3D Constitutive Model for Unbound Granular Materials

M. Oeser[1], S. Werkmeister[1], B. Möller[1], F. Wellner[1]

Summary

A phenomenologically-founded material model for unbound materials is presented in the present paper. This model, which takes account of elastic and plastic deformation rates, is calibrated on the basis of laboratory tests [1], [2] and, analogous to a material model for asphalt [3], is implemented in a numerical model [4]. The model is verified on the basis of a reference computation. The paper also considers the treatment of large load alternation numbers in the numerical analysis process.

Introduction

A realistic description of the load-bearing behaviour of pavements requires an appropriate geometrical and structural model of the layered construction as well as the foundation soil. Various numerical computational methods are available for this purpose. For example, the boundary element method may be effectively applied for modelling the load-bearing behaviour of the foundation soil. The method of finite elements offers a suitable means of accounting for the load-bearing action of pavement construction layers in the numerical analysis process. Besides the most exact description possible of the mechanics of layered pavement constructions it is also necessary to realistically model the behaviour of the construction materials used (asphalt and unbound materials).

Phenomenological and Mathematical Description of the Material Behaviour of Unbound Materials

Unbound materials exhibit nonlinear *elastic* and *plastic deformation properties*; initially they do not possess a "purely elastic" potential, i.e. reversible and irreversible deformation contributions always develop independent of the magnitude of the load. Temporal changes in material properties, as e.g. in the case of asphalt, do not occur. Due to the absence of bonding forces between individual grains this material is only capable of transferring compressive forces. An increase in compressive forces leads to an increase in the contact surfaces between neighbouring grains. This results in an increase in stiffness, i.e. with a linear increase in stress the elastic deformations increase degreessively. Material behaviour is not only influenced by the effects of stress but is also dependent on moisture content, in situ density, grain shape, grain diameter, and the composition and type of material. For the purpose of computing the *elastic deformation contributions* ε^{el} relationships between the elasticity modulus E [kN/m²] as well as Poisson's ratio μ and the rotationally symmetric principal stresses (σ_z [kN/m²], σ_r [kN/m²]) are given in [2]. The parameters of Eqs. (1a; b) for a granodiorite (4% moisture content) are listed in Table 1.

[1] Dresden University of Technology, Dresden, Germany

$$E = \left(Q + C \cdot \left(\sigma^r\right)^{Q_1} \right) \cdot \left(\sigma^z\right)^{Q_2} + D \; ; \qquad \mu = R \cdot \frac{\sigma^z}{\sigma^r} + A \cdot \sigma^r + B \qquad \text{(1a; 1b)}$$

Q	=	5,386.1	Q_1	=	0.593	D	=	20.000	A	=	-0.0024
C	=	2,315.6	Q_2	=	0.333	R	=	0.017	B	=	0.3520

Table 1 Parameters for elastic deformations

The *accumulated plastic deformations* $\varepsilon_z^{\Sigma pl}$ for an unbound material specimen are shown qualitatively in Fig. 1. For the purpose of describing the plastic deformation behaviour mathematically it is appropriate to subdivide the load regime into three regions (A, B and C). Small stresses are assigned to region A. In region A the increase in plastic deformations tails off with increasing load alternation number (consolidation). With an increase in the load alternation number at constant stress, plastic deformations no longer increase or only increase to a minor degree in region A. The material then exhibits purely elastic behaviour. Following initial consolidation a plastic deformation increase proportional to the load alternation number becomes established in region B. For large load alternation numbers so-called polishing effects lead to a progressive increase in plastic deformations. This is due to a reduction in the surface roughness of individual grains, which means that that lower intergranular shear stresses may be transferred from the grain matrix. Large stresses are assigned to region C, in which a progressive deformation increase occurs following consolidation (even for small load alternation numbers). With regard to the serviceability and durability of pavements the occurrence of progressive deformation increase must be eliminated by appropriate constructional measures.

