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EFFECT OF SURFACE WAVES ON DAM-RESERVOIR INTERACTION DURING THE EARTHQUAKE

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Summary

A time-domain exact solution for a coupled gravity dam-reservoir system is presented and applied to obtain the hydrodynamic pressure distributions on dams and dynamic responses when the gravity dam-reservoir system is subjected to ground motions. It is assumed that the water included in the reservoir is incompressible and irrotational. By these assumptions, the PDE governing the fluid domain is introduced and by satisfying relevant boundary conditions and solving the PDE, the equation depending on the dam displacements and by which the hydrodynamic pressure in the fluid domain can be calculated is obtained.

Explicit time-domain exact equation, by which the hydrodynamic pressure exerting on the gravity dam and the dynamic responses of the dam can be calculated, is obtained by solving the dynamic equilibrium equation governing the dam structure in which the hydrodynamic pressure caused by dam-reservoir interaction is taken into account as an external force. Numerical results based on the solution of an example are presented.

Introduction

Hydrodynamic pressure distribution on dams due to earthquake ground motion was first solved analytically by westergaard (1933) [1]. Simultaneously, Von karman proposed some semi analytic formulae to calculate the hydrodynamic pressure on dams during the ground motions. Their approach became famous as *added mass method* which was a basic solution to the dynamic problems and designs with which many engineers and designers dealt. Since then a number of studies have been carried out.

Zangar (1952) used electric analog to analyze the hydrodynamic pressure distribution. Chopra established a number of extensive investigations relevant to the subject in 1967. He attempted to take some factors into account such as water compressibility and absorptive effect of bottom of the reservoir. Chwang and Housner (1978a) extended the momentum-balance method developed by Von karman to investigate the hydrodynamic pressure distribution on dams with sloping faces. Chwang (1978b) obtained the hydrodynamic pressure distribution on sloping dams based on the exact theory. At the end of 1980s and the beginning of 1990s Tsai et al proposed a number of analytic and semi analytic time-domain formulae to calculate hydrodynamic pressure exerting on dams. In all the investigation mentioned, the effect of surface waves was neglected.

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Recently, because of the development and sophistication of the computers and numerical methods, investigators are interested in making contribution to the solution of the problem using numerical methods so, analytic and exact solutions are rare in the literature. In this paper effect of the surface waves is taken into account.

GOVERNING EQUATION AND BOUNDARY CONDITIONS

Neglecting internal viscosity and assuming an incompressible and irrotational fluid included in the reservoir of a dam-reservoir system (fig. 1), a potential function such as Φ governing the water motion in the reservoir and satisfying following equation called the Laplace equation can be introduced:

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial z^2}=0 \tag{1}$$

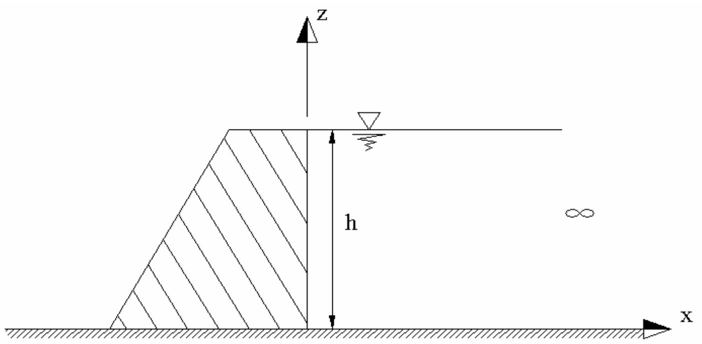

Figure 1: A gravity dam-reservoir system

The velocity components of the water particles can be obtained as below:

$$v=-\frac{\partial\Phi}{\partial x} \quad , \quad w=-\frac{\partial\Phi}{\partial z} \tag{2}$$

Where, v and w represent velocity components in the x and z direction respectively. Adopting linear theory of wave propagation and linearized form of Bernoulli equation and combining dynamic free surface boundary condition (DFSBC) and kinematic free surface boundary condition (KFSBC), free surface boundary condition is obtained:

$$\frac{\partial\Phi}{\partial z}+\frac{1}{g}\frac{\partial\Phi}{\partial t^2}=0 \tag{3}$$

