Fracture Mechanics of the Interface Crack in a Bimaterial Piezoelectric Wedge under Antiplane Deformation

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Summary

The antiplane electro-elastic analysis of the interface crack in a bimaterial piezoelectric wedge subjected to a pair of concentrated forces and surface charges is studied in this paper. The intensity factors at both crack tips are derived analytically. The energy density criterion is applied to examine the fracture behavior of the interface crack.

Introduction

Due to the capability of the transfer between mechanical and electric energy, piezoelectric materials are widely used in smart structures, sensors, and actuators. These structures usually involve wedge shape structures at some local regions, where the geometry and material are discontinuous. The singular stress field may occur at the wedge apex where the crack initiates. Chue et al. [1] and Wei et al. [2] used the Mellin transform to obtain the antiplane electro-elastic field and intensity factors of the single-material and bimaterial piezoelectric wedges subjected to a pair of concentrated forces and surface charges. Erdogan and Gupta [3] and Shahani and Adibnazari [4] used the Mellin transform and the singular integral equation to study the interface crack in a bimaterial elastic wedge under antiplane loadings. In this paper, we study the antiplane deformation of the bimaterial piezoelectric wedge with an interface crack subjected to a pair of antiplane concentrated forces and inplane electric surface charges. The intensity factors of stress, strain, electric displacement and electric field at crack tips are derived analytically. The energy density criterion is applied to examine the fracture behavior of the interface crack. Due to the mathematical difficulties posed by applying the Mellin transform to the piezoelectric wedge problem with an interface crack, the crack is assumed to be impermeable for this study.

Problem Statements and Basic Formulations

The structure shown in Fig. 1 is composed of two bonded piezoelectric wedges with same wedge angle α and an interface crack AB located on the common edge ($\theta = 0^{\circ}$) between $r = a$ and $r = b$. The crack surfaces are traction-free and impermeable [5]. A pair of longitudinal shearing forces F and another two inplane surface charges Q are applied on the edges $r = h$. Because the wedge has an infinite length along the z-axis,

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this problem becomes a generalized plane deformation problem. Since the piezoelectric materials are polarized in the z-direction, only the antiplane elastic field coupled with the inplane electric field is considered in the analysis.

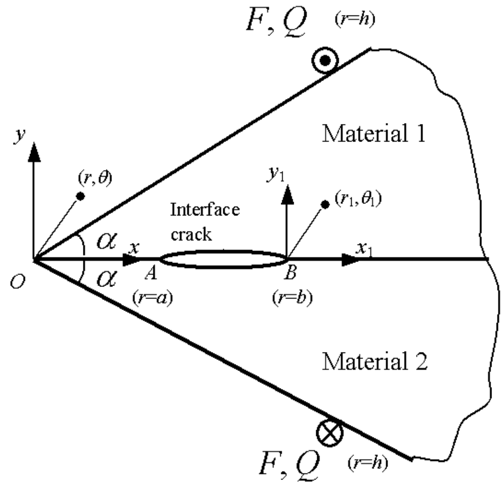

Fig. 1. A bimaterial piezoelectric wedge with an interface crack.

The constitutive equations of stresses (σ_{rz},$\sigma_{\theta z}$), strains (γ_{rz},$\gamma_{\theta z}$), electric displacements (D_r, D_θ) and electric fields (E_r, E_θ) can be written as follows

$$\begin{bmatrix} \sigma_{\theta z}^{(i)} \\ \sigma_{rz}^{(i)} \\ D_r^{(i)} \\ D_\theta^{(i)} \end{bmatrix} = \begin{bmatrix} c_{44}^{(i)} & 0 & 0 & -e_{15}^{(i)} \\ 0 & c_{44}^{(i)} & -e_{15}^{(i)} & 0 \\ 0 & e_{15}^{(i)} & \varepsilon_{11}^{(i)} & 0 \\ e_{15}^{(i)} & 0 & 0 & \varepsilon_{11}^{(i)} \end{bmatrix} \begin{bmatrix} \gamma_{\theta z}^{(i)} \\ \gamma_{rz}^{(i)} \\ E_r^{(i)} \\ E_\theta^{(i)} \end{bmatrix}, \quad i=1,2 \tag{1}$$