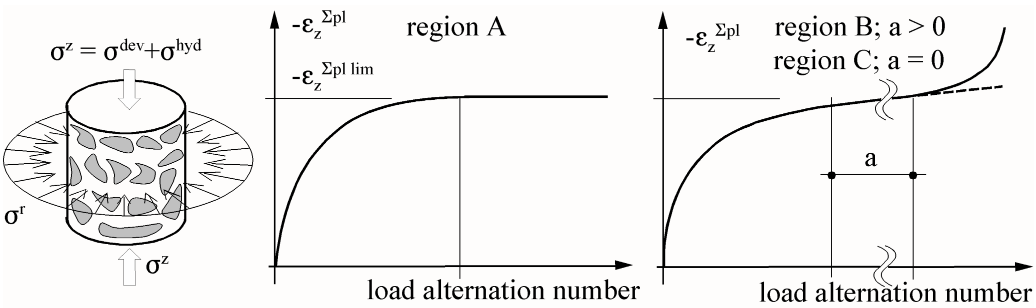

Fig. 1 Asphalt specimen and accumulated plastic deformations in regions A and B

Functional relationships between the accumulated plastic deformations ($\varepsilon_z^{\Sigma pl}$ [‰]) and the rotationally symmetric principal stresses (σ_z [kN/m²], σ_r [kN/m²]) for the loading regions A and B are given in [1].

$$\varepsilon^{\Sigma pl} = \mathfrak{A} \cdot \left(LW \cdot 10^{-3} \right)^{\mathfrak{B}} \qquad (2)$$

The following holds for region A:

$$\mathfrak{A} = a_{1A} \cdot e^{a_{2A} \cdot \sigma_r} \cdot \sigma_z^2 + a_{3A} \cdot \sigma_r^{a_{4A}} \cdot \sigma_z$$

$$\mathfrak{B} = b_{1A} \cdot e^{b_{2A} \cdot \sigma_r} \cdot \sigma_z + b_{3A} \cdot \sigma_r^{b_{4A}}$$

The following holds for region B:

$$\mathfrak{A} = a_{1B} \cdot \sigma_r^{a_{2B}} \cdot \left(\sigma_z / \sigma_r\right)^2 + a_{3B} \cdot \sigma_r^{a_{4B}} \cdot \left(\sigma_z / \sigma_r\right)$$

$$\mathfrak{B} = b_{1B} \cdot \sigma_r^{a_{2B}} \cdot \left(\sigma_z / \sigma_r\right) + b_{3B} \cdot \sigma_r^{b_{4B}}$$

The parameters a_{1A} to b_{4B} for a granodiorite (4% moisture content) are listed in Table 2. A criterion for the assignment of stresses occurring in regions A and B is given by Eq. (3). The condition pertaining to region A is that $\sigma_z \leq \sigma_{zlim}$, and for region B, that $\sigma_z > \sigma_{zlim}$.

a_{1A} = 0.00001	a_{2A} = -0.00970	a_{3A} = 0.00001	a_{4A} = 0.41340
b_{1A} = 0.00090	b_{2A} = -0.01070	b_{3A} = 0.00670	b_{4A} = 0.55790
a_{1B} = 0.00020	a_{2B} = 1.45140	a_{3B} = -0.00040	a_{4B} = 1.44070
b_{1B} = 0.01020	b_{2B} = 0.19950	b_{3B} = 0.00400	b_{4B} = 0.68440

Table 2 Parameters for accumulated plastic deformations

Three-dimensional Differential Formulation of the Material Model

In the three-dimensional formulation of the material model it is assumed that the stress rates $\dot{\sigma}_a$ (a = 1, …, 6) are dependent on the initial stresses σ_b and the deformation rates $\dot{\varepsilon}_b$, (b = 1, …, 6); $\dot{\sigma}_a = F(\sigma_b, \dot{\varepsilon}_b)$. Moreover, it is postulated [1] that plastic deformations result from relative displacements between neighbouring grains and are only initiated by the deviatoric part of the stresses σ_a^{dev}. Elastic deformations are caused by the hydrostatic and deviatoric stress contributions $\sigma_a^{hyd} + \sigma_a^{dev}$.

$$\dot{\sigma}_a = \dot{\sigma}_a^{hyd} + \dot{\sigma}_a^{dev} = C_{ab}^{el} \cdot f^{el} \cdot \left(\dot{\varepsilon}_b^{el,hyd} + \dot{\varepsilon}_b^{el,dev}\right) \qquad (3)$$

