HYDROELASTIC VIBRATION OF A RECTANGUAR PLATE IN CONTACT WITH A LIQUID FILLED IN A RIGID SEMI–CYLINDRICAL VESSEL

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ABSTRACT

An approximate method is proposed for the free vibration analysis of a rectangular plate clamped along the edges of a liquid–filled semi–circular cylindrical vessel. The vessel is transformed into an equivalent rigid hexahedron vessel for a simple theoretical formulation. It is assumed that the plate is fixed at the edges of the rigid vessel and it is in contact with a liquid contained in the vessel. In the theory, the transverse dynamic displacement of the wet plate is expressed by using a combination of the dynamic displacements of a dry rectangular plate with unknown coefficients. The liquid motion can be formulated with a displacement potential, since the liquid is assumed to be incompressible and inviscid. By considering the compatibility condition and by applying the Rayleigh−Ritz method, the natural frequencies and associated mode shapes of the wet plate are obtained. To verify the theory, a finite element analysis is carried out by using the ANSYS code. It is verified that the theoretical method can predict the wet natural frequencies well with an excellent accuracy.

1. INTRODUCTION

Dynamic characteristics of structures in contact with a contained liquid are very important in various engineering applications. There have been several theoretical and experimental researches on the free vibration problem of rectangular plates in contact with a liquid in recent years. Several researches on a hydroelastic vibration of a single rectangular plate in contact with a liquid have been published (Robinson et al. 1990; Kwak, 1996; Cheung et al. 2000; Zhou et al. 2000). Dynamic characteristics of a submerged plate in a liquid medium have also been studied (Fu et al. 1987; Haddara et al. 1996; Liang et al. 2001; Ergin et al. 2003; Yadykin et al. 2003).

However, studies on a hydroelastic vibration analysis of a rectangular plate in contact with a liquid contained in an uncommon vessel are scarce in the literature. Hence, a theoretical free vibration analysis of a flexible rectangular plate in contact with a liquid contained in a rigid semi−circular cylindrical vessel is developed here.

2. THEORY

2.1 Dynamic displacement of the plate

A flexible rectangular plate in contact with a liquid is supported by a rigid semi−circular cylindrical vessel as shown in Figure 1. A plate with length L and width $2R$, and thickness h is assumed to be clamped along its edges. The semi−circular cylindrical cavity is fully filled with an ideal liquid. The Rayleigh–Ritz method is introduced to obtain the approximate natural frequencies and associated mode shapes of the rectangular plate coupled with the liquid.

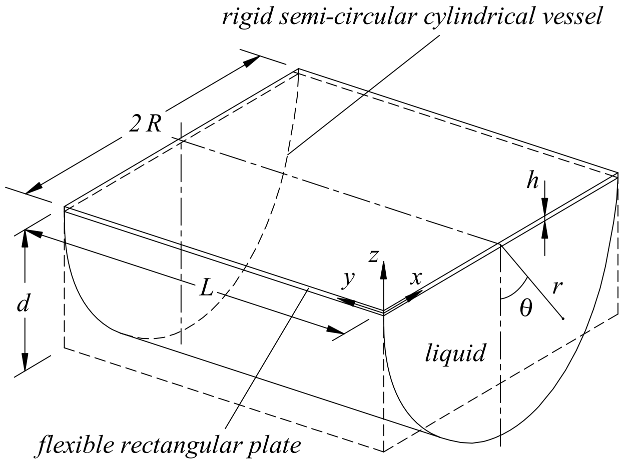


Figure 1: A rectangular plate in contact with a liquid contained in a rigid semi–circular cylindrical vessel.

The wet dynamic mode shapes of the plate can be approximated by a combination of a finite number of admissible modal functions, $W_{mn}(x,y)$ and unknown coefficients, q_{mn}.

$$w(x,y,t) = \sum_{m=1}^{M} \sum_{n=1}^{N} q_{mn}\, W_{mn}(x,y) \exp(\mathrm{i}\,\omega t), \quad (1)$$

where, $\mathrm{i} = \sqrt{-1}$ and ω is the circular natural frequency of the plate. A sufficiently large finite enough number of terms, M and N shall be considered. The transverse modal function for the rectangular plate can be defined by a multiplication of the x– and y– directional admissible beam functions.

$$W_{mn}(x,y) = X_m(x)\, Y_n(y). \quad (2)$$

Additionally, the slopes and the displacements must be zero for the clamped boundary condition of the plate.

