WAKE FORMATION FOR A PAIR OF CIRCULAR CYLINDERS IN CROSS-FLOW WITH ONE OSCILLATING

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ABSTRACT

The wake of a pair of equal diameter staggered circular cylinders, with pitch ratio $P/D = 2.0$ *and stagger angle* $\alpha = 45^0$*, and either one subject to forced harmonic oscillation transverse to the flow direction is investigated experimentally for Reynolds numbers within the range* $515 \le Re \le 730$*. Flow visualization of the wake formation region and hot-film measurements of the wake velocity are reported for cylinder excitation frequencies in the range* $0.07 \le f_eD/U \le 1.19$*. Results show that when the cylinders are stationary an integral relationship exists between two distinct Strouhal numbers. Oscillation of either cylinder with* $f_eD/U > 0.10$ *causes considerable modification of the wake vis-à-vis when the cylinders are stationary. In particular, there are distinct regions of synchronization between the dominant wake periodicities and the cylinder oscillation. For both upstream and downstream cylinder oscillations the number of the gap shear layers shed from the downstream cylinder which interact with the shear layers shed from the upstream cylinder plays a key role in the synchronization process of the periodicities on the mean-flow side of the upstream cylinder; whereas synchronizations on the mean-flow side of the downstream cylinder are more dependent on the shear layers shed from its outer surface.*

1. INTRODUCTION

The cross-flow around a single, isolated cylinder has been studied for years, and may now be considered to be reasonably well understood. Although the flow around small group of cylinders has only recently received significant attention, and is not so well understood, it has been shown that there are significant new complexities vis-à-vis the flow around an isolated cylinder. These include interactions between the shear layers, vortices and Kármán vortex streets shed by the individual cylinders.

The flow around a pair of staggered cylinders was first classified by Zdravkovich (1987), who, based on the relative position of the cylinders, identified three different flow regimes: (i) *no interference*, where the flow around each cylinder is effectively identical to that around a single cylinder; (ii) *wake interference*, where one cylinder is partially or completely submerged in the wake of the other; and (iii) *proximity interference*, where the cylinders are close to each other, but neither is submerged in the wake of the other. However, Sumner et al (2000) proposed that the flow around two cylinders is much more complex than this. They identified nine different flow patterns, which fall into three main categories: *single bluff-body* flows where there is almost no flow between the cylinders, *small-incidence* flows where there is a small gap flow between the cylinders and *large-incidence* flows where there is sufficient gap flow for both cylinders to shed vortices.

Recently Sumner and Richards (2003) measured the Strouhal numbers in the wake of a pair of cylinders for a wide range of P/D and α (see Figure 1 for the definition of P/D and α) in the subcritical Reynolds number range at $Re = 3.2 \times 10^4$. At $P/D = 2.0$ and $\alpha = 45^0$ they obtained two Strouhal numbers, $St = 0.15$ and 0.34, and suggested that the higher of these was associated with the shear layers shed from the upstream cylinder, whereas the lower one originated from shear layers shed by the downstream cylinder. Alam and Sakamoto (2005) also obtained two Strouhal numbers in an investigation carried out at $Re = 5.5\times10^4$ with $P/D = 2.0$ and $\alpha = 45^0$. However, in contrast to Sumner and Richards (2003), they concluded that the two shear layers originating from the upstream cylinder as well as that from the gap surface of the downstream cylinder were all shed at the same frequency—giving the higher of the two Strouhal numbers—while the shear layer formed on the outer surface of the downstream cylinder shed at a different frequency—resulting in the lower Strouhal number. Employing wavelet analysis they also suggested that the shedding frequency was multi-stable; however, no visualization showing this phenomenon was given.