C_{ab}^{el} is the elasticity matrix. If the increase in plastic deformations is only due to the deviatoric part of the stresses, it follows that the hydrostatic (volume-changing) contribution to plastic deformations is zero.

$$\dot{\varepsilon}_b = \dot{\varepsilon}_b^{el,hyd} + \dot{\varepsilon}_b^{el,dev} + \dot{\varepsilon}_b^{pl,dev} \quad \text{und} \quad \dot{\varepsilon}_b^{pl,hyd} = 0 \qquad (4)$$

In order to compute plastic deformation rates the yield law of v.MISES is applied. The yield law permits the computation of three-dimensional deformation rates from one-dimensional reference plastic strain rates $\dot{\varepsilon}^{pl}$.

$$\dot{\varepsilon}_b^{pl,dev} = \mathfrak{F}_b(\sigma_b^{dev}) \cdot \dot{\varepsilon}^{pl} \cdot f^{pl} \quad \text{mit} \quad \mathfrak{F}_b(\sigma_b^{dev}) = \sqrt{3} \Big/ \left(2 \cdot \sqrt{I_2^{dev}}\right) \cdot \sigma_b^{dev} \qquad (5)$$

I_2^{dev} is the second invariant of the stress deviator. The required relationship $\dot{\sigma}_a = F(\sigma_b, \dot{\varepsilon}_b)$ between stress rates and stresses as well as deformation rates is given by

Eqs. (3), (4) and (5). Eqs. (4) and (5) are substituted into Eq. (3) and subdivided into two summands.

$$\dot{\sigma}_a = \dot{\sigma}_a^R + \dot{\sigma}_a^I; \quad \dot{\sigma}_a^R = C_{ab}^{el} \cdot \dot{\varepsilon}_b \cdot f^{el}; \quad \dot{\sigma}_a^I = -C_{ab}^{el} \cdot \mathfrak{F}_b(\sigma_b^{dev}) \cdot \dot{\varepsilon}^{pl} \cdot f^{el} \cdot f^{pl} \qquad \text{(6a; 6b; 6c)}$$

Postulates regarding elasto-plastic material models

The requirements necessary to compute the stress rates $\dot{\sigma}_a = F(\sigma_b, \dot{\varepsilon}_b)$ are *material objectivity, first order homogeneity regarding deformation rates, n^th order homogeneity regarding stresses, and irreversibility*. Moreover, the *dissipation condition* must also be fulfilled. *Material objectivity* is ensured by implementing invariant variables in the set of Eqs. (6). The one-dimensional reference plastic strain rate $\dot{\varepsilon}^{pl}$ in Eq. (6c) may be computed with the aid of Eq. (2). Eq. (2) is (test-related) dependent on the rotationally - symmetric principal stress components σ_z and σ_r. In order to approximately compute σ_z and σ_r from the three-dimensional stress state and fulfil the material objectivity requirement, invariant variables of the rotationally symmetric principal stress state and the three-dimensional stress state (I_1 and I_2^{dev}) are determined and equated.

$$\sigma_z + 2 \cdot \sigma_r = -I_1; \quad \frac{1}{3}(\sigma_z - \sigma_r)^2 = I_2^{dev} \quad \rightarrow \quad \sigma_z = \frac{-I_1 + 2 \cdot \sqrt{3 \cdot I_2^{dev}}}{3}; \quad \sigma_r = \frac{-I_1 - \sqrt{3 \cdot I_2^{dev}}}{3} \qquad \text{(7a-d)}$$