$$X_m{}'(0) = X_m(2R) = X_m{}'(0) = X_m(2R) = 0, \quad (3)$$

$$Y_n{}'(0) = Y_n(L) = Y_n{}'(0) = Y_n(L) = 0. \quad (4)$$

The orthogonal admissible functions satisfying Eqs. (3) and (4) can be selected as modal functions of a dry beam with clamped boundary conditions. Therefore, they will be written

$$X_m(x) = \cosh\left[\lambda_m (x)/2R\right] - \cos\left[\lambda_m (x)/2R\right] - \sigma_m\left\{\sinh\left[\lambda_m (x)/2R\right] - \sin\left[\lambda_m (x)/2R\right]\right\}, \quad (5)$$

$$Y_n(y) = \cosh\left[\lambda_n (y)/L\right] - \cos\left[\lambda_n (y)/L\right] - \sigma_n\left\{\sinh\left[\lambda_n (y)/L\right] - \sin\left[\lambda_n (y)/L\right]\right\}, \quad (6)$$

where the frequency parameters λ_m, λ_n and the other parameters σ_m, σ_n are delineated in a text book (Blevins, 1979).

2.2 Formulation for the liquid

As shown in Figure 1, the liquid surrounding the rectangular plate and the rigid semi–circular cylindrical vessel wall is assumed as invisid, irrotational and incompressible. However it is very difficult to describe a liquid motion in the cylindrical coordinates, since a singularity at $r = 0$ cannot be reflected. Therefore, a liquid motion can be described in Cartesian coordinates by the Laplace equation for the velocity potential:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0. \quad (7)$$

The velocity potential can be separated with respect to the time and the space. Thus the velocity potential can be replaced with the displacement potential ϕ of Eq. (8).

$$\Phi(x,y,z,t) = \mathrm{i}\,\omega\,\phi(x,y)\, f(z) \exp(\mathrm{i}\,\omega t). \quad (8)$$

For the formulation of the theory in Cartesian coordinates, the equivalent liquid depth, d is determined on the basis of the same liquid volume.

$$d = \pi R/4. \quad (9)$$

The boundary conditions along the rigid vessel walls of the transformed hexahedron cavity, by assuring a zero liquid displacement, lead to:

$$\partial\phi(x,0,z)/\partial y = \partial\phi(x,L,z)/\partial y = 0, \quad (10)$$

$$\partial\phi(0,y,z)/\partial x = \partial\phi(2R,y,z)/\partial x = 0, \quad (11)$$

$$\partial\phi(x,y,d)/\partial z = 0. \quad (12)$$

The displacement potential of the liquid, $\phi(x,y,z)$, satisfying the boundary conditions of Eqs. (10)–(11), can be written as the following:

$$\phi(x,y,z) = \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} \cos(\alpha_r x)\cos(\beta_s y) \times\left\{B_{rs}\sinh(\gamma_{rs} z) + C_{rs}\cosh(\gamma_{rs} z)\right\}, \quad (13)$$

where

$$\alpha_r = (r-1)\pi/2R, \qquad \beta_s = (s-1)\pi/L, \quad (14, 15)$$

$$\gamma_{rs} = \sqrt{\alpha_r^2 + \beta_s^2}. \quad (16)$$

The unknown coefficients, B_{rs} and C_{rs}, can be determined by the boundary condition of Eq. (12). Therefore, the displacement potential of the liquid will be reduced to:

$$\phi(x,y,z) = \sum_{r=1}^{\infty}\sum_{s=1}^{\infty} \cos(\alpha_r x)\cos(\beta_s y) \times\left\{\sinh(\gamma_{rs} z) - \cosh(\gamma_{rs} z)/\tanh(\gamma_{rs} d)\right\}. \quad (17)$$

On the other hand, as the structural displacement and the liquid displacement must be identical in the normal direction to the interface surface between the liquid and the plate, the compatibility condition at the wet surface yields:

$$\sum_{m=1}^{M}\sum_{n=1}^{N} q_{mn}\, W_{mn}(x,y) = \partial\phi(x,y,0)/\partial z. \quad (18)$$

Substituting Eqs. (2), (5), (6) and (17) into Eq. (18) results in:

$$\sum_{n=1}^{N}\sum_{m=1}^{M} q_{mn}\Big[\cosh(\lambda_m x/2R)-\cos(\lambda_m x/2R) \\ -\sigma_m\{\sinh(\lambda_m x/2R)-\sin(\lambda_m x/2R)\}\Big] \\ \times\Big[\cosh(\lambda_n y/L)-\cos(\lambda_n y/L) \\ -\sigma_n\{\sinh(\lambda_n y/L)-\sin(\lambda_n y/L)\}\Big] \\ =\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\gamma_{rs} B_{rs}\cos(\alpha_r x)\cos(\beta_s y)\cdot \quad (19)$$

