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Numerical Simulation of Beach Erosion Due to Storm Wave Actions[1]

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Summary

This paper presents a numerical modeling for analysis of coastal development problem based on the beach erosion mechanism. A coupled model is developed from the theory of cross-shore and long-shore sediment transport. The concept of the erosion-damage of coastal media and the wash out shear stresses on the sea bed surface have been taken into account to express the process of erosion due to storm wave actions. A hybrid approach combining numerical and analytical methods has been used to analyze the development of the coast-line (long-shore) and the coastal profile (cross-shore).

Introduction

The Flow and sediment transport near the coastal line are important in relation to several engineering topics. During the last decade the development in beach sediment transport research has changed from simple phenomenological descriptions to sophisticated numerical models[1,2]. The flow and the resulting sediment transport rate have been described in detail by [1,3,4,6]. The natural causes of beach erosion are due to deglaciation and greenhouse effect; sever storms, hurricanes and typhoons contribute to beach erosion by amplifying wind conditions, and wave conditions as well as causing sea-level to rise; and reduction in sediment supply. The coastal characteristics along the beach will also affect the rate of erosion. If the material along the coast is very hard, such as rock, they will have stronger resistance to mechanical wave erosion and chemical weathering. However, if the material is very soft, such as soil, it will be eroded easily. These natural phenomena inspired us to develop the new concept of mechanics to the area of the ocean engineering for modelling the mechanism of beach erosion problem. Numerical models of beach change have the potential of quantitatively representing in time-dependent fashion the influence of all primary variables [1~3].

In this study a coupled sediment transport model based on the cross-shore and long-shore sediment transport theories will be employed into a numerical modeling of the development of coastline and topographic evolution of beach bottom based on the concept of beach erosion under storm wave actions

Modeling of Beach Development

The kinematics of a shoreline change model ensures the conservation of volume of sediment. The associated equation can be expressed in a very compact mathematical form,

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however, the form widely used is one which relates the time rate of change in sand volume V in a beach profile to spatial gradients in the components (Q_x, Q_y) of sediment transport [6] as

$$\frac{\partial V}{\partial t} = -A_x \frac{\partial Q_x}{\partial x} - A_y \frac{\partial Q_y}{\partial y} \qquad (1)$$

where the coefficients, A_x and A_y are generally difficult to determine.

The main assumption herein is that the net discharge in direction parallel to the coastline is zero. The morphological consideration of onshore/offshore sediment transport model can be taken into account by calculating the variation of sediment transport across the profile (see Fig.1 and Fig.2).

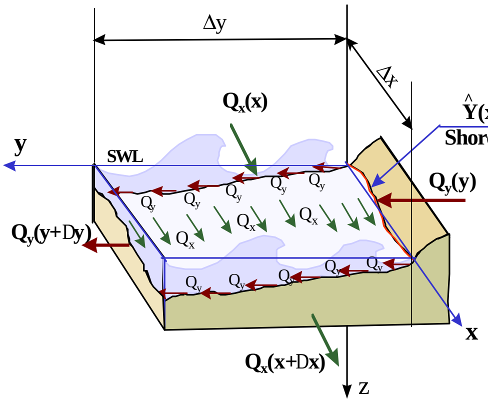


Fig.1 Illustration of sediment transport near shore

The long-shore sediment transport is often manifesting itself through the coastal erosion or accretion around coastal areas. If the beach is long enough, the accretion and erosion should continue and the coastline may move offshore on the up-drift side. The change in shore-line position $\hat{\mathbf{Y}}(x,t)$ can be calculated from the equation of sand conservation near the shoreline [6]

$$r_s \frac{\partial \hat{\mathbf{Y}}}{\partial t} + \frac{1}{(1-a^*)D^*} \frac{\partial Q_x}{\partial x} = 0 \qquad (2)$$

where D^* is the active water depth of the beach profile near the coastal line; r_s is the mass density of coastal bed; a^* is the effective porosity of the erosion-damaged medium, and it can be assumed by an appropriate form, wherein it may be suggested as, $\alpha^* = \alpha/(1-\gamma\alpha)$, in which a is the porosity of the un-eroded medium and g is an erosion parameter[5]; Q_x is the sediment transport rate in the long-shore direction. It is evident that Q_x and Q_y are components of the sediment mass transport rate. The transport rate of the eroded sediment mass per unit area on the beach bed can be derived from the mass conservation based on the topographical change of the coastal bed as shown in Fig.2.