Considering the additional complexities arising from the oscillation of a single cylinder compared with when it is stationary it is expected that the flow around a pair of cylinders will also become more complex when one or both are subject to forced oscillation. For a single cylinder one of the most signification phenomena associated with forced oscillation is the synchronization of the shedding frequency, f_0, with the excitation frequency, f_e. This synchronization can take the form of a fundamental lock-in ($f_0 = f_e$), although both sub- and superharmonic relationships between the frequencies are also observed. For example, Stansby (1976) obtained both 2-superharmonic ($f_0 = ½ f_e$) and 3-superharmonic ($f_0 = 1/3 f_e$) synchronizations. Price et al (2005) obtained similar synchronizations for two nearly in-line cylinders with $L/D = 2.0$.

The aim of the present study is to investigate the near wake structure and synchronization phenomena for a pair of staggered circular cylinders when one of them, either the upstream or the downstream, is subject to forced harmonic oscillation transverse to the flow direction. The configuration chosen has a centre-to-centre pitch ratio $P/D = 2.0$ and stagger angle $\alpha = 45^0$ at the mean position of the cylinders. The cylinder oscillation employed has a ½ peak-to-peak magnitude of $0.22D$, which results in the relative transverse separation between cylinders being in the range $1.2 \leq T/D \leq 1.63$, while the longitudinal separation between cylinders remains constant at $L/D = 1.41$ (the transverse, T, and longitudinal , L, separation between cylinders are defined in Figure 1).

According to Sumner et al (2000), when the cylinders are stationary this configuration belongs to the *large-incidence* regime, with the specific flow patterns being what was referred to as *vortex pairing, splitting and enveloping* (VPSE) for $T/D < 1.35$ and *synchronized vortex shedding* (SVS) for $T/D > 1.35$. For a VPSE flow the two shear layers shed in the gap between the cylinders, which have opposing circulation, initially pair up and are then partially enveloped by the outer shear layer shed from the upstream cylinder. The gap shear layer shed by the upstream cylinder then splits into two, one portion of which is enveloped, along with the gap shear layer shed by the downstream cylinder, by the shear layer shed from the outer surface of the upstream cylinder. The SVS flow pattern is characterized by a strong synchronization of the shear layers shed on either side of the gap between the cylinders as well as the resulting Kármán vortices. Within the combined wake, these two opposite-sign vortices pair up, resulting in two adjacent Kármán vortex streets that exhibit anti-phase synchronization. The most important feature and commonality of these two flow patterns is that the gap flow between the cylinders results in two different-width near-wakes, a narrow wake behind the upstream cylinder and a wide one behind the downstream cylinder. Consequently, two distinct Strouhal numbers exist which are associated with the vortex shedding frequencies of the two differently sized near-wake regions.

It is expected that oscillation of either of the two cylinders will have considerable effect on these vortex formation regions as well as the flow in the gap between the cylinders; the primary motivation of the present study was to investigate these effects.

2. EXPERIMENTAL PROCEDURE

Experiments were carried out in a closed-circuit Kempf & Remmers water tunnel which has a test section 110 cm long and 254 mm × 254 mm in cross-section; the flow within the tunnel has a streamwise turbulence intensity of 0.5% and a velocity profile uniformity of better than 95%. The two cylinders were mounted vertically in the test section, and a scotch-yoke mechanism was employed to oscillate either one of them with an amplitude of 3.5 mm or $0.22D$ (the other cylinder being held stationary). A complete description of the oscillation mechanism is given in Krishnamoorthy et al (2001). The cylinders, which were made of plexiglas, had a diameter of 16 mm and an aspect ratio of 16; each individual cylinder had a blockage ratio of 6.3%.

A two-fold experimental approach consisting of flow visualization and spectral measurements of the wake velocity was employed. Flow visualization was carried out using Rhodamine dye which was injected at the mid-span of each cylinder from two ports of 0.8 mm diameter and 120^0 apart. The flow images revealed by the dye were recorded by a professional S-VHS video camera and digitized. Spectral measurements of the wake flow were conducted using a constant temperature hot-film anemometer. The analog output of the anemometer system was acquired, and velocity spectra were calculated with a dimensionless frequency resolution of $f_e D/U \approx 0.02$.

