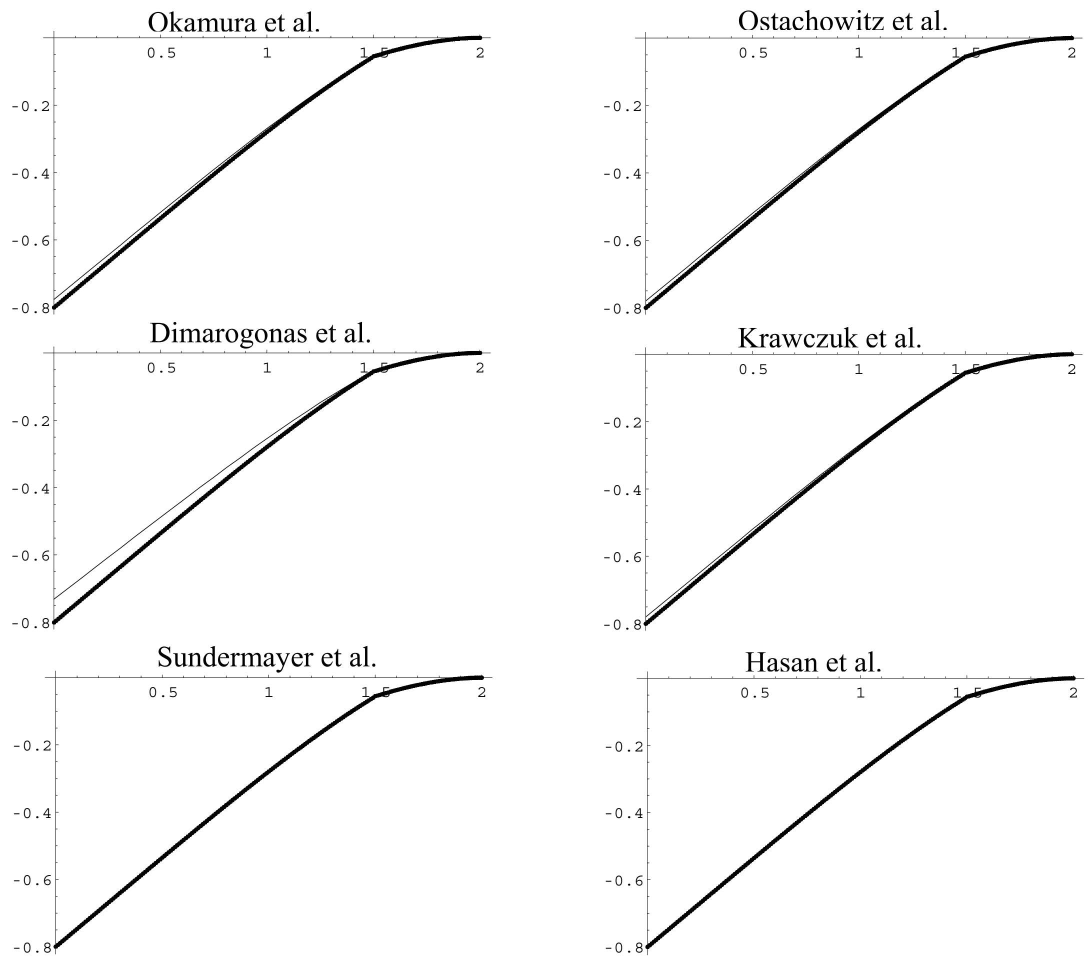

Figure 2: Cantilever beam subjected with uniform transverse load. The crack is located at the distance 1.5 m from the left free end.

Several types of structures were considered and within each type of the structure the geometrical dimensions of the structure were altered. In these models several positions and depths of the crack were considered. The obtained results were further compared with the results obtained with the simplified model where the governing differential equations were solved for each type of the structure. Typical examples for the depth of the crack equal to the half of the height of the cross section are summarized in Figure 2 (for cantilever beam), Figure 3 (for simply supported beam) and Figure 4 (for beam, simply supported at the left end and clamped at the right end). In all presented cases the length of the structural element was 2 m. In Figures 2-4 the continuous slim line presents solutions, obtained with the differential equations, and the continuous double line present the values, obtained with the finite element meshes.

