Thermomechancial Wrinkling in Sandwich Panels with a Nonuniform Temperature Distribution

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Summary

The paper outlines an approach to the evaluation of combinations of mechanical compressive stresses and elevated temperature resulting in wrinkling of composite facings of a sandwich panel with foam core. A nonuniform temperature distribution through the thickness considered in the paper is typical in applications. The analysis is based on the extension of the Hoff method, accounting for variations of material properties affected by temperature. The results obtained for a linear temperature distribution through the thickness illustrate an increasing danger of wrinkling in sandwich structures affected by elevated temperature.

Introduction

Elevated temperature can significantly reduce the magnitude of applied stress that causes wrinkling in sandwich panels [1,2]. As was shown in these studies, while thermally induced compressive stresses represent an important factor, a degradation of material properties due to elevated temperature can be at least as important for the onset of wrinkling.

In numerous applications, one of the surfaces of a panel is subject to heat flux, resulting in temperature that is elevated on this surface and gradually decreases throughout the thickness. If the conductivity of the facing and core materials depends on temperature, a distribution of temperature through the thickness is nonlinear [2]. If the effect of temperature on the conductivity can be discounted, temperature varies linearly through the thickness of the facings and core (the gradients of temperature variation in the facings and core differ). This case is considered in the present paper that illustrates the solution of thermomechanical wrinkling problem using an extension of the Hoff method [3].

Analysis

Consider a sandwich panel subject to compression along the x-axis and a uniform temperature T_1 on the surface of the affected facing. The facing being

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thin, it is possible to assume that temperature does not vary through its thickness. However, temperature is a linear function of the thickness (z) coordinate in the core, as long as the conductivity of the core can be assumed constant. While the properties of the facing are affected by a uniform temperature, their degradation is uniform through the thickness. However, in the core the properties become a function of the thickness coordinate, reflecting a temperature distribution.

Although the solution presented here can be expanded to arbitrary temperature and property variations through the thickness of the core, due to space limitations, it is limited to the case where the stiffness of the core is a linear function of temperature. It is further assumed that compression in the x-direction is sufficiently large, so that the wrinkling wave is oriented perpendicular to this axis. Furthermore, the Poisson effect is neglected.

As was shown in a recent paper by Kardomateas [4], the solution of the wrinkling problem by the Hoff method is sufficiently accurate for sandwich beams with a thick and/or compliant core. The Hoff solution [4] neglecting the interaction between the facings is questionable in numerous practical situations [1]. However, this approach may be justified in panels affected by temperature resulting in a severe degradation of the stiffness of the core.

Let us assume that the stiffness of the core is a linear function of temperature, while the conductivity and the Poisson ratio are not affected by temperature. Ten the moduli of the core can be represented by

$$E_c = E_0 + E_1 z \qquad\qquad G_c = G_0 + G_1 z \tag{1}$$

where z is counted from the facing-core interface and E_i and G_i are determined according to the temperature distribution.

The mode shape of a wrinkle formed by the facing subject to a combination of compressive stresses σ_1 and temperature T_f is sinusoidal, so that the deflections in the facing and core are

$$w_f = W \sin\frac{\pi x}{l} \qquad\qquad w_c = W\frac{h-z}{h}\sin\frac{\pi x}{l} \tag{2}$$

respectively, where l is the length of the wrinkling wave and h is a depth of the affected zone in the core. According to the standard approach in the wrinkling analysis, in-plane displacements of the core along the x-axis are neglected.

The effect of pre-wrinkling deformations associated with compression and thermally induced bending on wrinkling loads is neglected. This assumption is justified if the length of the wrinkle is small, as is the case in the present problem where the core is compliant due to elevated temperature. Accordingly, strains in the core associated with wrinkling, i.e. strains superimposed on the pre-wrinkling strain state are $\varepsilon_z = w_c,_z$ and $\gamma_{xz} = w_c,_x$. The facing is treated as a slender plate (or beam), so that wrinkling related bending strain in the facing is $\varepsilon_x = -z' w_f,_{xx}$ where the coordinate z' is counted from the middle plane of the facing.

The core strain energy accumulated as a result of wrinkling is

$$U_c = \frac{1}{2}\int_0^l\int_0^h \left(E_c\varepsilon_z^2 + G_c\gamma_{xz}^2\right)dzdx = \frac{W^2}{4}\left[l\left(\frac{E_0}{h}+\frac{E_1}{2}\right)+\frac{\pi^2 h}{3l}\left(G_0+\frac{G_1 h}{4}\right)\right] \quad (3)$$

The strain energy in the facing added as a result of wrinkling is

$$U_f = \frac{1}{2}\int_0^l\int_0^{t_f} E_f\varepsilon_x^2 dz' dx = \frac{\pi^4 t_f^3 E_f W^2}{48 l^3} \quad (4)$$

where t_f is the thickness of the facing and E_f is its bending modulus reduced by a uniform temperature in the facing.