Initially experiments were conducted with the cylinders stationary at the T_{min}, T_{nom} and T_{max} (minimum, nominal and maximum transverse separations) positions. In the second phase of the experiments the upstream and downstream cylinders were forced to oscillate (one at a time with the other cylinder held stationary) transverse to the flow. The oscillation frequency f_e was incremented such that the various synchronization regimes were spanned. Based on cylinder diameter, D, and mean upstream flow velocity, U, the Reynolds number for the results presented here was in the range $515 \leq \mathrm{Re} \leq 730$.

To investigate all of the wake periodicities the hot-film probe was placed at locations *a* and *b* when either the cylinders were stationary or the upstream cylinder was being oscillated, and at locations *c* and *d* for downstream cylinder oscillation, see Figure 1. Measurements at positions *a* and *c* are expected to reveal the periodicities resulting from the interaction of the two gap shear layers and the outer shear layer shed from the upstream cylinder, whereas the measurements at positions *b* and *d* are expected to reveal those shed from the outer shear layer of the downstream cylinder. The locations of positions *a*, *b*, *c* and *d* are shown schematically in Figure 1.

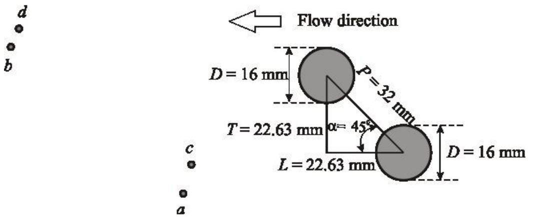

Figure 1. Schematic of cylinder positions and hot-film probe locations.

3. RESULTS

3.1 Stationary cylinders

Two sample wake spectra measured at the T_{min} position for probe locations *a* and *b* are shown in Figure 2. From these and other results the Strouhal numbers obtained for different orientations of the cylinders are summarized in Table 1.

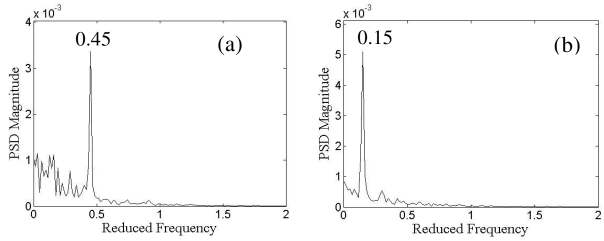

Figure 2. Wake spectra measured at (a) location a and (b) location b with both cylinders held stationary at the T_{min} position. The magnitudes of the PSDs are in arbitrary units.

	Probe-position *a*	Probe-position *b*
T/D = 1.20 (T_{min})	$St_{SL} = 0.45$	$St_{CW} = 0.15$
T/D = 1.41 (T_{nom})	$St_{SL} = 0.46$ & $St_{CW} = 0.15$	$St_{CW} = 0.14$
T/D = 1.63 (T_{max})	$St_{SL} = 0.45$	$St_{CW} = 0.15$

Table 1: Strouhal numbers measured in the wake of the stationary cylinders.

The results presented in Table 1 show the existence of two distinct Strouhal numbers $St_{CW} \approx 0.15$ and $St_{SL} \approx 0.45$ over the complete range of *T/D*. The spectral measurements conducted at probe-position *a* yielded the higher Strouhal number, St_{SL}, which as ascertained from the flow visualization (to be discussed later) is associated with the shear layer periodicities; the measurements at probe-position *b* yielded the lower Strouhal number, St_{CW}, which is associated with a dominant periodicity in the combined wake of the cylinders. It is evident that for the whole range of *T/D*, $St_{CW} = St_{SL}/3$. Alam and Sakamoto (2005) and Sumner and Richards (2003) also obtained two distinct Strouhal numbers for similar cylinder configurations. Alam and Sakamoto (2005) obtained St ≈ 0.14 and 0.34 for *P/D* = 2.0 and $\alpha = 45^0$ at Re = 5.5×10^4 and Sumner and Richards (2003) obtained St = 0.15 and 0.34 for *P/D* = 2.0 and $\alpha = 45^0$ at Re = 3.2×10^4. Although the lower Strouhal number is almost the same for all three sets of experiments, the higher Strouhal number obtained by either Alam and Sakamoto (2005) or Sumner and Richards (2003) is smaller than that obtained in the present case. Also of significance is that the present experiments yield an integral relationship between the two Strouhal numbers ($St_{CW} = St_{SL}/3$), whereas no such integral relationship was found by the other authors. An integral relationship of 3 between the two Strouhal numbers was also observed by Hayder and Price (2007) for *P/D* = 2.5 and $\alpha \approx 21^0$ at Re = 525-750.

