Simulation of 3D Solid Tumour Angiogenesis Including Arteriole, Capillary and Venule

Jie Wu^{1,2}, Quan Long², Shixiong Xu¹, Anwar R. Padhani³, Yuping Jiang⁴

Summary

In this paper, a 3D mathematical model of tumour angiogenesis is developed, to generate a circulative tumour vasculature functional for blood microcirculation. The model follows that of Anderson and Chaplain $(1998)^{[1]}$ with three exceptions: (a) Extension of the model from 2D to 3D, one arteriole and one venule are induced as two parent vessels with the purpose to form an intact circulation network for blood flow; (b) Generation of the network able to penetrate into the tumour interior rather than the exterior only, allowing for the heterogeneous spatial mechanical environments in different tumour regions; (c) Consideration of the branching generations with different vessel diameters, based on which three groups of vessels, such as arterioles, venules and capillaries are classified. The present study mainly contains four steps: 1. Generation of 3D angiogenic vasculature induced from one arteriole and one venule, with branching generations considered. 2. Examination of vessel connectivity among each other to construct a functional network for blood circulation, investigation of sensitivity of network architectures to changes in some model parameters. 3. Simulation of blood flow in the developed vasculature. 4. Comparisons of blood flow calculated on the network induced from an arteriolevenule system and from a single parent vessel.

The results show that the networks from simulations could present basic geometric and morphological features of tumour vasculatures, such as tortuosity, branching and anastomosis. The sensitivity analysis exhibits the flexibility and controllability of the created network structures, and indicates that the present model could produce vasculatures of specific architectures, corresponding to not only various types of tumours, but also same tumour with different phases of its development or during anti-angiogenesis therapy, by adjusting the values of related parameters. Moreover, comparisons of blood flow calculated on the two networks mentioned above not only demonstrate the impact of network structures on blood flow, but also highlight the validity for incorporating the arteriole-veinule system into the angiogenesis model. (The figure below is one angiogenic vasculature from simulation, with three groups of vessels defined in the present study according to different branching generations).

¹Department of Mechanics and Engineering Science, Fudan University, Shanghai, China ²Brunel Institute for Bioengineering, Brunel University, Uxbridge, Middlesex, UK

³Paul Strickland Scanner Centre, Mount Vernon Hospital, Northwood, Middlesex, UK

⁴Department of neurology, Huashan hospital, Fudan University, Shanghai, China

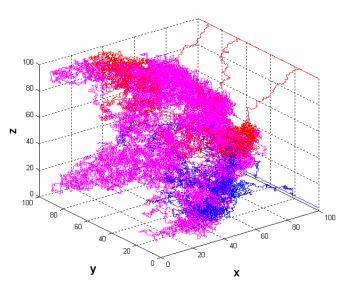


Figure 1: 3D angiogenic vasculature from simulation with three groups of vessels: red—arterioles, blue—venules and pink—capillaries, according to their branching generations defined in the model.

References

1. Anderson, A.R.A.; Chaplain, M.A.J. (1998): Continuous and Discrete Mathematical Models of Tumor-induced Angiogenesis. Bulletin of Mathematical Biology 60, 857-900.