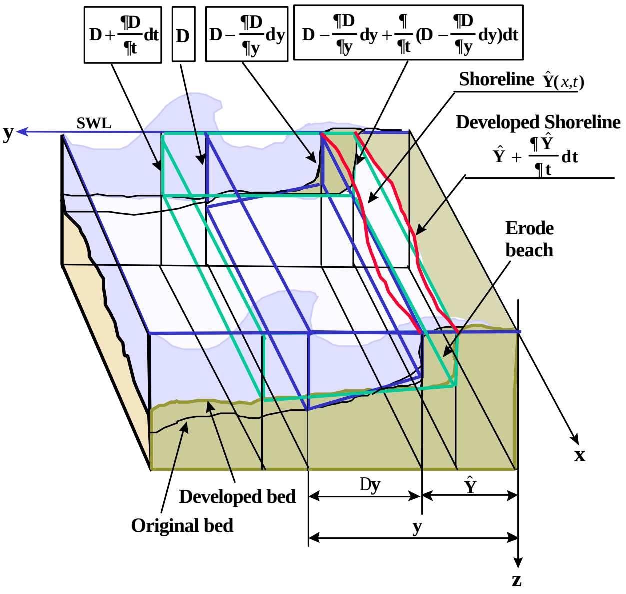


Fig. 2 Topographical development of coastal profile

Let us consider an elemental volume along the shoreline with a bottom area of *Ds*. The most interesting for the problem of coastal development is to study the topographical change of the coastal bottom near the shoreline. Therefore, the analyzed elemental volume should be taken near the shoreline with thickness, $Dy = y - \hat{Y}$, where $\hat{Y}$ is the equation of the shore-line position represented by a function of time *t* and coordinate *x* as $\hat{Y}(x,t)$; *y* is the observed distance variable of the elemental volume from the *x* axis. Despite *Dy* is an elemental quantity, the size of *Dy* can be taken as required, because that size along shoreline can be considered as infinite. According to Fig.2, the changed volume should be equal to that of topographical profile of the coastal bed. Neglecting the higher order term , $(\partial^2 D / \partial y \partial t)\, DyDsDt$ and substituting $Dy = y - \hat{Y}$, the volume changed due to the development of the beach depth during *Dt* as

$$\frac{1}{Ds}\frac{dV}{dt} = \frac{\partial D}{\partial t} + \frac{1}{2}\frac{\partial D}{\partial y}\frac{\partial \hat{Y}}{\partial t} - \frac{1}{2}\frac{\partial D}{\partial y}\frac{dy}{dt} \tag{3}$$

where the elemental base area *Ds* approximately equals to that *Dy* times the length of the elemental along *x* direction; $\partial \hat{Y} / \partial t$ is the rate of shoreline change as presented in Eq.(2); $\partial D/\partial t$ is the rate of cross-shore profile; *dy/dt* actually is the drift velocity of water/sediment particle on the surface of the beach bottom near the coastline along *y* direction. It should equal to the rate of sediment transported at same point due to washing away effected by stormy wave. The drift velocity can be assumed equal to $\bar{v}_b$, which is the time-averaging rate of water /sediment particles on the surface of the beach bed over one wave period as $dy/dt = \bar{v}_b$.

The eroded mass transport rate per unite area on the surface of the beach bed near the coastline can be defined as $dm/dt = r_s(1-a^*)dV/dt/DS$ [4,5], thus we have

$$\frac{dm}{dt} = (1-a^*)\left[r_s\frac{\partial D}{\partial t} + \frac{r_s}{2}\frac{\partial D}{\partial y}\frac{\partial \hat{Y}}{\partial t} - \frac{r_s\bar{v}_b}{2}\frac{\partial D}{\partial y} \right] \tag{4}$$

Considering the effect of sedimentation and erosion-damage parameters on material properties, an erosion criterion can be developed from the expression presented by Dyer (1986) as

$$\frac{dm}{dt} = M_c\left(\frac{\bar{t}_b}{(1-\Omega)^g\, t_c} - 1 \right) \tag{5}$$

where $\bar{\tau}_b$ is the average shear stress on the surface of the coastal bed; $(1 - W)^g t_c$ is the effective critical shear stress of the erosion-damaged sediment medium on the surface of the coastal bed. The coefficient M_c has an unit of the erosion-transport rate (transport of eroded mass per unit area per unit time), and it varies with other factors such as temperature and the presence of organic matter. Ref.[4] has investigated the relationship between M_c and the exchange capacity, sodium adsorption ratio, pore fluid concentration and temperature. Values were generally in the range 0.005 to 0.015, but varied particularly steeply with temperature, being greater at high temperatures.

