POD-GALERKIN METHOD IN FLUID STRUCTURE INTERACTION

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ABSTRACT

This paper describes Reduced Order Modeling (ROM) in Fluid Structure Interaction (FSI) and discusses Proper Orthogonal Decomposition (POD) utilization. In fact to use POD in a moving domain, a reference fixed domain, with a fixed uniform grid, is introduced. Next the solution is interpolated from the time-variant grid to the fixed uniform grid to obtain the global velocity field (fluid and structure). Thus PODs modes are obtained for the global velocity field and not only for the fluid velocity. Then a method to reduce dynamical system in rigid body fluid interaction is developed. This method uses the fictitious domain method approach, which consists in treating the entire fluid-solid domain as a fluid by adding a distributed Lagrange multiplier in the weak formulation on solid domain. The method is tested on a two-dimensional case of rigid body immersed in a fluid. The results are compared with computational solution and discussed.

1. INTRODUCTION

Constructing a Reduced-Order Model (ROM), in order to reduce the size of the model and the computational cost and also to obtain a good simulation, is essential in this Fluid Structure Interaction (FSI) domain. We chosed to study POD capacities in fluid structure interaction. In fact, this method, introduced by Lumley (1967) in fluid mechanic, has been intensively used since 90's in many applications. In structure mechanics, POD is a recent investigation domain similar to modal analysis. Its study in moving domain case is recent. The moving carater of fluid domain is the main difficulty for application, because this characteristic is incompatible with the POD classic formulation.

2. THE PROPER ORTHOGONAL DECOMPOSITION (POD)

In this section, the POD method is briefly introduced. A detailed methodology is already stipulated in literature (Allery (2002); Berkooz and al. (1993)). The POD consists in finding for a field $v(t) \in H$ (H a Hilbert space, $t \in [0,T]$, a function $\Phi \in H$ which gives the optimum representation for v on $[0,T]$ in H norm. In the case of $H = L^2(\Omega)$, it leads to solving the following eigenvalue problem :

$$\int_\Omega \mathcal{R}(x,y)\,\Phi(y)\,dy = \lambda\Phi(x) \qquad (1)$$

where $\mathcal{R}$ is the symetric spatial correlation tensor, defined non-negative, $\mathcal{R}(x,y) = \langle v(\bullet,x) \otimes v(\bullet,y)\rangle$, where $\langle\bullet\rangle$ denotes the temporal average or the statistic average operator for a random field (in the case of process ergodic this average are the same). Morever if the associated operator to (1) is compact, the Hilbert-Schmidt theory assures that there exists a set of positive eigenvalues $(\lambda_i)_{i\geq 1}$ which decrease to 0 and a set of eigenmodes $(\Phi_i)_{i\geq 1}$ which is a Hilbertien basis for H. The eigenvalue λ_i is the energy captured by the mode Φ_i. Thus, in practice only one ten modes are enough to keep more than 99.99% of the total energy. Thus a reduced space basis and a reduced dynamic system, obtained by projecting Navier-Stokes equation on this basis, can be constructed.
In the case of fluid structure interaction, the Ω domain would be the fluid domain, which moved. A POD basis, dependent on time, would be obtained, what is unconsistent.

3. POD APPLICATION IN MOVING DOMAIN

To resolve this difficulty, a reference fixed domain Ω, with a fixed uniform grid, is introduced, which

contains all the different configurations of the movings domains ($\Omega = \Omega_f(t) \cup \Omega_s(t)$, where $\Omega_f(t)$ denotes the fluid domain and $\Omega_s(t)$ the solid domain). Thus fluid-rigid body interaction problems can be study using fictious domain method (Glowinski and al. (1999); Laure and al. (2005)). This method consists in treating the entire fluid-solid rigid domain (the fictious domain) as a fluid, by using Navier-Stokes equations for solid rigid domain. Thus a weak formulation for the global domain Ω can be used. Find a field u such as div $u = 0$ and :
For all Φ a virtual velocity field, div $\Phi = 0$:

$$\int_\Omega \rho\left(\frac{\partial v}{\partial t} + v\cdot\nabla v\right)\Phi dx \quad + \int_\Omega 2\nu \mathbf{tr}\left(\mathbf{D}\left(v\right)\mathbf{D}\left(\Phi\right)\right)dx + \int_\Omega \mathbb{I}_{\Omega_S}\mathbf{tr}\left(\mathbf{D}\left(\lambda\right)\mathbf{D}\left(\Phi\right)\right)dx = 0 \tag{2}$$