Flow visualization experiments carried out at all three positions, T_{min}, T_{nom} and T_{max}, revealed that all four shear layers from the two cylinders are shed at the same frequency, giving the Strouhal number $St_{SL} \approx 0.45$, whereas the lower Strouhal number, $St_{CW} \approx 0.15$, is associated with the wake on the mean-flow side of the downstream cylinder. For every three successive shear layers shed from the outer surface of the downstream cylinder the first one sheds and forms an independent vortex, while the next two coalesce to form a composite vortex. This pattern repeats itself every three cycles and is responsible for producing the dominant combined wake periodicity with a frequency of ⅓ of the shear layer shedding frequency, leading to an integral relationship of 3 between the two Strouhal numbers.

3.2 Cylinder oscillation

In the second stage of the experiments, transverse oscillation was imposed on one of the cylinders with the other being held stationary. Flow visualization experiments suggest that when $f_eD/U \leq 0.10$ oscillation of either the upstream or downstream cylinder causes the wake structure to shift from a VPSE- to a SVS-type flow as the cylinder separation changes from T_{min} to T_{max}. Also, the frequencies of the dominant wake periodicities at these lower oscillation frequencies are approximately equal to those measured when the cylinders are stationary.

The wake starts undergoing modification caused by the increased cylinder acceleration for f_eD/U > 0.10. For either upstream or downstream cylinder oscillation the shear layers shed from the mean-flow surface of the downstream cylinder synchronize with the cylinder oscillation. On the other side of the wake the flow is synchronized with upstream cylinder oscillation via either a complete enveloping of the two "paired-up" gap shear layers by the outer shear layer shed from the upstream cylinder, referred to as a vortex pairing and enveloping (VPE) process, or by a VPSE process. For downstream cylinder oscillation, however, synchronization of the wake is caused by the VPSE process only.

3.2.1 Upstream cylinder oscillation

Experiments with the upstream cylinder subject to forced oscillation were carried out within the frequency range $0.07 \leq f_eD/U \leq 1.19$. The wake spectral measurements obtained at probe-positions *a* and *b* are summarized in Figure 3, where the nondimensional frequencies of the wake periodicities, f_0D/U, are presented as a function of the nondimensional cylinder excitation frequency, f_eD/U; the figure also shows which was the most dominant of the frequencies in the wake spectrum. Based on the results shown in Figure 3 and the flow visualization it is concluded that there are a number of distinct frequency regimes where the wake periodicities resonate with the cylinder oscillation.

A ½-subharmonic synchronization ($f_0/f_e = 2$) occurs for the periodicities shed on the mean-flow side of the downstream cylinder at $f_eD/U \approx 0.08$. As f_eD/U is increased, this is followed by a fundamental synchronization ($f_0/f_e = 1$) for $0.13 \leq f_eD/U \leq 0.57$, where the periodicities shed on the mean-flow side of the upstream cylinder resonate with the cylinder oscillation, see Figure 3(a). However, as seen from Figure 3(b), the fundamental synchronization of periodicities on the mean-flow side of the downstream cylinder exists up to $f_eD/U = 0.24$ only. Moreover, a 2-superharmonic synchronization ($f_0/f_e = 1/2$) of the periodicities on the mean-flow side of the downstream cylinder occurs for $0.32 \leq f_eD/U \leq 0.43$. Figure 3(a) indicates that the wake switches to a 2-superharmonic synchronization for $0.61 \leq f_eD/U \leq 0.79$, where only the periodicities on the mean-flow side of the upstream cylinder resonate with the cylinder motion.

