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Numerical Analysis of Laminated Composite Beam Based on Micromechanical Multi-Level Damage Model

S. H. Pyo[1], H. K. Lee[2], E. H. Lyu[3]

Summary

A micromechanics-based evolutionary damage model[1] is implemented into a finite element program ABAQUS to numerically predict the overall elastic behavior and damage evolution of laminated composite beam. A multi-level damage modeling process in accordance with the Weibull's probabilistic function is incorporated into the micromechanical framework to describe the sequential evolution of imperfect interfaces in the composites. To verify the implemented computational model, computational studies are undertaken by comparing the predicted load-displacement curve and experimental data available in literature under bending. The implemented computational model provides results that show good agreement with available benchmark problems. A parameter analysis is also carried out to illustrate the influence of the stacking sequences.

Introduction

Fiber-reinforced composite materials are being used increasingly in the area of high performance mechanical structures (aerospace, automotive, offshore, and other industrial applications) due to their high specific strength and stiffness[2]. However, composite materials are still relatively limited compared with their counterparts, i.e. homogeneous and isotropic materials such as metals, ceramics, and polymers[3]. A major limitation is in the lack of an efficient and versatile constitutive description for the composite materials[4]. Specifically, the composite load carrying capacity has not been well understood[5,6]. The present author has recently proposed a micromechanics-based evolutionary damage model for fiber-reinforced brittle matrix composites with microcracks and imperfect interface[1]. The present study implements the previous work[1] into the finite element program ABAQUS to numerically predict the overall elastic behavior and damage evolution of laminated composite beam.

A Constitutive Damage Model for Unidirectional Laminated Composites

The three-dimensional, micromechanical constitutive model for unidirectional laminated composites proposed by Lee and Pyo[1] is here summarized. Let us start by considering an initially perfectly bonded three-phase composite consisting of an elastic matrix (phase 0) with the bulk modulus k_0 and shear modulus μ_0, and

[1]Graduate Student, Dept. of Civil & Environ. Eng., Korea Advanced Institute of Science and Technology (KAIST), South Korea

[2]Associate Professor, Dept. of Civil & Environ. Eng., KAIST, South Korea

[3]Undergraduate Student, Dept. of Civil & Environ. Eng., KAIST, South Korea

randomly located yet unidirectionally aligned elastic circular fiber (phase 1) with the bulk modulus k_1, and shear modulus μ_1, and penny-shaped microcracks (phase 5). The fibers are assumed to be non-interacting and initially embedded firmly in the matrix with perfect interface. When loadings or deformations are applied and gradually increased, some initially perfectly bonded fibers are transformed to fibers with mild imperfect interface (phase 2), some fibers with mild imperfect interface are then transformed to fibers with severe imperfect interface (phase 3), and all fibers are transformed to completely debonded fibers that are regarded as cylindrical voids (phase 4), and microcracks are nucleated asymptotically within the proposed framework.

The effective stiffness tensor $\mathbf{C}^*$ for multi-phase, linear elastic composites containing arbitrarily non-aligned and/or dissimilar inclusions can be derived as[7]

$$\mathbf{C}^* = \mathbf{C}_0 \cdot \left[\mathbf{I} + \sum_{r=1}^{5} \left\{ \phi_r (\mathbf{A}_r + \mathbf{S}_r)^{-1} \cdot \left[\mathbf{I} - \phi_r \mathbf{S}_r \cdot (\mathbf{A}_r + \mathbf{S}_r)^{-1} \right]^{-1} \right\} \right] \quad (1)$$

where is $\mathbf{C}_r$ the elasticity tensor of the r-phase, "" is the tensor multiplication, $\mathbf{I}$ is the fourth-rank identity tensor, ϕ_r and denotes the volume fraction of the r-phase inclusion, and $\mathbf{S}_r$ signifies the Eshelby's tensor for the r-phase. The fourth-rank tensor $\mathbf{A}_r$ is defined as

$$\mathbf{A}_r \equiv (\mathbf{C}_r - \mathbf{C}_0)^{-1} \cdot \mathbf{C}_0 \quad (2)$$

With the help of the Eshelby's tensor for perfectly bonded cylindrical fibers, completely debonded cylindrical voids, cylindrical fibers with mild imperfect interface, cylindrical fibers with severe imperfect interface and penny-shaped microcracks, the stress-strain relation for the (unidirectional) continuous, fiber-reinforced brittle matrix composite was explicitly derived in our previous work[1].