I_1 is the first stress invariant. The requirement of *first order homogeneity with respect to deformation rates* means that stress rates and deformation rates must be linearly proportional; $\dot{\sigma}_a(\sigma_b, \lambda \cdot \dot{\varepsilon}_b) = \lambda \cdot \dot{\sigma}_a(\sigma_b, \dot{\varepsilon}_b)$. In the case of *n^th order homogeneity with respect to stresses* it is required that stresses and stress rates are related by n^{th} order proportionality; $\dot{\sigma}_a(\lambda \cdot \sigma_b, \dot{\varepsilon}_b) = \lambda^n \cdot \dot{\sigma}_a(\sigma_b, \dot{\varepsilon}_b)$. Both of these requirements were established by GUDEHUS [6] and are based on a wide variety of tests on unbound materials. *Irreversibility* implies that deformation reversal must not take place along the stress path established during deformation development; $\dot{\sigma}_a(\sigma_b, -\dot{\varepsilon}_b) \neq -\dot{\sigma}_a(\sigma_b, \dot{\varepsilon}_b)$. In order to fulfil the irreversibility requirement the two functions f^{el} and f^{pl} are defined as follows:

$$f^{el} = \left(1 - \frac{I_1(\sigma_b)}{|I_1(\sigma_b)|}\right) \cdot \frac{1}{2} + \left(1 + \frac{I_1(\sigma_b)}{|I_1(\sigma_b)|}\right) \cdot \left(1 - \frac{I_1(\dot{\varepsilon}_b)}{|I_1(\dot{\varepsilon}_b)|}\right) \cdot \frac{1}{4}; \quad f^{pl} = \left(1 - \frac{I_1(\dot{\varepsilon}_b)}{|I_1(\dot{\varepsilon}_b)|}\right) \cdot \frac{1}{2} \qquad \text{(8a; 8b)}$$

The first summand in Eq. (a) is always unity in the case of compressive stresses ($\sigma_b < 0$) whereas the second summand is zero. Tensile stresses ($\sigma_b > 0$) cannot be sustained by unbound material. If a compressive stress state is fully depleted by positive deformation increases, the material yields for $\sigma_b > 0$ in the event that further positive deformation increases are effective. In this case both of the summands of Eq. (8a) are zero. If negative deformation increases become effective once the yield limit $\sigma_b > 0$ has been attained, the second summand of Eq. (8a) is unity, i.e. a repeated compressive stress state develops in the material.

Three-dimensional Incremental Formulation of the Material Model

In order to solve the differential equations (6a) and (6b) the loading process is modelled incrementally. The system reactions w (stresses, deformations; w = σ, ε) within an increment are computed by interpolating between the initial and final values over the increment; $w(\xi) = w^{[i+1]}\cdot\xi + w^{[i]}\cdot(1-\xi)$. At the beginning of the increment $\xi = 0$ whereas at the end of the increment $\xi = 1$, cf. [3]. The equality $\dot{w}(\xi) = \Delta w$ applies for the derivatives of the system reactions. The differential equations (6a,b) are then:

$$\Delta\sigma_a^R = C_{ab}^{el} \cdot \Delta\varepsilon_b \cdot f^{el}; \qquad \Delta\sigma_a^I = -C_{ab}^{el} \cdot \mathfrak{F}_b(\sigma_b^{dev}) \cdot \Delta\varepsilon^{pl} \cdot f^{el} \cdot f^{pl} \qquad (9a; 9b)$$

Eq. (9) must be formulated as an eigenvalue problem and developed as a TAYLOR series with respect to the terms $\partial\Delta\sigma_a$ and $\partial\Delta\varepsilon_b$. The solution is obtained iteratively by the NEWTON-RAPHSON method. A disadvantage of the method presented here is that each load alternation must be computed separately. The plastic deformation increase per load alternation is:

$$\Delta\varepsilon^{pl} = \mathfrak{A} \cdot \left(\left(LW+1\right)\cdot 10^{-3}\right)^{\mathfrak{B}} - \mathfrak{A} \cdot \left(LW \cdot 10^{-3}\right)^{\mathfrak{B}} \qquad (10)$$

In order to reduce the amount of numerical computations the cyclic loading process may be subdivided into load alternation increments. Each load alternation increment is thereby comprised of ΔLW load alternations. The load alternation number during the incremental computation is $LW(\xi) = LW^{[i]} + \Delta LW \cdot \xi$. $LW^{[i]}$ is the number of load alternations already applied at the beginning of the i^{th} increment. The deformation rate is obtained by differentiating Eq. (2) with respect to ξ.

$$\dot{\varepsilon}^{pl} = \frac{\partial\varepsilon^{\Sigma pl}}{\partial\xi} = \frac{\partial\varepsilon^{\Sigma pl}}{\partial LW} \cdot \frac{\partial LW}{\partial\xi} = \mathfrak{A} \cdot \mathfrak{B} \cdot \left(LW(\xi) \cdot 10^{-3}\right)^{\mathfrak{B}} \cdot \frac{\Delta LW}{LW(\xi)} = \Delta\varepsilon^{pl} \qquad (11)$$

The method presented here accelerates the numerical analysis process and may be applied under the assumption that geometrical changes in the construction to be analysed are either negligibly small or are mainly caused by the accumulated plastic deformation contributions.