where the material constants c_{44}, ε_{11}, and e_{15} are the elastic stiffness constant, dielectric constant, and piezoelectric constant, respectively. The superscript i denotes materials 1 and 2. The governing equations and the solutions for the antiplane displacement w and inplane electric potential ϕ are

$$c_{44}^{(i)}\nabla^2 w^{(i)} + e_{15}^{(i)}\nabla^2\phi^{(i)} = 0, \quad e_{15}^{(i)}\nabla^2 w^{(i)} - \varepsilon_{11}^{(i)}\nabla^2\phi^{(i)} = 0 \tag{2}$$

$$\nabla^2 w^{(i)} = 0, \ \nabla^2\phi^{(i)} = 0 \tag{3}$$

The boundary conditions on the edges of the wedge ($\theta = \pm\alpha$) are as follows

$$\sigma_{\theta z}^{(1)}(r, \ \alpha) = F\,\delta(r-h), \ \sigma_{\theta z}^{(2)}(r,-\alpha) = F\,\delta(r-h),$$

$$D_{\theta}^{(1)}(r,\ \alpha)=Q\delta(r-h),\ D_{\theta}^{(2)}(r,-\alpha)=Q\,\delta(r-h) \tag{4}$$

where δ is the Dirac-Delta function. Without loss of generality, the distance h satisfies the relation $a\le b\le h$. The continuity conditions along the bonded interface ($\theta = 0^{\circ}$) are

$$\begin{aligned}&w^{(1)}(r,0)=w^{(2)}(r,0)\ ,\quad \phi^{(1)}(r,0)=\phi^{(2)}(r,0),\\ &\sigma_{\theta z}^{(1)}(r,0)=\sigma_{\theta z}^{(2)}(r,0)\ ,\quad D_{\theta}^{(1)}(r,0)=D_{\theta}^{(2)}(r,0),\qquad 0\le r\le a\,,\ b\le r<\infty\end{aligned} \tag{5}$$

On the crack surfaces, the traction-free and impermeable conditions are

$$\sigma_{\theta z}^{(1)}(r,0)=\sigma_{\theta z}^{(2)}(r,0)=0,\ \ D_{\theta}^{(1)}(r,0)=D_{\theta}^{(2)}(r,0)=0\ ,\quad a\le r\le b \tag{6}$$

Solutions

Applying the Mellin transform with integration by parts on Eq. (3), gives

$$\frac{d^2\overline{w}^{(i)}}{d\theta^2}+p^2\overline{w}^{(i)}=0,\ \text{provided that}\ \left[r^{p+1}\frac{\partial w^{(i)}}{\partial r}-pr^{p}w^{(i)}\right]_0^{\infty}=0 \tag{7}$$

where the bar over w denotes the transformed quantity and p is a complex transform parameter. The solutions of $\overline{w}$ in Eq. (7) and $\overline{\phi}$ for materials 1 and 2 are

$$\begin{aligned}&\overline{w}^{(1)}=C_1(p)\cos p\theta+C_2(p)\sin p\theta\ ,\quad \overline{w}^{(2)}=C_5(p)\cos p\theta+C_6(p)\sin p\theta\,,\\ &\overline{\phi}^{(1)}=C_3(p)\cos p\theta+C_4(p)\sin p\theta,\quad \overline{\phi}^{(2)}=C_7(p)\cos p\theta+C_8(p)\sin p\theta\end{aligned} \tag{8}$$

The functions C_j (j =1~8) can be deduced from Eqs. (4)-(6). Applying the inverse Mellin transform on Eqs. (8), $w^{(1)}$ and $\phi^{(1)}$ in material 1 are obtained. The residue theorem and appropriate path of integration [4] are applied to solve for the integrals. Furthermore, the stresses and electric displacements are obtained in the region $a \le r \le b$ of material 1 as follows

$$\sigma_{\theta z}^{(1)}(r,\theta)=\frac{F}{r}\left[-X_1-X_2+X_3\right],\ D_{\theta}^{(1)}(r,\theta)=\frac{Q}{r}\left[-X_1-X_2+X_3\right],\ a\le r\le b \tag{9}$$