A sample set of flow visualization images for two complete cycles of upstream cylinder oscillation at $f_eD/U \approx 0.42$ is presented in Figure 4.

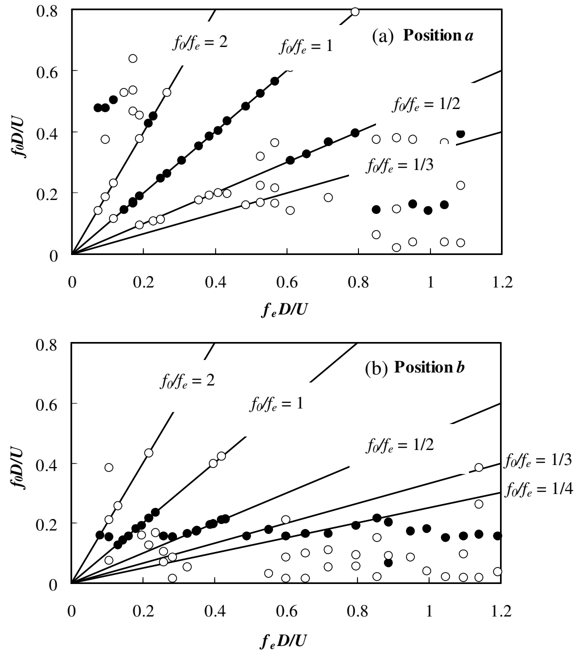

Figure 3. Distinct frequency peaks, f_oD/U, obtained from the wake spectra measured at positions (a) a and (b) b for upstream cylinder oscillation f_eD/U : ●, dominant wake periodicity; ○, other wake periodicity; ——, lines showing the integral relationships between the wake and excitation frequency

At this f_eD/U a fundamental synchronization of the periodicities on the mean-flow side of the upstream cylinder exists along with a 2-superharmonic synchronization of those on the mean-flow side of the downstream cylinder. Shear layers X_1 and X_2 are shed from the gap surface of the downstream cylinder during the first and second cycles of oscillation, respectively. The upstream cylinder also sheds one shear layer from both of its two surfaces during each cycle of oscillation. Shear layers Y_1 and Z_1 and Y_2 and Z_2 separate from the gap and outer surfaces of the upstream cylinder while it completes the first and second cycles of oscillation, respectively. Frame (c) shows that during the first cycle of oscillation the gap-vortex-pair, consisting of vortices X_1 and Y_1, is partially enveloped by the outer shear layer shed from the upstream cylinder, Z_1. Vortex X_1 then splits into two concentrations of vorticity, one of which propagates as a small vortex in the middle of the combined wake [frames (d) and (e)] whereas the other, along with vortex Y_1, is enveloped by the shear layer Z_1 and forms a large composite vortex W_{C1}. Thus, the three shear layers shed on the mean-flow side of the upstream cylinder during its first cycle of oscillation yield the