$\mathbf{D}(v) = \dfrac{1}{2}\left(\nabla v + {}^t\nabla v\right)$, ρ et ν are defined on the global domain Ω :

$$\rho = \mathbb{I}_{\Omega_f}\rho_f + \left(\mathbb{I} - \mathbb{I}_{\Omega_f}\right)\rho_s \ ; \ \nu = \mathbb{I}_{\Omega_f}\nu_f + \left(\mathbb{I} - \mathbb{I}_{\Omega_F}\right)\nu_s$$

with $\mathbb{I}_{\Omega_s}(x,t)$, the characteristic function of solid domain (1 if $xin\Omega_s(t)$, else 0). ν_S is a penalization factor of the contrainst $\mathbf{D}(v) = 0$, which is solid rigid contrainst, and the symmetrical tensor $\mathbf{D}(\lambda)$ is the Lagrange multiplier associated to this contrainst. The solution at each time step is interpolated from the time-variant grid to the fixed uniform grid, and the basis obtained by solving (1) is truncated at N such as more than 99.99% of energy is captured. Thus the velocity field v is evaluated by using this truncated basis

$$v = \sum_{i=1}^{N} a_i(t)\,\Phi_i(x)$$

This decomposition is used in (2) to obtain the following reduced dynamical system :
For $n = 1..N$:

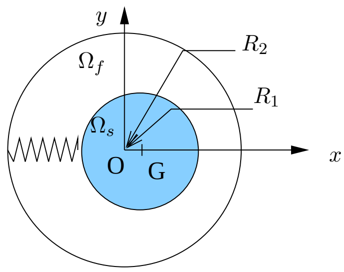
$$\begin{cases} \displaystyle\sum_{i=1}^{N}\frac{da_i}{dt}A_{in} + \sum_{i=1}^{N}a_iB_{in} + \sum_{i=1}^{N}\sum_{j=1}^{N}a_ia_jC_{ijn} = \sum_{i=1}^{N}b_iE_{in} \\ \dfrac{\partial\mathbb{I}_{\Omega_f}}{\partial t} + v\cdot\nabla\mathbb{I}_{\Omega_f} = 0 \end{cases} \tag{3}$$

with

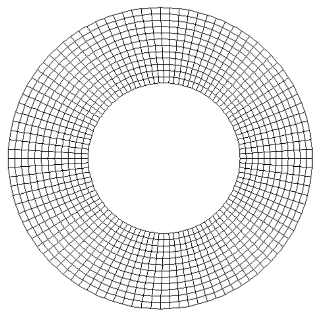
$$\begin{aligned} A_{in} &= \int_\Omega \rho\Phi_i\Phi_n dx \\ B_{in} &= \int_\Omega 2\mu\ \mathrm{Tr}\left(\mathbf{D}\left(\Phi_i\right)\mathbf{D}\left(\Phi_n\right)\right)dx \\ C_{ijn} &= \int_\Omega \rho\Phi_i\cdot\nabla\Phi_j\Phi_n dx \\ E_{in} &= -\int_{\Omega_S}\mathrm{Tr}\left(\mathbf{D}\left(\Phi_i\right)\mathbf{D}\left(\Phi_n\right)\right)dx \end{aligned} \tag{4}$$

The initial problem is transformed in a more simple system of ordinary differential equation in $a_i(t)$ with low degrees of freedom. Indeed, in practice the POD method gives a basis which is maximal in energy sens with only few functions.

4. TEST ON A TWO DIMENSIONAL CASE



(a)



(b) fluid mesh

Figure 1: Schematic description of the problem domain

The studied case considers a two-dimensional rigid cylinder immerged in an annular cavity (Figure 1(a)). At the beginning, the fluid is at rest and the rigid cylinder is removed from its equilibrium position. Due to the spring effect the solid moves, starts to oscillate and generates a fluid flow. The fluid flow generates fluide forces, which damp the fluid oscillations.