where

$$X_1=\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}\left(\frac{r}{h}\right)^{p_n}p_n\cos(p_n\theta)\ ,\quad X_2=-\sum_{n=1}^{\infty}\frac{\left(\int_a^r\left(\frac{\nu}{r}\right)^{p_n}f^*(\nu)d\nu\right)\sin(p_n\theta-p_n\alpha)}{\alpha\sin(p_n\alpha)}\ ,$$

$$X_3 = -\sum_{n=1}^{\infty} \frac{\left(\int_r^b \left(\frac{\nu}{r} \right)^{-p_n} f^*(\nu) d\nu \right) \sin(p_n\theta - p_n\alpha)}{\alpha \sin(p_n\alpha)} \tag{10}$$

$$p_{\pm n} = \pm \frac{2n-1}{2\alpha}\pi \ , \quad n = 1,2,\cdots,\infty \tag{11}$$

$$f^*(r) = \frac{1}{\alpha}\sqrt{h^\zeta (a^\zeta + h^\zeta)(b^\zeta + h^\zeta)} \left[\frac{1}{r^\zeta + h^\zeta} - \frac{1}{b^\zeta + h^\zeta} \frac{\Pi(k, m, \pi/2)}{K(k, \pi/2)} \right] \left(r^{(\zeta/2)-1} \right) \frac{1}{\sqrt{(r^\zeta - a^\zeta)(b^\zeta - r^\zeta)}} \ , \quad a \le r \le b \tag{12}$$

$$\Pi(k, m, \pi/2) = \int_0^{\pi/2} \frac{d\theta}{(1 + m\sin^2\theta)\sqrt{1 - k^2\sin^2\theta}} \quad , \quad K(k, \pi/2) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2\sin^2\theta}} \tag{13}$$

$$k^2 = \frac{b^\zeta - a^\zeta}{b^\zeta}, \quad m = -\frac{b^\zeta - a^\zeta}{b^\zeta + h^\zeta}, \quad \zeta = \frac{\pi}{\alpha} \tag{14}$$

The function $f^*(r)$ in Eq. (12) is derived by the mathematical method proposed in [4]. It should be noted that $f^*(r)$ is independent of material properties. From Eqs. (9), the stress and electric displacement fields are uncoupled and independent of material properties. This conclusion is true when the following conditions are satisfied: (1) These two wedge angles are equal; (2) A pair of equal longitudinal shear forces are applied at same distance; (3) Two equal electric charges are applied at same distance; (4) The crack is impermeable. The same conclusion was met in previous studies [6,7]. According to the concepts in [3,4] and the uncoupled phenomenon in Eqs. (9), the stress and electric displacement intensity factors at the interface crack tips ($r = a$ and $r = b$) can be determined in the following equations:

$$K_{III}^{\sigma}(a) = \lim_{r\to a+} \sqrt{2\pi(r-a)} F f^*(r) \, , \; K_{III}^{\sigma}(b) = -\lim_{r\to b-} \sqrt{2\pi(b-r)} F f^*(r) \tag{15}$$

$$K_{III}^{D}(a) = \lim_{r\to a+} \sqrt{2\pi(r-a)} Q f^*(r) \, , \; K_{III}^{D}(b) = -\lim_{r\to b-} \sqrt{2\pi(b-r)} Q f^*(r) \tag{16}$$

By using the constitutive equations, the strain and electric field intensity factors are:

$$K_{III}^{\gamma(i)}(a) = \frac{Qe_{15}^{(i)} + F\varepsilon_{11}^{(i)}}{e_{15}^{(i)2} + c_{44}^{(i)}\varepsilon_{11}^{(i)}} \lim_{r\to a+} \sqrt{2\pi(r-a)} f^*(r) \ , \quad K_{III}^{\gamma(i)}(b) = -\frac{Qe_{15}^{(i)} + F\varepsilon_{11}^{(i)}}{e_{15}^{(i)2} + c_{44}^{(i)}\varepsilon_{11}^{(i)}} \lim_{r\to b-} \sqrt{2\pi(b-r)} f^*(r) \tag{17}$$

$$K_{III}^{E(i)}(a) = \frac{Qc_{44}^{(i)} - Fe_{15}^{(i)}}{e_{15}^{(i)2} + c_{44}^{(i)}\varepsilon_{11}^{(i)}} \lim_{r\to a+} \sqrt{2\pi(r-a)} f^*(r) \ , \quad K_{III}^{E(i)}(b) = -\frac{Qc_{44}^{(i)} - Fe_{15}^{(i)}}{e_{15}^{(i)2} + c_{44}^{(i)}\varepsilon_{11}^{(i)}} \lim_{r\to b-} \sqrt{2\pi(b-r)} f^*(r) \tag{18}$$