Where, g and t represent the gravity acceleration and the time respectively. There is no flow in direction perpendicular to the bottom of the reservoir so, it yields:

$$w=-\frac{\partial\Phi}{\partial z}=0 \tag{4}$$

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Progressive waves and standing waves, generated by the dam structure vibration, travel away from the dam satisfying following periodic boundary condition in far field extending to infinity:

$$\Phi(x\ ,\ t+T) = \Phi(x\ ,\ t) \tag{5}$$

Standing waves decay with increasing x so, they should satisfy following condition:

$$\Phi(x,t) \approx 0 \quad ,\ x \to \infty \tag{6}$$

Progressive waves satisfy following periodic boundary condition:

$$\Phi(x+L\ ,\ t) = \Phi(x\ ,\ t) \tag{7}$$

In the equations above; L and T represent the wave length and period of the waves respectively. Applying theory of wave makers [2] and assuming simultaneous water particles vibration with the dam by a frequency equal to that of the first natural mode of the dam structure vibration, following kinematic boundary condition is obtained:

$$v(x=0,z,t) = -\frac{\partial\Phi(x=0,z,t)}{\partial x} = \left(\dot{u}_g(t) + \sum_{n=1}^{\infty}\phi_n(z)\dot{y}_n(t)\right)\cos\omega_1 t \tag{9}$$

Where, $\dot{u}_g(t)$ and $\dot{y}(t)$ represent velocity of the ground and the particles of the structure along the upstream face respectively and $\phi_n(z)$ represents nth natural mode shape function of the structure and ω_1 represents frequency of the first natural mode. Applying separation of variables to the Eq. 1 with respect to the linearity of the Φ:

$$\Phi = A_p \cosh k_p z \sin(k_p z - \omega_1 t) + \sum_{n=1}^{\infty} G_n \exp(-k_n^s x)\cos k_n^s z \cos\omega_1 t \tag{10}$$

Where, A_p and G_n are arbitrary coefficients. k_p and k^s_n are the progressive and standing wave numbers respectively which, can be obtained by the following equations:

$$\omega_1^{\ 2} = g k_p \tanh k_p h \quad , \quad \omega_1^{\ 2} = -g k_s \tan k_s h \tag{11}$$

There are clearly an infinite number of solutions as k_s to Eq. 11 and all are possible. Substituting Eq. 10 into Eq. 9 yields:

$$A_p = \frac{\int_0^h \dot{u}_g(t)\cosh k_p z dz + \sum_{r=1}^{\infty}\int_0^h \phi_r(z)\cosh k_p z dz \dot{y}_r(t)}{-k_p\int_0^h \cosh^2 k_p z dz} \tag{12}$$

$$G_n = \frac{\int_0^h \dot{u}_g(t)\cos k_n^s z dz + \sum_{r=1}^{\infty}\int_0^h \phi_r(z)\cos k_n^s z dz \dot{y}_r(t) dz}{k_n^s\int_0^h \cos^2 k_n^s z dz} \tag{13}$$

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The hydrodynamic pressure field is related to the potential function by following equation:

$$p = \rho \frac{\partial \Phi}{\partial t} \tag{14}$$

Where, ρ represents density of the water. Substituting Eq. 12 and Eq. 13 into Eq. 10, Eq. 14 can therefore yield the pressure distribution on the upstream face of the dam. The generalized load, p_n, exerting on the dam due to the hydrodynamic pressure, is shown as:

$$p_n(t) = \phi_n(z) p(x = 0, z, t) \tag{15}$$

SOLUTION TO THE INTERACTION EQUATION

According to the fig. 1, the modal equilibrium equation governing the dam structure is expressed by following equation [3]:

$$M_n \ddot{y}_n(t) + \omega_n^2 M_n y_n(t) = -V_n(t) - P_n(t), n = 1,2,3,...,\infty \tag{16}$$