Substituting Eq.(5) into (4), we have

$$r_s \frac{\partial D}{\partial t} + \frac{r_s}{2}\left(\frac{\partial \hat{Y}}{\partial t} - \overline{v}_b\right)\frac{\partial D}{\partial y} = \frac{M_c}{(1-a^*)}\left[\frac{\bar{t}_b}{(1-W)^g t_c} - 1\right] \tag{6}$$

On a long shoreline coast the long-shore sediment transport rate, Q_x, can be determined from the wave climate that is statistics for wave height and direction. If the coast is given a different orientation, the entire calculation can be carried out once more, and in this way the long-shore sediment transport rate can be determined as a function of the coastline orientation, such as $Q_x = Q_x(\P \hat{Y} / \P x)$ [6]. Eq.(2) becomes

$$r_s \frac{\partial \hat{Y}}{\partial t} + \frac{1}{(1-a^*)h_p}\frac{\partial Q_x}{\partial(\partial \hat{Y}/\partial x)}\frac{\partial^2 \hat{Y}}{\partial x^2} = 0 \tag{7}$$

The establishment of the coast orientation and the long-shore sediment transport rate will require a large number of individual calculations of the long-shore sediment transport rate. It will be not possible to establish any analytical solutions, instead a data base can be established with corresponding values of Q_x and $\P \hat{Y} / \P x$ [6]. One example for long-shore sediment transport rate can be assumed in the form as $Q_x = a_q(b_q - \partial\hat{Y}/\partial x)^l, (m^3/year)$, where a_q, b_q and l are the coefficients to be determined from the observed data, Ref.[6] suggested the values of $a_q = 2.27\text{x}10^6$; $b_q = 0.15$ and $l = 1$.

It should be noted that the model for the coastal profile developed herein, Eqs.(6) and (7), have provided an approach, which couples the cross-shore, and long-shore sediment transport models and the total simulation may possibly be carried out by the interaction between the coastal profile and coastline development.

Modeling of Storm Wave Actions

Wave in the ocean actually serve as a mechanism which can attract energy from wind system, store it in the form of potential and kinetic energies, and transmit it toward shoreline. The dissipation of wave energy occurs near shore in a relatively narrow zone. In relation to sediment transport, it is essential to distinguish between the mean velocity $\overline{v}$ measured at a fixed point and the drift velocity V (mass-transport speed), which is the time-averaging velocity of a fluid particle over a wave period as shown in Fig.3 given by

$$V = \bar{v} + \overline{\frac{\P v}{\P y}\int v dt} + \overline{\frac{\P v}{\P z}\int w dt} \tag{8}$$

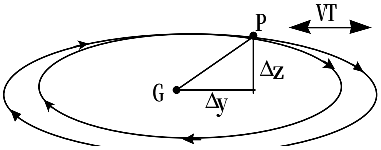


Fig.3 Drift velocity observed from a Lagrangian point of view

where P is a point on the orbit of a particle, the mean position of which is G (see Fig.3); v, w are the velocity components of fluid particle in y and z direction.. Actually the theory of linear wave is not suitable to apply in shallow beach simply, because the shallow wave has a strong non-linear property. This study developed the relative expressions of wave drift velocity and energy for Stokes second order approximated non-linear wave model as

$$V=\bar{v}+\frac{1}{2C}\left(\frac{p\,H}{T}\right)^2\frac{\cosh[2k(D-z)]}{\sinh^2(kD)}+\frac{9}{32C}\left(\frac{p\,H}{T}\right)^4\left(\frac{T}{L}\right)^2\frac{\cosh[4k(D-z)]}{\sinh^8(kD)} \tag{9}$$

in which H is the wave height; k is the wave number; T is the wave period; C is the wave speed. The first and second terms in equation are contributions of linear wave, the third is that of non-linear wave. Close to the beach bed (z $=D$), the mean drift velocity on the surface of the bed becomes $\bar{v}_b = V|_{z=D}$.