Eqs. (17) and (18) show the coupling behaviors between mechanical and electric effects. For studying the crack behavior, the energy density theory [8] is adopted. The energy density dW/dV near the crack tip under antiplane mechanical loads and inplane electric loads can be expressed by

$$\left(\frac{dW}{dV}\right)^{(i)} = \frac{S^{(i)}}{r_1} = \frac{1}{4\pi r_1}\left[K_{III}^{\sigma} K_{III}^{\gamma(i)} + K_{III}^{D} K_{III}^{E(i)}\right] \text{ or } S^{(i)} = \frac{1}{4\pi}\left[K_{III}^{\sigma} K_{III}^{\gamma(i)} + K_{III}^{D} K_{III}^{E(i)}\right] \quad (19)$$

where (r_1, θ_1) is the localized coordinate system at crack tip B shown in Fig. 1. Note that the energy density factor $S^{(i)}$ is independent of θ_1 in materials 1 or 2 in this antiplane study.

Results and Discussions

The effects of the wedge angle α on the energy density factors are discussed in this section. Two typical piezoelectric ceramics PZT-4 and PZT-5H are considered to show the variations of $S^{(i)}(a)$ and $S^{(i)}(b)$ at crack tips A and B.

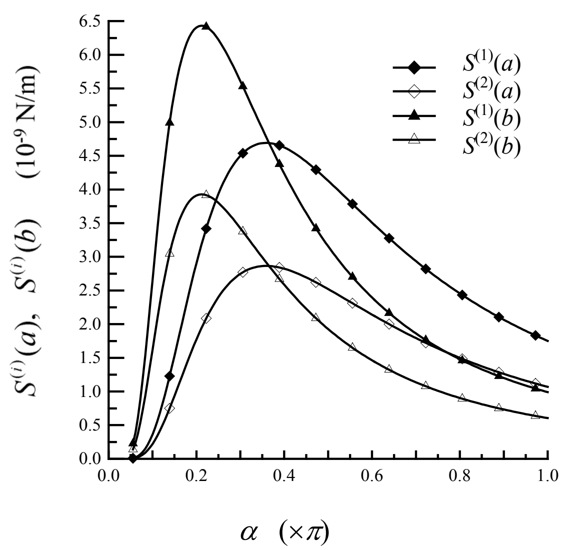

Fig. 2. Variation of energy density factors with α.

The material properties are c_{44}= 2.56×10^{10} N/m^2, e_{15}= 12.7 C/m^2, ε_{11}= 64.6×10^{-10} C/Vm for PZT-4 and c_{44}= 2.3×10^{10} N/m^2, e_{15}= 17 C/m^2, ε_{11}= 150.4×10^{-10} C/Vm for PZT-5H. The wedge with a=0.01m and b-a=0.01m is subjected to the external force F=10 N/m and charge Q=1×10^{-8} C/m applied at the edges with distance h=0.03m. The variations of $S^{(i)}(a)$ and $S^{(i)}(b)$ with α in materials 1 and 2 at both interface crack tips are shown in Fig. 2. The energy density factors in material 1 are greater than those in material 2.

For a given α, the energy density factors at crack tip B ($S^{(1)}(b)$, $S^{(2)}(b)$) in materials 1 and 2 are independent of θ_1, respectively. It is also true for $S^{(1)}(a)$ and $S^{(2)}(a)$ at crack tip A. For example, consider the crack tip B. The crack will propagate along the direction of the least fracture resistance S_C. The material property S_C may represent the fracture toughness along the interface $\theta_1=0^o$ or any other directions in the piezoelectric material PZT-4 or PZT-5H.

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