Verification of the Material Model

The material model for unbound materials was verified by large-scale laboratory tests [5]. The results presented in [5] are based on a series of tests on a 0/32 gravel-sand mixture. The plan area of the test rig is 2.5 x 2.5 x 0.9 m. A cyclic load of 5 Hz was applied; the maximum applied load was 14.1 kN. By taking advantage of symmetry it was only necessary to model one quarter of the test specimen. The implemented FE mesh is shown in Fig. 2. The mesh consists of 1,985 isoparametric displacement elements with 28,515 displacement degrees of freedom.

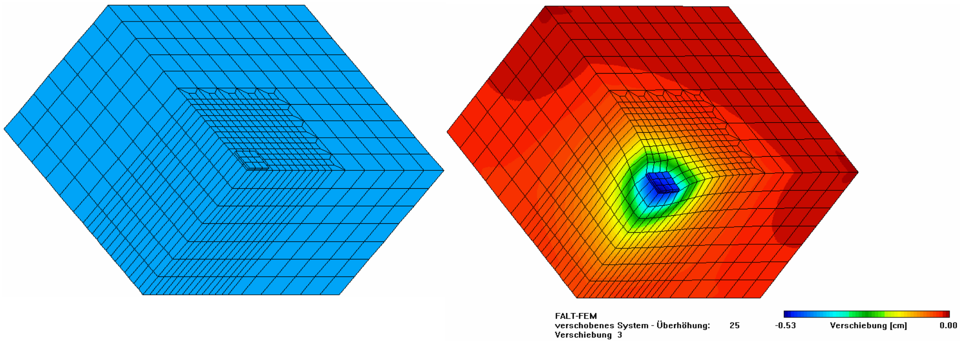

Fig. 2 FE mesh and displacements after one million load alternations

A comparison between the computed elastic and plastic vertical displacements FE(u^{el}); FE(u^{pl}) and the measured displacements LV(u^{el}); LV(u^{pl}) in the region of load application is presented in Table 3. The percentage differences AB are also given in the Table.

LW	FE(u^{el}) [mm]	LV(u^{el}) [mm]	AB	FE(u^{pl}) [mm]	LV(u^{pl}) [mm]	AB
$5 \cdot 10^5$	-0.161	-0.160	0.6	-3.857	-3.750	2.9
10^6	-0.161	-0.150	7.3	-5.250	-4.950	6.1

Table 3 Elastic and plastic displacements in the region of load application

Literature

1 Werkmeister, S. (2003): "Permanent Deformation Behavior of Unbound Granular Materials", *Ph D thesis, Dresden University of Technology*, Dresden

2 Numrich, R. (2003): "Non-linear-resilient Deformation Behavior of Unbound Granular Materials", *Ph D thesis, Dresden University of Technology*, Dresden

3 Möller, B. and Oeser, M (2003): "A New Numerical Model for Asphalt Pavements", *ICCES'03, July 2003, Corfu*

4 Möller, B. and Oeser, M (2002): "FE-BE Computational Model for Muli-Layered Road Construction", *3rd International Symposium – 3D Finite Element Modeling of Pavement Structures*, Amsterdam, pp. 209-234

5 Wellner, F. (1996): "Untersuchungen zum Einsatz von Geokunststoffen im Straßenbau", *Veröffentlichungen des Lehrstuhls für Straßenbau, TU Dresden*, Dresden, H. 4

6 Wu, W. (1992): "Hypo-plasticity as a Mathematical Model on the Mechanical Behaviour of Unbound Materials" *Ph D thesis, University Karlsruhe*, Karlsruhe