Substituting Eq. 15 into Eq. 16 yields:

$$M_n \ddot{y}_n(t) + \sum_{m=1}^{\infty} W'_{mn} \dot{y}_n(t) + \omega_n{}^2 M_n y_n(t) = -V_n(t) - P_n^r(t) - F_n(t) - \sum_{m=1}^{\infty} W''_{mn} \ddot{y}_n(t) - F'_n(t) \tag{17}$$

The coefficients: W'_{mn} and F'_n are defined as:

$$W'_{mn} = \frac{\rho \omega_1 \cos \omega_1 t Q'_m Q'_n}{A} - \rho \omega_1 \sin \omega_1 t \sum_{k=1}^{\infty} \frac{Q''_{mk} Q''_{nk}}{A_k} \tag{18}$$

$$F_n^{'}(t) = P_n^p(t) + P_n^s(t) \tag{19}$$

The coefficients: $p^p{}_n$ and $p^s{}_n$ indicating the contribution of the surface waves are defined as:

$$P_n^p(t) = \frac{\rho \omega_1 Q'_n \cos \omega_1 t \int_0^h \cosh k_p z dz \dot{u}_g(t) + \rho Q'_n \sin \omega_1 t \int_0^h \cosh k_p z dz \ddot{u}_g(t)}{k_p \int_0^h \cosh^2 k_p z dz} \tag{20}$$

$$P_n^s(t) = \rho \cos \omega_1 t \sum_{k=1}^{\infty} \frac{Q''_{nk} \int_0^h \cos k_k^s z dz \ddot{u}_g(t)}{k_k^s \int_0^h \cos^2 k_k^s z dz} - \rho \omega_1 \sin \omega_1 t \sum_{k=1}^{\infty} \frac{Q''_{nk} \int_0^h \cos k_k^s z dz \dot{u}_g(t)}{k_k^s \int_0^h \cos^2 k_k^s z dz} \tag{21}$$

The functions: Q'_n and Q''_{nk} are expressed as:

$$Q'_n = \int_0^h \phi_n(z) \cosh k_p z dz \tag{22}$$

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$$Q''_{nk} = \int_0^h \phi_n(z)\cos k_k^s z dz \tag{23}$$

If the first M natural modes, contributing to the response of the dam, are taken into account, the equation for solving the dam-reservoir interaction will be expressed as:

$$\begin{bmatrix} m_{11} & m_{12} & . & m_{1M} \\ m_{21} & m_{22} & . & m_{2M} \\ . & . & . & . \\ m_{M1} & m_{M2} & . & m_{MM} \end{bmatrix} \begin{Bmatrix} \ddot{y}_1(t) \\ \ddot{y}_2(t) \\ . \\ \ddot{y}_M(t) \end{Bmatrix} + \begin{bmatrix} W'_{11} & W'_{12} & . & W'_{1M} \\ W'_{21} & W'_{22} & . & W'_{2M} \\ . & . & . & . \\ W'_{M1} & W'_{M2} & . & W'_{MM} \end{bmatrix} \begin{Bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \\ . \\ \dot{y}_M(t) \end{Bmatrix} + \begin{bmatrix} K_{11} & 0 & 0 & 0 \\ 0 & K_{22} & 0 & 0 \\ 0 & 0 & . & 0 \\ 0 & 0 & 0 & K_{MM} \end{bmatrix} \begin{Bmatrix} y_1(t) \\ y_2(t) \\ . \\ y_M(t) \end{Bmatrix} = \begin{Bmatrix} L_1(t) \\ L_2(t) \\ . \\ L_M(t) \end{Bmatrix} \tag{24}$$

The coefficients: m_{ij} and K_{ii} and L_i are defined by following equations [3]:

$$m_{ij} = \frac{\rho \sin \omega_1 t Q'_i Q'_j}{k_p \int_0^h \cosh^2 k_p z dz} + \rho \cos \omega_1 t \sum_{k=1}^{N} \frac{Q''_{ik} Q''_{jk}}{k_k^s \int_0^h \cos^2 k_k^s z dz}, i \neq j \tag{25}$$

$$m_{ij} = M_i + \frac{\rho \sin \omega_1 t Q'_i Q'_j}{k_p \int_0^h \cosh^2 k_p z dz} + \rho \cos \omega_1 t \sum_{k=1}^{N} \frac{Q''_{ik} Q''_{jk}}{k_k^s \int_0^h \cos^2 k_k^s z dz}, i = j \tag{26}$$

$$L_i(t) = -V_i(t) - P_i^r(t) - F_i(t) - F'_i(t) \quad , \quad K_i = \omega_i^{\ 2} M_i \tag{27}$$