The energy of cumulative mechanical and thermal stresses $\overline{\sigma}$ is

$$U_s = -\frac{1}{2}\int_0^l \overline{\sigma} t_f w,_x^2\, dx = -\frac{\pi^2 \overline{\sigma} t_f W^2}{4l} \quad (5)$$

Minimizing the total energy with respect to the amplitude of the wrinkle yields the expression for the magnitude of the stress $\overline{\sigma}$ resulting in wrinkling:

$$\overline{\sigma} = \frac{E_0 l^2}{\pi^2 t_f h} + \frac{G_0 h}{3t_f} + \frac{\pi^2 E_f}{12}\left(\frac{t_f}{l}\right) + \frac{E_1 l^2}{2\pi^2 t_f} + \frac{G_1 h^2}{12 t_f} \quad (6)$$

Note that the first three terms in the right side of (6) coincide with the corresponding solution obtained by Hoff [3], without accounting for temperature.

The stress given by (6) is minimized with respect to the depth of the affected core, h, and the length of the wrinkle, l, to determine geometry and actual stress. This results in a system of two nonlinear algebraic equations:

$$\frac{l}{h} = \pi\sqrt{\frac{G_0 + \frac{G_1 h}{2}}{3E_0}} \qquad \frac{2E_0 l}{\pi^2 t_f h} - \frac{\pi^2 E_f t_f^2}{6l^3} + \frac{E_1 l}{\pi^2 t_f} = 0 \qquad (7)$$

Given geometry and material constants values, l and h can be determined from (7). Subsequently, the total wrinkling stress is available from (6). In the absence of temperature, the solution converges to the classical Hoff result.

The total wrinkling stress is of little interest in applications. Instead, it is important to determine the value of the applied mechanical stress causing wrinkling in the panels subject to a prescribed temperature or heat flux. The value of the applied stress is available from (6) by subtracting the thermal stress from $\overline{\sigma}$. In sandwich beams, i.e. neglecting the Poisson effect, the latter stress in a quasi-isotropic facing subject to a constant through the thickness temperature is given by $E_f \alpha_f T_f$ where α_f and T_f are the effective (average) coefficient of thermal expansion and the facing temperature, respectively

Numerical Examples and Discussion

Wrinkling stress was found for sandwich panels with quasi-isotropic glass-epoxy facings subject to an elevated temperature. The bending modulus of elasticity of the facings is based on data published in [5]. There is a lack of data on the variations of the stiffness of a typical foam core with temperature. Accordingly, the properties of the core used in the analysis correspond to those of Divinylcell H-30 at room temperature. These properties are assumed to be reduced by half at 100^{o}C. The results are presented in Figs. 1-3 for the case where the core is 30mm thick. Note that the facings considered in these examples are quite thin compared to the core. The choice is dictated by limitations of the applicability of the method of Hoff that neglects an interaction between the facings [1].

Results shown in Figs. 1-3 were obtained for the case where temperature of the opposite facing is equal to 20^{o}C. Three values of stress are shown in these figures. The total wrinkling stress, $\overline{\sigma} = S$, includes both mechanical and thermal components. The applied mechanical stress (S1) is found by subtracting the thermal stress $E_f \alpha_f T_f$ from the total stress. The applied mechanical stress S2 is calculated by subtracting $E_f \alpha_f (T_f - 20)$ from the total stress, i.e. by assumption that the "stress- free" temperature is 20^{o}C, rather than 0^{o}C.

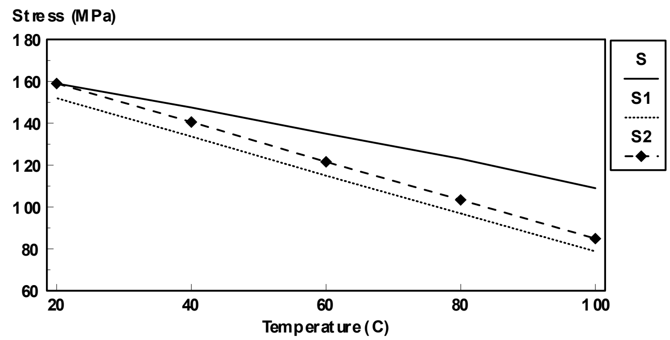

Fig. 1. Wrinkling stress for a quasi-isotropic 1.35 mm thick facing.