Now we consider rotations through an angle θ about the X_2-axis to derive the stiffness of off-axis unidirectional fiber-reinforced composites. The angle θ is measured positive counterclockwise from the X_3-axis to the x_3-axis. Following the stiffness transformation law by Herakovich[8], the stiffness matrix of off-axis unidirectional fiber-reinforced composites $\bar{\mathbf{C}}$, which is the transformed stiffness matrix through an arbitrary angle θ about the 2-axis, is derived as

$$\bar{\mathbf{C}} = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} & 0 & \bar{C}_{15} & 0 \\ \bar{C}_{21} & \bar{C}_{22} & \bar{C}_{23} & 0 & \bar{C}_{25} & 0 \\ \bar{C}_{31} & \bar{C}_{32} & \bar{C}_{33} & 0 & \bar{C}_{35} & 0 \\ 0 & 0 & 0 & \bar{C}_{44} & 0 & \bar{C}_{46} \\ \bar{C}_{51} & \bar{C}_{52} & \bar{C}_{53} & 0 & \bar{C}_{55} & 0 \\ 0 & 0 & 0 & \bar{C}_{64} & 0 & \bar{C}_{66} \end{bmatrix} \quad (3)$$

Here, the components of $\bar{\mathbf{C}}$ are given in Appendix of Pyo and Lee[9]. A more detailed description on the micromechanical formulations for the effective elastic stiffness tensor for the off-axis unidirectional fiber-reinforced composites can be found in Lee and Pyo[1] and Pyo and Lee[9].

Multi-level elastic damage model is considered for a complete description of the sequential progression of imperfect interface in the composites. Following Lee and Pyo[10], the probability of imperfect interface is modeled as a two-parameter Weibull process and the average internal stresses of fibers are the controlling factor of the Weibull function. The current volume fractions of fibers are derived for multi-phase composite state (see Lee and Pyo[10] and Pyo and Lee[9]). Furthermore, microcrack nucleation in the matrix is modeled using the continuum damage model proposed by Karihaloo and Fu[11]. The constitutive model incorporating the multi-level damage model and microcrack nucleation model is then implemented into the finite element program ABAQUS using a user defined subroutine UMAT to characterize numerical prediction of elastic behavior and damage evolution.

Numerical Simulations for Laminated Composite Beam under Bending

For verification of the proposed constitutive model, we conduct comparison between the present prediction and experiments on a 12-ply carbon fiber-reinforced, epoxy matrix laminated beam, $[0/\pm45/0/90/0]_s$, subjected to three-point bending tested by Huang[12]. The beam has a span of 84 mm, a width of 15.1 mm and a thickness of 2.76 mm. Detail descriptions of material properties and model parameters are described in Pyo and Lee[9]. Figure 1 presents the comparison of load-displacement behavior between the present prediction and experimental data[12] under bending for laminated composite beam. As a whole, the present and the experimental data match well. A parameter analysis is also carried out to illustrate the influence of the stacking sequences (Quasi-isotropic:

The load-displacement curves of the laminated composite beam with various stacking sequence. It is observed from the figure that quasi-isotropic and cross-ply laminated composite beams are stiffer than angle-ply laminated composite beams under three-point bending test.