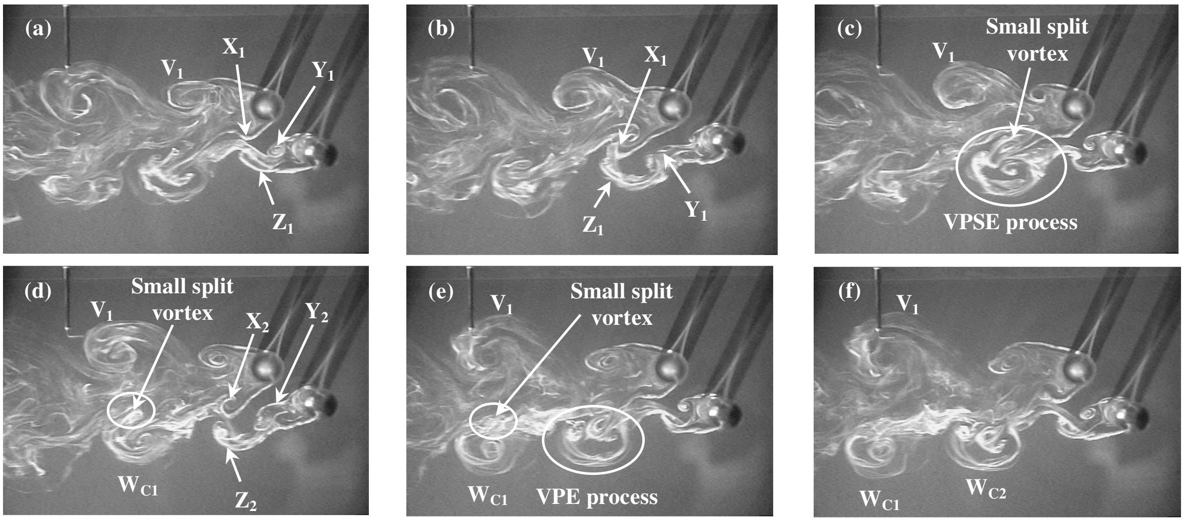

Figure 4. Flow visualization images with upstream cylinder oscillating at $f_eD/U \approx 0.42$. *Two complete cycles of oscillation are shown, time as a fraction of one period of the cylinder oscillation* (P^*)*: (a)* $t/P^* \approx 0.0$, T_{max} *position; (b)* $t/P^* \approx 0.50$*; (c)* $t/P^* \approx 0.83$*; (d)* $t/P^* \approx 1.33$*; (e)* $t/P^* \approx 1.83$*; (f)* $t/P^* \approx 2.0$, T_{max} *position.*

composite vortex W$_{C1}$ via a VPSE process. However, during the second cycle of oscillation shear layers X$_2$, Y$_2$ and Z$_2$ take part in a VPE process where the gap-vortex-pair, X$_2$ and Y$_2$, is enveloped completely by the outer shear layer shed from the upstream cylinder, Z$_2$, yielding the second composite vortex, W$_{C2}$ [frames (e) and (f)]. Therefore, during two cycles of upstream cylinder oscillation two composite vortices are formed from the three shear layers shed on its the mean-flow side, but via two different processes.

Figure 4 also shows one cycle of vortex formation on the mean-flow side of the downstream cylinder, V$_1$. Every two cycles of cylinder oscillation two vortices propagate on the mean-flow side of the upstream cylinder along with another on the mean-flow side of the downstream cylinder—resulting in both a fundamental and 2-superharmonic synchronization in the combined wake.

Examination of the complete set of flow visualization images revealed that the number of gap shear layers shed from the downstream cylinder decreased with increasing frequency. For $0.13 \leq f_eD/U \leq 0.24$ two shear layers separate from the gap surface of the downstream cylinder every cycle of oscillation, but only one of them takes part in a VPSE process and produces a composite vortex—inducing a fundamental synchronization on the mean-flow side of the upstream cylinder. If the oscillation frequency is increased to $0.32 \leq f_eD/U \leq 0.43$ one shear layer separates from the gap surface of the downstream cylinder during each cycle of oscillation, producing the lone composite vortex by either a VPSE or VPE process—resulting in a fundamental synchronization. Finally, for the 2-superharmonic synchronizations which occurred for $0.61 \leq f_eD/U \leq 0.79$ only one shear layer separates from the gap side of the downstream cylinder during every two cycles of cylinder oscillation—producing one composite vortex. Thus, the number of gap shear layers shed from the downstream cylinder which interact with those shed from the upstream cylinder, via either the VPSE or VPE process, plays the key role in determining the type of synchronization which occurs on the mean-flow side of the upstream cylinder, whereas it is the outer shear layers shed from the downstream cylinder which are dominant in the synchronization process on the other side of the combined wake.