After a multiplication of $\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\cos(\alpha_r x)\cos(\beta_s y)$ for both sides of Eq. (19), integrations along [0, $2R$] and [0, L] are performed for the finite Fourier transform. A relationship between the unknown coefficients B_{rs} and q_{mn} can be obtained by using the orthogonal property of the sinusoidal functions:

$$B_{rs}=\frac{1}{\gamma_{rs}\,\xi_r\,\eta_s}\sum_{n=1}^{N}\sum_{m=1}^{M} q_{mn}\,\Gamma_{mr}\,\Gamma_{ns}, \quad (20)$$

where

$$\xi_r=\int_0^{2R}\cos^2(\alpha_r x), \qquad \eta_s=\int_0^{L}\cos^2(\beta_s y)\,dy, \quad (21, 22)$$

$$a_{mr}=\int_0^{2R}\cosh(\lambda_m x/2R)\cos(\alpha_r x)\,dx, \quad (23)$$

$$b_{mr}=\int_0^{2R}\cos(\lambda_m x/2R)\cos(\alpha_r x)\,dx, \quad (24)$$

$$c_{mr}=\int_0^{2R}\sinh(\lambda_m x/2R)\cos(\alpha_r x)\,dx, \quad (25)$$

$$h_{mr}=\int_0^{2R}\sin(\lambda_m x/2R)\cos(\alpha_r x)\,dx, \quad (26)$$

$$\Gamma_{mr}=a_{mr}-b_{mr}-\sigma_m\{c_{mr}-h_{mr}\}, \quad (27)$$

$$\Gamma_{ns}=a_{ns}-b_{ns}-\sigma_n\{c_{ns}-h_{ns}\}. \quad (28)$$

The coefficients, a_{ns}, b_{ns}, c_{ns} and h_{ns} can be obtained by a similar integration of Eqs. (23)–(26). Eventually, the displacement potential of the liquid satisfying all the liquid boundary conditions and the compatibility condition will become:

$$\phi(x,y,z)=\sum_{m=1}^{M}\sum_{n=1}^{N} q_{mn}\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\frac{\Gamma_{mr}\,\Gamma_{ns}}{\gamma_{rs}\,\xi_r\,\eta_s}\cos(\alpha_r x) \\ \times\cos(\beta_s y)\left\{\sinh(\gamma_{rs} z)-\frac{\cosh(\gamma_{rs} z)}{\tanh(\gamma_{rs} d)}\right\}\Bigg]. \quad (29)$$

2.3 Rayleigh−Ritz method

A sufficiently large finite number of terms, N and M, are considered to obtain a converged solution, and a vector $\boldsymbol{q}$ of the unknown parameters is defined as,

$$\boldsymbol{q}=\{\, q_{11} \quad q_{12} \quad q_{13}\cdots q_{1N} \quad q_{21} \quad q_{22} \quad q_{23}\cdots q_{MN}\,\}^T. \quad (30)$$

The reference kinetic energy, T^* of the plate can be obtained by using the orthogonal property of the modal functions of a clamped dry beam:

$$T^*=\frac{\rho h}{2}\boldsymbol{q}^T\,\boldsymbol{Z}\,\boldsymbol{q}, \quad (31)$$

where ρ is the mass density of the plate. The matrix $\boldsymbol{Z}$ of Eq. (31) will be an $MN\times MN$ diagonal matrix and it can be written as:

$$Z=\int_0^{2R}\int_0^{L} W_{mn}\,W_{jk}\,dy\,dx=2RL\cdot \quad (32)$$

Similarily, the indices j and k also indicate the j-th mode in the x–direction, and the k-th mode in the y–direction of the admissible functions, respectively.