The motion of the fluid is governed by the incompressible Navier Stokes equations in the

ALE formulation Sarrate and al. (2001); Abouri and al. (2004) and computed using Castem code (CEA, 2005) during 6.28 s using the followings : $R_1 = 0.1\ m$, $R_2 = 0.2$ m, $\rho_f = 1000$ kg.m^{-3}, $\rho_s = 31.83$ kg.m^{-3}, $\mu_f = 0.001$ kg.m.s. The initial coordinates of the rigid body center are $x_0 = (0, 005, 0)$, and "at rest" the length of spring is equal to 0.1 m.

For the spatial discretisation of the Navier Stokes equations the finite element Crouzeix-Raviart ($\mathbb{Q}_2 - \mathbb{P}_1$) hasb been used. For the velocity-pression coupling, a projection method has been applicated, and a SUPG method for the convection term stabilisation has been employed.

The rigid body displacement follows the following equation :

$$x\left(t\right) = x_0 \cos\left[\omega\left(\xi\right)t\right]e^{-\xi\omega t} \tag{5}$$

where $\omega\left(\xi\right) = \omega\sqrt{1-\xi^2}$.

The damping parameter ξ can be computed on one pseudo-oscillation period, which allows to evaluate the numerical solution obtained by Castem. Indeed,

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \tag{6}$$

where δ denotes the decrement logarithmic curve of rigid body displacement. On Figure 2, the analytical solution and that obtained by Castem show a good fit, which validates the solution.

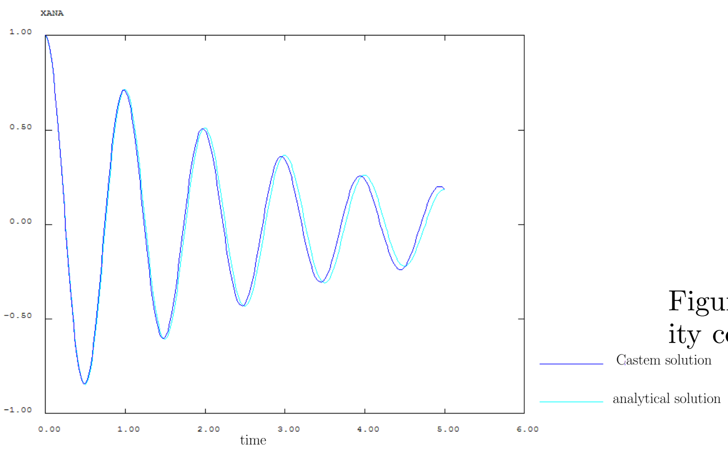


Figure 2: Solid gravity center displacement

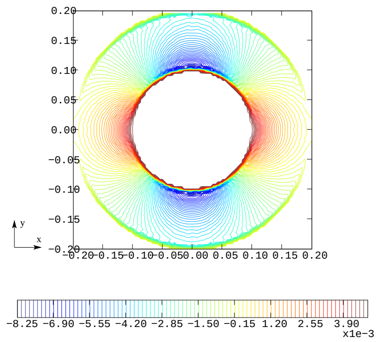
4.0.1. Reduced order modeling

The method presented in section 3 is tested with this results. Hundred snapshots have been taken during one pseudo-oscillation period.

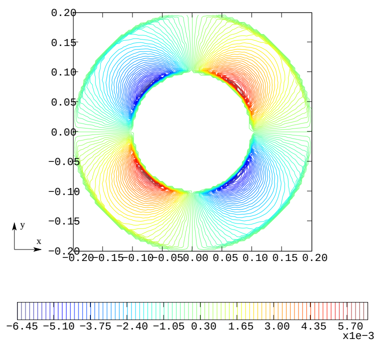
First, the POD vectors have been searched and the reconstructed velocity has been defined as

$$v^N\left(x,t\right) = \sum_{n=1}^{N} a_n\left(t\right)\Phi_n\left(x\right)$$

and is compared to the initial velocity. On Figure 4, we can see that we have a good reconstruction with 3 modes and a maximal error near to the fluid-solid interface.