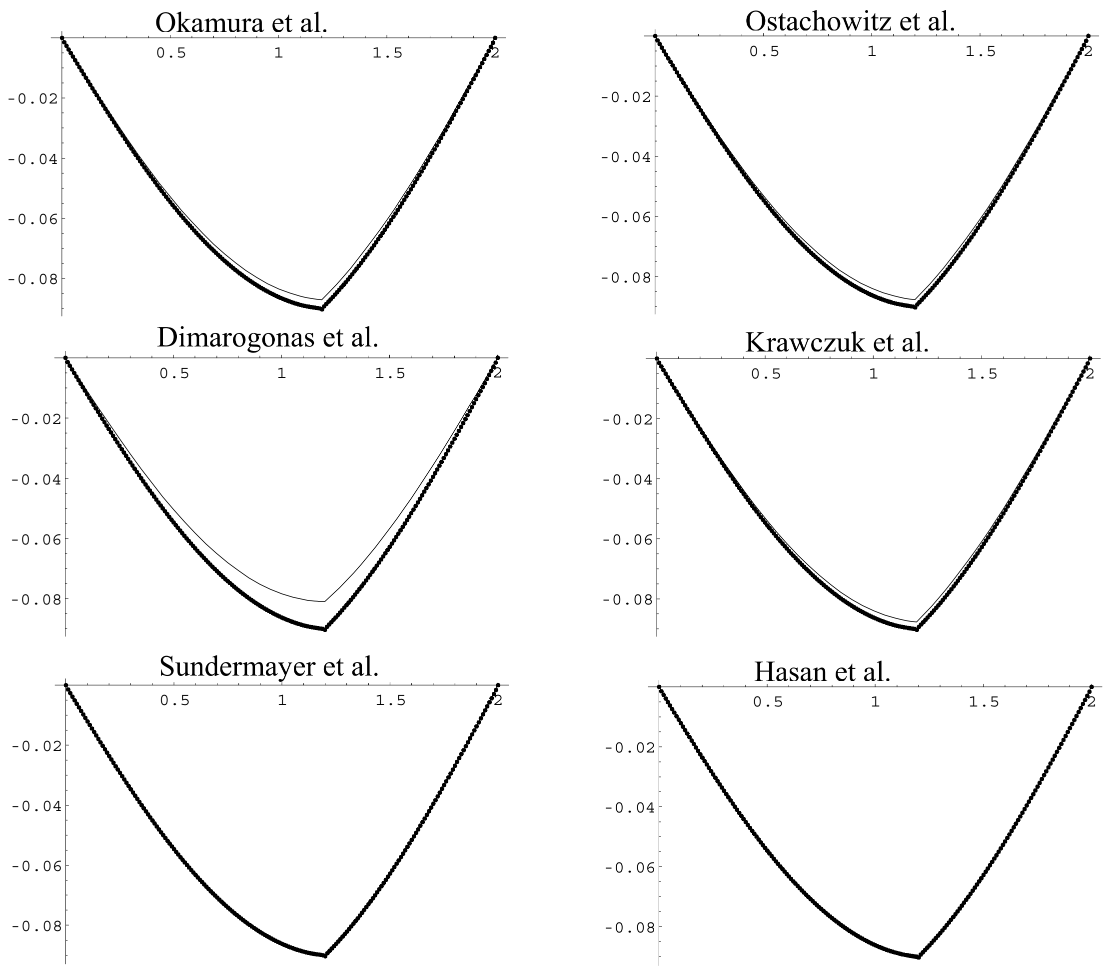

Figure 3: Simply supported beam subjected with uniform transverse load. Crack is located at the distance 1.2 m from the left end.

Discussion of the results

Although just a limited amount of the analyzed cases was presented, some conclusions can be drawn. From presented cases it can be concluded that the definition given by Krawczuk & Ostachowitz slightly overestimate the results; while definitions given by Sundermayer & Weaver, Dimarogonas & Papetis, and Ostachowitz & Krawczuk underestimate the displacements, with the first being the worst and the last being the best. The same can be concluded also from cases that are not presented. However, it should be noted that the difference between the 2D finite element solutions and differential equations solutions increase simultaneously with the depth of the crack.