Solution to an example

Here a dam-reservoir system subjected to 1940 Elcentro earthquake (fig. 2) is analyzed. This system is specified as: h=180 m , m=36 ton/m , $EI = 9.8437 \times 10^7 \, ton.m^2$. Solution to this example has been presented in the reference [2] considering compressibility of the water and neglecting the surface waves. Here this example is solved by proposed equations, taking five natural modes for the structure and 35 modes (N=35) for the reservoir into account. Figure 3 shows the results obtained by the solution.

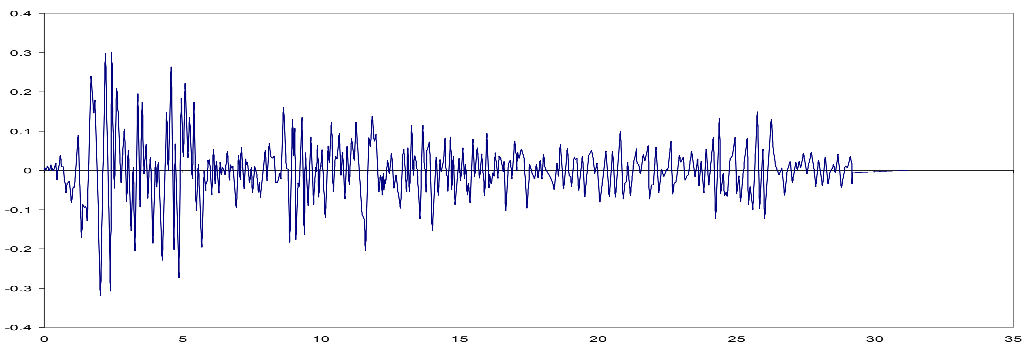

Figure 2: **1940 North-South Elcentro earthquake records**

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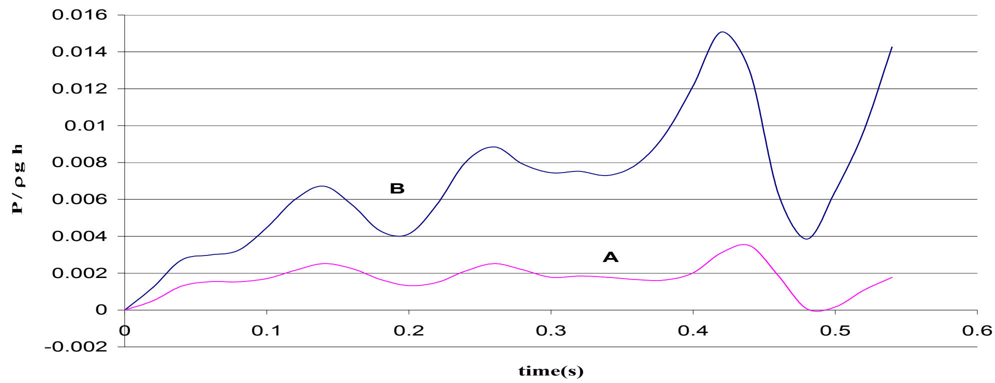

Figure 3:Hydrodynamic pressure calculated by considering surface waves (B) , Hydrodynamic pressure calculated by neglegting surface waves(A)

CONCLUSION

From fig. 3 it can be concluded that the magnitude of the hydrodynamic pressure calculated by considering surface wave effects is sometimes three times as great as that calculated by neglecting surface waves. Although surface waves affect on the hydrodynamic pressure noticeably, their effect on the total pressure obtained from the sum of the hydrostatic and hydrodynamic pressure is very low which approximately equals 2 percent of that. (This can be deduced from fig. 1 by adding a unit to the vertical axis).

Reference

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