The maximum potential energy V of the rectangular plate can be computed by integrating the derivatives of the admissible modal functions:

$$V=\frac{D}{2}\int_0^{L}\int_0^{2R}\Bigg[\left\{\frac{\partial^2 W_{mn}}{\partial x^2}\frac{\partial^2 W_{jk}}{\partial x^2}\right\}+\left\{\frac{\partial^2 W_{mn}}{\partial y^2}\frac{\partial^2 W_{jk}}{\partial y^2}\right\} \\ +\mu\left\{\frac{\partial^2 W_{mn}}{\partial x^2}\frac{\partial^2 W_{jk}}{\partial y^2}+\frac{\partial^2 W_{jk}}{\partial x^2}\frac{\partial^2 W_{mn}}{\partial y^2}\right\} \\ +2(1-\mu)\left\{\frac{\partial^2 W_{mn}}{\partial x\partial y}\frac{\partial^2 W_{jk}}{\partial x\partial y}\right\}\Bigg]dx\,dy, \quad (33)$$

where $D=Eh^3/12(1-\mu^2)$ is the flexural rigidity of the rectangular plate; μ and E are the Poisson's ratio and the modulus of elasticity, respectively. Inserting the admissible functions described in Eqs. (2), (5) and (6) into Eq. (33) gives the maximum potential energy of the rectangular plate as a matrix form:

$$V=\frac{D}{2}\boldsymbol{q}^T\boldsymbol{U}\,\boldsymbol{q}, \quad (34)$$

where $\boldsymbol{U}$ is also a square matrix which can be derived as:

$$U = \Lambda_{4mj}\, \Xi_{1nk} + \Lambda_{1mj}\, \Xi_{4nk} + \mu\,(\Lambda_{2mj}\, \Xi_{3nk} + \Lambda_{3mj}\, \Xi_{2nk}) + 2\,(1-\mu)\,\Lambda_{5mj}\, \Xi_{5nk}\,, \qquad (35)$$

and

$$\Lambda_{1mj} = \int_0^{2R} X_m(x)\, X_j(x)\, dx\,, \qquad (36)$$

$$\Lambda_{2mj} = \int_0^{2R} X_m''(x)\, X_j(x)\, dx\,, \qquad (37)$$

$$\Lambda_{3mj} = \int_0^{2R} X_m(x)\, X_j''(x)\, dx\,, \qquad (38)$$

$$\Lambda_{4mj} = \int_0^{2R} X_m''(x)\, X_j''(x)\, dx\,, \qquad (39)$$

$$\Lambda_{5mj} = \int_0^{2R} X_m'(x)\, X_j'(x)\, dx\,. \qquad (40)$$

The apostrophe in the above equations indicates a derivative with respect to the corresponding variable. The coefficients with respect to the y–diection can also be similary defined.

The relationship between the reference kinetic energy of each mode multiplied by its square circular frequency and the maximum potential energy of the same mode is used to find the natural frequencies of the dry plate. The Rayleigh quotient for the plate vibration in the dry condition is given as V/T^*. Minimizing the Rayleigh quotient with respect to the unknown parameters $\boldsymbol{q}$, the Galerkin equation can be obtained:

$$D\,\mathbf{U}\,\boldsymbol{q} - \omega^2 \rho\, h\, \mathbf{Z}\,\boldsymbol{q} = \boldsymbol{0}\,. \qquad (41)$$

On the other hand, the reference kinetic energy of the liquid can be evaluated from its boundary motion.

$$T_o^* = \frac{-\rho_o}{2} \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{N} q_{jk} \int_0^L \int_0^{2R} W_{jk}\, \phi(x,y,0)\, dx\, dy\,, \qquad (42)$$

where ρ_o is the mass density of the contained liquid. Substituting Eqs. (2), (5), (6), and (29) into Eq. (42), we obtain:

$$T_o^* = \frac{\rho_o}{2} \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{j=1}^{M} \sum_{k=1}^{N} q_{mn}\, q_{jk}$$

$$\times \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} \frac{\Gamma_{mr}\, \Gamma_{jr}\, \Gamma_{ns}\, \Gamma_{ks}}{\gamma_{rs}\, \xi_r\, \eta_s \tanh\left(\gamma_{rs}\, d\right)} = \rho_o\, \boldsymbol{q}^T \boldsymbol{G}\, \boldsymbol{q}\,. \qquad (43)$$

where

$$G = \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} \frac{\Gamma_{mr}\, \Gamma_{jr}\, \Gamma_{ns}\, \Gamma_{ks}}{\gamma_{rs}\, \xi_r\, \eta_s \tanh\left(\gamma_{rs}\, d\right)}\,. \qquad (44)$$

Finally, the Galerkin equation can therefore be obtained for the wet case:

$$D\,\mathbf{U}\,\boldsymbol{q} - \omega^2 \left\{ \rho\, h\, \mathbf{Z} + \rho_o\, \boldsymbol{G} \right\} \boldsymbol{q} = \boldsymbol{0}\,. \qquad (45)$$

The natural frequencies of the rectangular plate in contact with the liquid can be obtained from the determinant of Eq. (45).