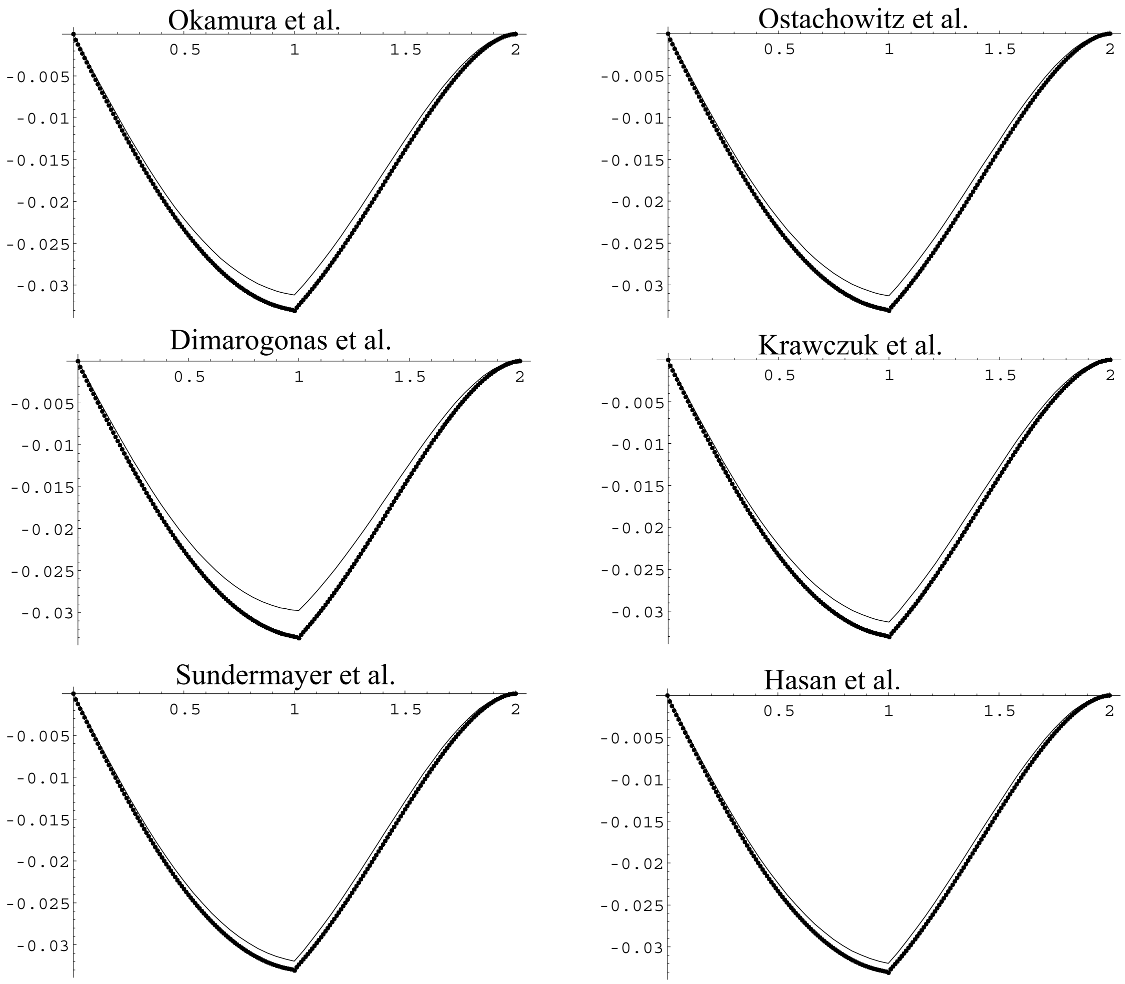

Figure 4: Beam, simply supported at the left end and clamped at the right end subjected with uniform transverse load. Crack is located at the distance 1.0 m from the left end.

For deep cracks the definitions given by Okamura and Hasan generally produced the best agreement with the finite elements results for all types of structures, regardless the position of the crack.Although no essential difference was detected between the results obtained by the two solutions, the definition given by Okamura produced slightly better results.

Conclusions

The key to the efficient implementation of an idealised model certainly lies in the appropriate stiffness definitions of linear springs that is utilised to model the crack and therefore the paper presented a numerical comparison of various definitions for linear springs stiffness implemented for transverse displacement computation. The definitions presented yield valid engineering results as long as it can be considered that the structure behaves within the linear theory (i.e. small displacements) that is usually a correct assumption for most civil engineering structures. Among presented definitions two of them yield great agreement with the results obtained with huge finite elements solutions and despite a drastic difference in computational effort the difference between the best results obtained was within 2%.

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