From the principle of that, the variation of energy flux equals the rate of energy dissipation [2], we have $\partial E_{av}/\partial y = -\overline{t_b v_b}$, in which E_{av} is the average energy flux per unit of wave crest through a fixed vertical plane parallel to the wave crest near the shoreline. Thus, the shear stress on the bottom surface of the coastal bed can be evaluated by $\bar{t}_b = r\,c_f\,\bar{v}_b|\bar{v}_b|$, where ρ is the mass density of water,

$$\bar{t}_b = -\frac{8T^5\sinh^6(kD)\tanh(kD)}{9\,r\,p^5H^4}\bar{v}_b|\bar{v}_b|\frac{\partial}{\partial y}(E_{av}) \tag{10}$$

$$E_{av}=\frac{1}{16}r\,gCH^2\left(1+\frac{\sinh(2kD)+2kD}{\sinh(2kD)}+\left(\frac{3kH}{4}\right)^2\frac{\sinh(4kD)+4kD}{\sinh(2kD)\sinh^6(kD)}\right\} \tag{11}$$

The wave parameter can be generated by stormy wind model in dimensionless form [2] as

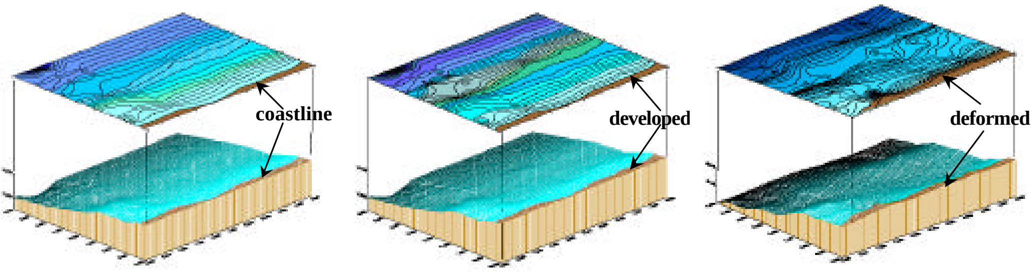
$$\frac{C}{W}=y_1\left[\frac{gF}{W^2},\frac{gt}{W}\right],\qquad \frac{gH}{W^2}=y_2\left[\frac{gF}{W^2},\frac{gt}{W}\right] \tag{12}$$

where W is the wind speed; t is the duration time; F is the equivalent length of wind area; y_1 and y_2 are functional relations that must be determined from observed data [2].

Numerical Results

The above developed models have been applied to the numerical analysis based on the finite difference method. Total (N+1)*M first order non-linear ordinary differential equations with respect to time can be solved by Runge-Kutta method in time domain under the given initial water depth to the surface of the coastal bed by $D(x_i, y_j, 0)$ (i.e. the initial topography of beach bottom), and the initial position of coastal-line by $\hat{Y}(x_i,0)$. The simulated region is taken as $5\times 4(Km)^2$ along the coastal line to sea is located at the Qiantang River's Seaport in the Hangzhou Gulf near the Hainin Town in China. The required data calibrated from the database presented by [1,7,8] and summarized as ρ_s=2.65kg/cm^3, ρ=1kg/cm^3, g=9.81, M_c=0.08 m^{2}sec^{-1}, a_q= 2.27x106m^3/a, b_q = 0.15m^3/a, τ_c = 0.24kg/cm^2, H = 4~8m, T=3~7sec, γ=0.25, λ=1.0, α=0.34, Ω=0.18.

Some results are plotted in Fig.4 and 5. Fig.4 shows the three dimensional surface and contours for the initial, developed and deformed topographic of beach profile under wave and stormy tidal actions. Fig.5(a) presents a comparison of the original and effected coastline by general wave and stormy tidal wave Fig.5(b) presents a comparison of the



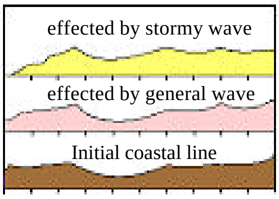
(a) initial topographic of the beach profile (b) developed beach profile under general wave (c) deformed beach profile under stormy tidal wave

Fig.4 Three dimensional surface and contours for the initial, developed and deformed topographic of beach profile under wave and stormy tidal actions

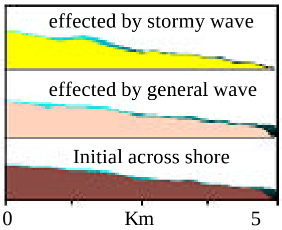
cross section at $x=0.75Km$ among the original and effected by general and stormy wave. It can be found from these figures that the stormy wave has a significant effect to the development of the coast both for shoreline and cross-shore and the evolution of coastal line is coupled with the changes of beach bottom.

Conclusions

The model developed in this paper presents a feasibility study of numerical simulation for coastal profile evolution (erosion/deposition). The significance of this model is coupling among pores-erosions sediment transports and stormy wave actions. The numerical simulation of coastal-topography change may carry out a full description of the interactions between evolution of beach profile and coastal line development.



(a) comparison of coastal



(b) comparison of across shore

Fig.5 Comparison of coastal profile in different cases

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