3. EXAMPLE AND DISCUSSION

3.1 Verification of the approximate method

The eigenvalues of Eq. (45) were extracted, on the basis of the analysis, in order to obtain the natural frequencies of a rectangular plate in contact with a liquid contained in a semi−circular rigid cylindrical vessel. A commercial software, Mathcad (version 2000 Professional) was used for the calculation. The frequency equation derived in the previous sections involves double infinite series of algebraic terms. The series expansion terms r and s were set at 50, and the number of admissible functions is m = 10 in the x–direction and n = 10 in the y–direction respectively, to obtain a converged solution.

In order to check on the validity of the proposed theory, a finite element analysis was also carried out for the same liquid–coupled system by using a commercial computer code, ANSYS (release 10.0). A finite element model was constructed with the same plate geometry, boundary conditions and material properties used in the theoretical calculation. The dimensions and physical properties of the rectangular plate and the liquid are listed in Table 1.

Dimension or properties	Value
Width of plate	360 mm
Length of plate	480 mm
Thickness of plate	3 mm
Poisson's ratio of plate	0.3
Young's modulus of plate	69.0 GPa
Density of plate	2700 kg m^{-3}
Density of liquid	1000 kg m^{-3}

~~(55)~~

Table 1: Dimensions and properties of the system.

The viscosity and compressibility of the liquid were neglected in both the theoretical calculation and the finite element analysis.

Finite element analysis using a commercial computer code, ANSYS software, was carried out to obtain the natural frequencies and mode shapes of the rectangular plate in contact with the liquid. A three–dimensional finite element model was composed of three–dimensional contained liquid elements (FLUID80) and elastic shell elements (SHELL63). The liquid movement along the rigid walls was restricted to the normal direction only, in order to realize Eqs. (10)–(12). The displacement of the liquid element nodes adjacent to the surface of the wetted rectangular plate coincided with that of the rectangular plate so that the finite element model could simulate Eq. (18).

The rectangular plate was divided into 1600 elastic shell elements with the same size, and the liquid region of the finite element model was segmented into 25600 fluid elements. A clamped boundary condition along the plate edges was applied in the finite element model by constraining all the displacements and rotations. The number of 50 modal frequencies was extracted in the finite element analysis and the associated mode shapes plotted, by employing the Block Lanczos method.

3.2 Comparison of the results

The theoretical natural frequencies of the rectangular plate are listed in Table 2 and compared with the FEM results for the dry condition, and in Table 3 for the wet condition. The discrepancy between the theoretical and FEM results is less than 0.3% within the 12th serial mode for the dry condition. This result shows that a combination of the dry beam modes can approximate the plate mode shapes excellently for the dry rectangular plate with clamped boundary conditions.

Mode		Natural frequency (Hz)		
m'	*n'*	ANSYS	Theory	Error (%)
0	0	161.7	161.8	0.06
0	1	271.2	271.4	0.07
1	0	380.4	380.7	0.08
0	2	451.6	452.0	0.09
1	1	481.3	481.9	0.12
1	2	651.7	652.9	0.18
0	3	697.9	698.6	0.10
2	0	713.0	713.7	0.10
2	1	810.1	811.5	0.17
1	3	890.2	892.3	0.24
2	2	974.1	976.8	0.27
0	4	1008.3	1009.6	0.13

Table 2: Natural frequencies of the dry clamped rectangular plate.

Mode		Natural frequency (Hz)		
m'	*n'*	ANSYS	Theory	Error (%)
0	1	69.2	69.7	0.72
1	0	114.8	114.0	– 0.70
0	2	147.1	147.9	0.54
1	1	165.2	166.6	0.84
1	2	251.5	254.2	1.06
2	0	259.8	262.8	1.14
0	3	265.9	267.6	0.64
2	1	323.8	328.4	1.40
1	3	376.2	380.3	1.08
2	2	409.3	416.1	1.63
0	4	438.9	441.9	0.68
3	0	491.2	498.0	1.36

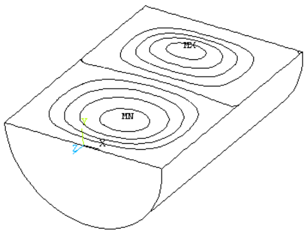
Table 3: Natural frequencies of the clamped rectangular plate in contact with water filled with a semi−circular cylindrical vessel.