Concluding Remarks

A computational investigation of the behavior of laminated composite beam under three-point bending has been conducted. A constitutive model for laminated composites[1] is implemented into the finite element program ABAQUS for an accurate characterization of the elastic behavior and damage evolution of laminated composites. The present prediction is compared with available experimental data in the literature to verify the proposed computational model. The predicted load-displacement behavior of laminated composite beam is shown to have a good corre-

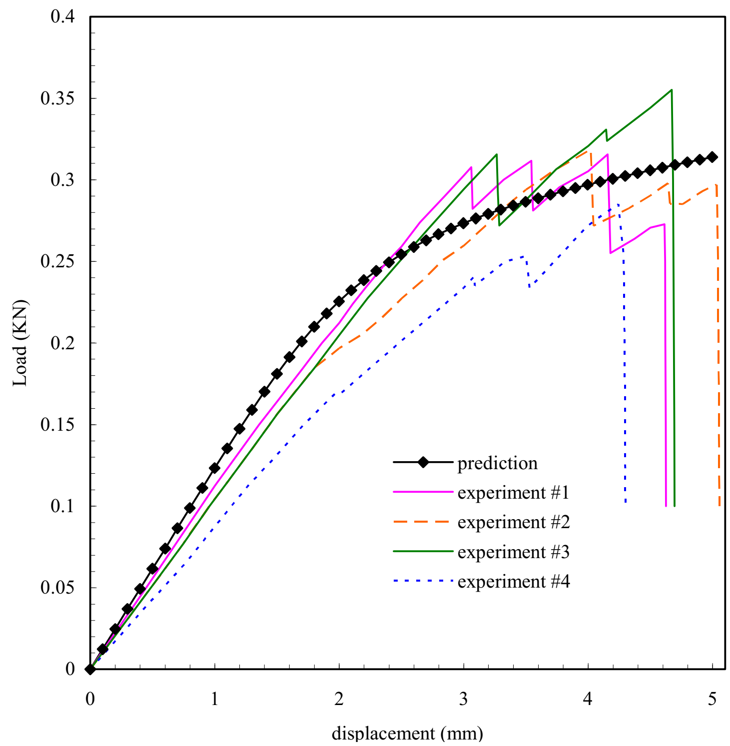

Figure 1: The comparison of load-displacement curves between the present prediction and experimental data[12] under three-point bending[9]

lation with the experimental data[12]. From a parametric study, stacking sequences of quasi-isotropic and cross-ply are stiffer than that of angle-ply under bending.

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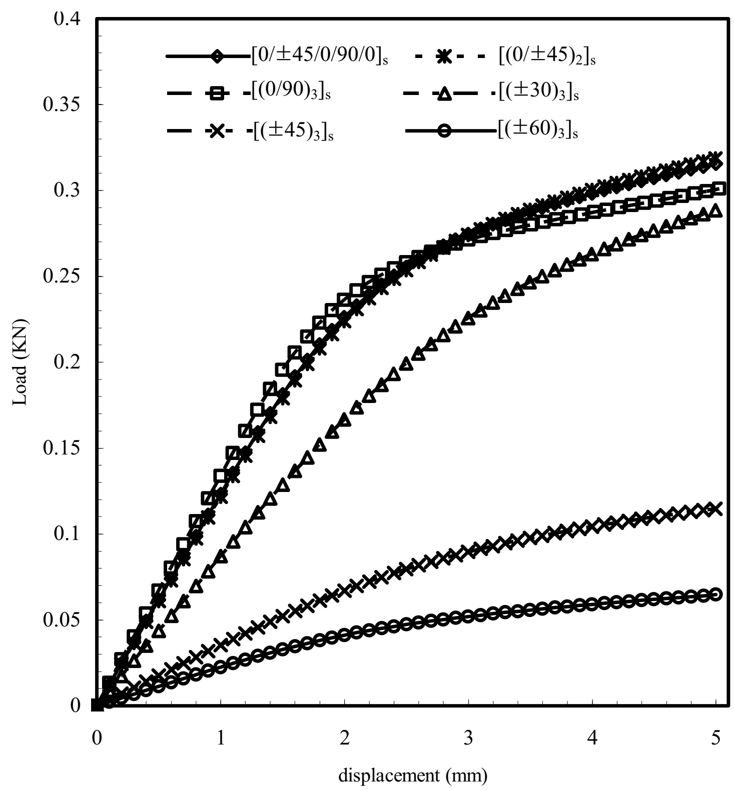

Figure 2: The load-displacement curves of laminated composite beam with various stacking sequence

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