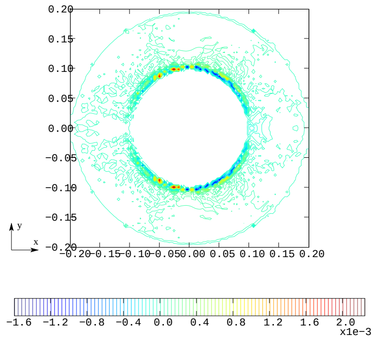


(a) v_x

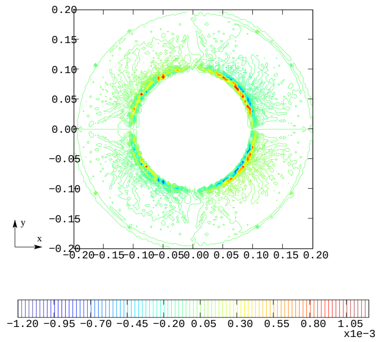


(b) v_y

Figure 3: Isovalues of the first and second velocity component at the snapshot 70



(a) $\left(v^3 - v\right)_x$



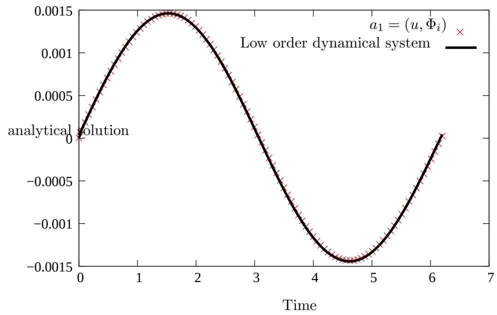
(b) $\left(v^3 - v\right)_y$

Figure 4: Isovalue of the difference between the reconstructed solution with 3 modes and the initial at the 70 snapshot

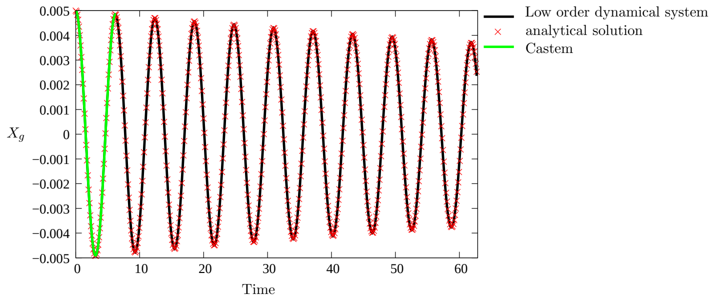
In fact the first POD eigenvalue contains 99.2% of the total kinetic energy and with three vectors almost 99.99% of the energy is captured.
That is why with only three POD vectors the reconstructed velocity is a good approximation of the initial velocity.

Next, the low order dynamical system with three modes is constructed and the temporal coefficients obtained are compared with those obtained by computing the POD vector (at each snapshot t_i, $a_n(t_i) = (v(\bullet, t_i), \Phi_i)$). There is a good conformity between them, for example for the first temporal coefficient a_1 (Figure 5(a)).

In the following the low order dynamical system during a longer period than the snapshot period will be presented. The solution obtained has not been compared to a numerical solution, but the gravity center displacement can be predicted by an analytical solution and compared to those computed. In fact, in this case the analytical



(a) First temporal coefficient obtain by POD and solving reduced order model



(b) Original displacement of rigid gravity center and obtained by reduced system

Figure 5: Low order dynamical system result

solution is $X_g(t) = X_g(0)\cos\left(\omega\sqrt{1-\varepsilon^2}\right)\mathrm{e}^{\varepsilon\omega t}$, ε being evaluated on the snapshot period. The solution for a period longer than 10 times the snapshot period gives good prediction for the gravity center position (Figure 5(b)). It is an adequate criteria to conclude that the reduced system obtained gives a good result with a few degrees of freedom and this case gives a good prediction for simulate a period longer than the snapshot period.

5. CONCLUSION

We have presented POD methodology and its application in fluid-structure interaction (FSI). The principal difficulties to apply it were the spatial properties of POD modes and the fact that in FSI the fluid domain moves in time. Thus we introduce a fictious fixed domain whose contains all movings domains. Next a Lagrangian multiplier is used formulate Navier-stokes equation on solid domain. Thus we can formulate a reduced dynamic system in case of FSI. This system is tested on a two dimensional case and good adequation are found beetween initial solution and reduced

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