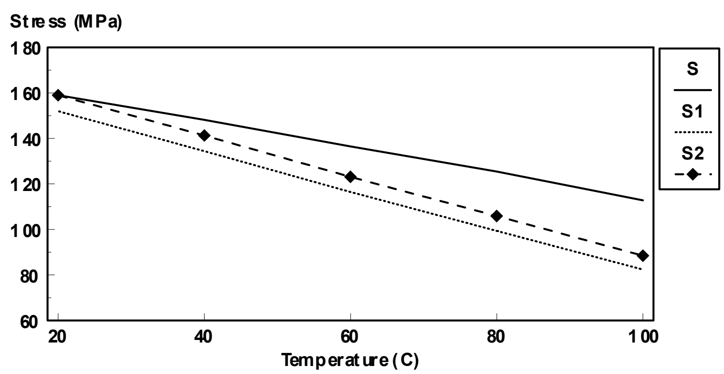

Fig. 2. Wrinkling stress for a quasi-isotropic 1.75 mm thick facing.

Numerical results shown in the paper illustrate that elevated temperature has a profound effect on the applied stress resulting in wrinkling. Surprisingly, the effect of the thickness of the facing on the magnitude of the wrinkling stress was relatively small. This can be explained by predominant effects of reduced stiffness of the facing and core as well as the contribution of thermally induced stresses. It was further confirmed that the effect of temperature on material properties is at least as important as thermally induced stresses, i.e. neglecting this effect may result in a dangerous overestimate of stability of the facing. The magnitude of the applied stress resulting in wrinkling is calculated by subtracting the thermally induced stress. The latter stress should be determined accounting for the actual "stress-free" temperature. As shown in the figures, even a small difference in the "stress-free" temperature may yield a noticeable error. In general, the applied wrinkling stress decreases almost proportionally to the applied temperature acting on the facing.

In conclusion, the paper illustrates that wrinkling may become a dominant mode of failure at high temperature, even if it does not cause damage at room temperature. This conclusion may have a profound effect on design process of sandwich structures operating in high-temperature conditions.

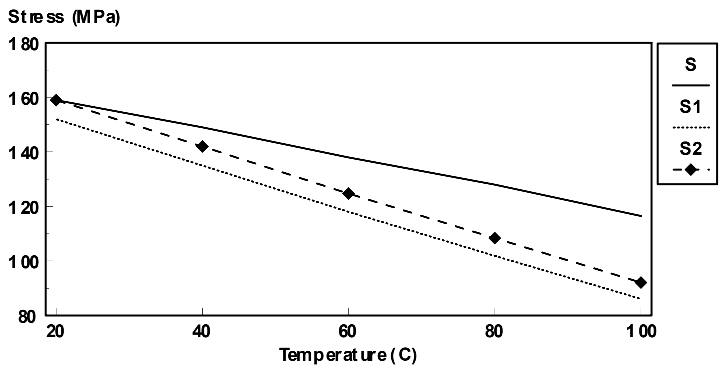

Fig. 3. Wrinkling stress for a quasi-isotropic 2.10 mm thick facing.

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References

1. Birman, V. (2004): "Thermomechanical Wrinkling in Composite Sandwich Structures," *AIAA Journal*, in press.
2. Birman, V. (2004): "Effect of Elevated Temperature on Wrinkling in Composite Sandwich Panels," *Proc. Annual SAMPE Conference*, May 2004.
3. Hoff, N.J. and Mautner, S.E. (1945): "The Buckling of Sandwich-Type Panels," *Journal of Aeronautical Sciences*, Vol. 21, pp. 285-297.
4. Kardomateas, G.A. (2003): "Wrinkling of Wide Sandwich Panels/Beams by an Elasticity Approach," *Proc. 6th Int. Conf. Sandwich Struct.,* Eds. Vinson, J.R., Rajapakse, Y.D.S., Carlsson, L.A., CRC Press, Boca Raton, 142-153.
5. Kulkarni, A.P. and Gibson, R.F. (2003): "Nondestructive Characterization of Effects of Temperature and Moisture on Elastic Moduli of Vinylester Resin and E-glass/vinylester Composite," *Proc. Amer. Soc. Composites 18th Annual Technical Conf.,* October 20-23, 2003, University of Florida, Gainesville.