The indices *m'* and *n'* in Tables 1 and 2 indicate the number of nodal lines in the *y*– and *x*– directions, respectively. It is found that the theoretical natural frequencies agree excellently with the finite element results within a 2 % discrepancy range for the wet case. It was observed that most of the theoretical natural frequencies slightly overestimate the FEM results. Therefore, an approximation based on the equivalent depth can be useful in engineering applications. The fundamental natural frequency of the wet plate has a mode with *m'* = 0 and *n'* = 1 instead of a mode with *m'* = 0 and *n'* = 0 so that the liquid volume can be conserved. Also we observed that the natural frequency of the fundamental wet mode decreased approximately by 25% of that of the corresponding dry mode due to the added mass effect of the liquid.

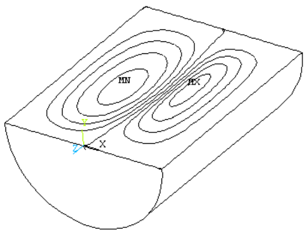
The typical wet mode shapes of a wet rectangular plate are illustrated in Figure 2. It was observed that the wet mode shapes of the higher modes, such as the 6th, 7th, 8th and 10th modes, are distorted from the classical dry mode shapes of a clamped rectangular plate.

4. CONCLUSIONS

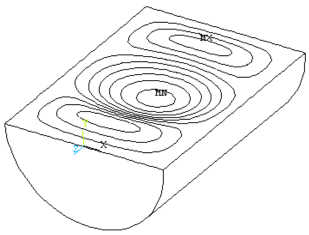
An analytical method based on the Rayleigh −Ritz approach to calculate approximate natural frequencies of a rectangular plate in contact with a bounded liquid was developed. The wet dynamic modal functions of the plate were expanded in terms of the finite Fourier series for a compatibility requirement along the contacting surface between the plate and the liquid. The proposed analytical method was verified by observing an excellent agreement with three–dimensional finite element analysis results. It was found that the wet natural frequencies decreased owing to the added mass of the liquid.



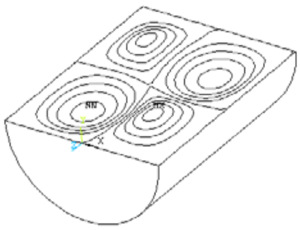
(1st mode, 69.2 Hz)



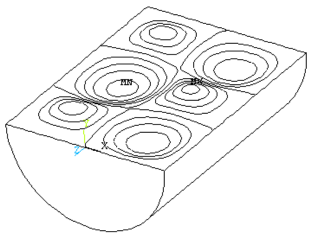
(2nd mode, 114.8 Hz)



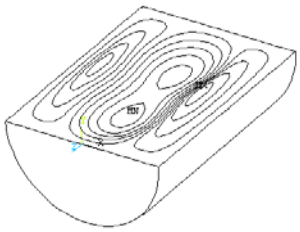
(3rd mode, 147.1 Hz)



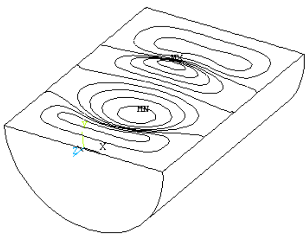
(4th mode, 165.2 Hz)



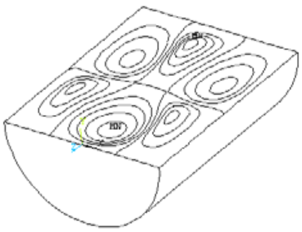
(5th mode, 251.5 Hz)



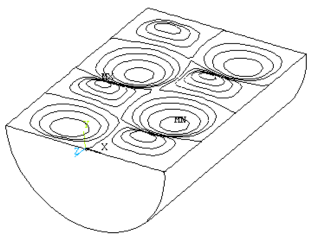
(6th mode, 259.8 Hz)



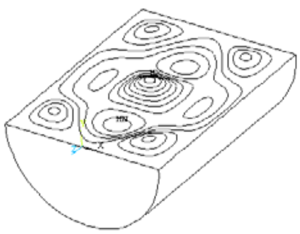
(7th mode, 265.9 Hz)



(8th mode, 323.8 Hz)



(9th mode, 376.2 Hz)



(10th mode, 409.3 Hz)

Figure 2: Mode shapes of the rectangular plate in contact with water filled in the rigid semi–circular cylindrical vessel (ANSYS results).

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