3.2.2 Downstream cylinder oscillation

Experiments with the downstream cylinder subject to forced oscillation were carried out in the frequency range $0.07 \leq f_eD/U \leq 1.13$. In this case, the wake synchronizes with the cylinder oscillation for $f_eD/U \geq 0.09$ approximately. Both fundamental and 2-superharmonic synchronizations occur in two different frequency ranges. The first fundamental synchronization exists for $0.09 \leq f_eD/U \leq 0.23$. Within this range of fundamental synchronization the wake periodicities shed on the mean-flow side of the downstream cylinder resonate with the cylinder motion for $0.09 \leq f_eD/U \leq 0.17$ while the periodicities shed on the mean-flow side of the upstream cylinder resonate with the cylinder motion for $0.13 \leq f_eD/U \leq 0.23$. This is followed by a 2-superharmonic synchronization of the periodicities shed on both sides of the combined wake for $0.24 \leq f_eD/U \leq 0.29$. There is then a second region of fundamental synchronization for $0.39 \leq f_eD/U \leq$

0.50, where only the periodicities shed on the mean-flow side of upstream cylinder resonate with the cylinder motion. For $0.82 \leq f_eD/U \leq 1.01$ the wake switches to a second 2-superharmonic synchronization where the periodicities shed on the mean-flow side of upstream cylinder resonate with the motion of the downstream cylinder. Besides the fundamental and 2-superharmonic synchronizations, 1.5-superharmonic and 3-superharmonic synchronizations also exist at $f_eD/U \approx 0.62$ and 0.43, respectively.

Flow visualization experiments conducted with downstream cylinder oscillation showed that for $0.13 \leq f_eD/U \leq 0.23$ two gap shear layers separate from the downstream cylinder during one cycle of oscillation; however, only one of them forms a vortex, via a VPSE process, resulting in a fundamental synchronization. For the 2-superharmonic synchronizations which occur for $0.24 \leq f_eD/U \leq 0.29$ two gap shear layers are shed from the downstream cylinder during two successive cycles of oscillation; however, once again only one of them yields a vortex via a VPSE process. For $0.39 \leq f_eD/U \leq 0.50$ the downstream cylinder sheds only one shear layer from its gap surface during each cycle of oscillation, this shear layer takes part in a VPSE process resulting in the second fundamental synchronization. Finally, for $0.82 \leq f_eD/U \leq 1.01$ one shear layer separates every two cycles of oscillation producing one cycle of vortex propagation by the VPSE process, leading to the second 2-superharmonic synchronization. It is, therefore, concluded that, similar to upstream cylinder oscillation, for downstream cylinder oscillation too, the number of gap shear layers involved in the VPSE process plays the key role in the type of synchronization of the wake periodicities shed on the mean-flow side of the upstream cylinder.

4. CONCLUSIONS

For stationary cylinders two distinct Strouhal numbers are observed in the wake; the higher of which, $St_{SL} = 0.45$, is associated with the shedding frequency of the four shear layers while the lower, $St_{CW} = 0.15$, is associated with the combined wake of the cylinder pair. An integral relationship between the two Strouhal numbers was obtained with $St_{SL} = 3\ St_{CW}$.

Oscillation of either cylinder for $f_eD/U > 0.10$ causes considerable modification of the flow vis-à-vis the static case. Distinct regions of synchronization between the dominant wake periodicities and the cylinder oscillation are obtained, these synchronization regions involve fundamental, as well as sub- and superharmonic resonances. For both upstream and downstream cylinder oscillations, the number of the gap shear layers shed from the downstream cylinder per cycle of oscillation which interact with the upstream cylinder shear layers plays a key role in determining the type of synchronizations on the mean-flow side of the upstream cylinder. Synchronizations on the mean-flow side of the downstream cylinder, however, are associated more with the shear layers shed from its outer surface.

5. ACKNOWLEDGEMENTS

The financial support of NSERC is acknowledged.

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