CLASSIFICATION OF THE WAKE OF TWO SIDE-BY-SIDE SQUARE CYLINDERS

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ABSTRACT

The wake of two side-by-side square cylinders is experimentally investigated at a Reynolds number (Re) of 4.7×10^4. The cylinder center-to-center spacing ratio T/d (d is the cylinder characteristic dimension) is varied from 1.02 to 6. Four distinct flow regimes, i.e., A, B, C and D, are identified based on measured Strouhal number (St) and forces. Regime A or single-body regime occurs at T/d < 1.3, where the two cylinders act like a single body in terms of St. This regime may be further divided into two sub-regimes: the perfect single-body regime (T/d ≤ 1.02), where drag, lift and St all are all the same as in a single cylinder wake and the single-body-like regime (1.02 < T/d < 1.3), where only St is the same as in the single cylinder wake. Regime B or two-frequency regime (1.3 < T/d < 2.2). Regime C or transition regime (2.2 < T/d < 3), where vortex shedding from the cylinders switches from two different frequencies to coupled shedding at the same frequency or vice versa. Regime D or coupled vortex-shedding regime, T/d > 3. This regime is further divided into two, i.e. the anti-phase-dominated regime (3 < T/d < 4.6), and the anti-phased and in-phased regime (4.6 < T/d < 6).

1. INTRODUCTION

Circular and square cross-sections are most frequently seen in engineering and considered as the basic shapes of structures. There have been extensive investigations on the wake of two side-by-side circular cylinders. Three flow regimes have been identified (Zdravkovitch 1987). The single-bluff-body regime occurs at the cylinder center-to-center spacing ratio *T/d* < 1.2 -1.3, where the two cylinders behave like one structure, generating a single vortex street with one predominant *St*. The asymmetrical wake regime occurs at *T/d* =1.2-1.3 ~ 2.2-2.5, where the gap flow between the cylinder is biased, resulting in one wide and one narrow wakes, characterized by two distinct frequencies. At *T/d* > 2.2 - 2.5, two coupled vortex streets of the same *St*, either in-phased or anti-phased, were observed. A great attention has also been given to the forces and flow structures on two side-by-side circular cylinders in different regimes (e.g., Kim and Durbin, 1988; Alam et al, 2003; Zhou et al. 2001, 2002)

It is well known that the non-stationary flow separation point oscillates on a circular cylinder; in contrast, the flow separation point on a square cylinder is fixed. One may surmise that aerodynamic interference associated with the two different types of cylinders should be rather different. There have been scattered investigations on the wake of two side-by-side square cylinders. Using a two-component laser Doppler velocimetry, Kolar et al (1997) measured the wake of two side-by-side square cylinders at *T/d* = 3 ($Re = 2.3\times10^4$) at *x/d* = 1~ 9, where *x* was the streamwise distance from the cylinder centers. They only examined the flow at *T/d* = 3, corresponding to the coupled two-vortex-street regime, without covering other possible flow regimes. Agrawal et al (2005) investigated the laminar wake (*Re* = 73) of two side-by-side square cylinders at *T/d* = 1.7 and 3. They observed both in-phased and anti-phased vortex shedding from the two cylinders at *T/d* = 3, and a biased gap flow at *T/d* = 1.7. The observation is similar to that behind two side-by-side circular cylinders. Sakamoto and Haniu (1988) measured time-averaged and fluctuating forces on the full length of two square cylinders submerged in a turbulent boundary layer ($Re = 1.52\times10^5$). In spite of these investigations, a systematic study of flow-induced forces on two side-by-side square cylinders in a uniform flow is scarce. The dependence of these forces and *St* on *T/d* is of practical importance. The forces on two side-by-side square cylinders in a boundary layer or circular cylinders in a uniform flow cannot be used to predict those on two side-by-

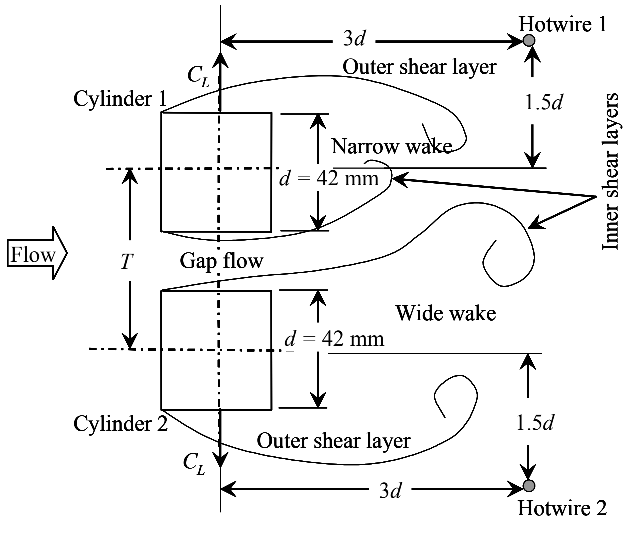

Figure 1: *Arrangement of cylinders and hotwire positions.*

side square cylinders in a uniform flow. Many important issues have yet to be addressed. For example, how would flow separation, which is different from the circular cylinders, impact on the interference between two side-by-side square cylinders? In particular, how does it affect flow classification?

This work aims to measure systematically *St* and fluid forces on two side-by-side square cylinders and to address the issues raised above. *T/d* is varied from 1.02 to 6, which covers all possible flow regimes. Time-averaged drag and lift coefficients (C_D and C_L) were measured. So were their fluctuating (rms) components (C_D' and C_L'). Fluctuating velocities (u_1 and u_2) in the outer sides of the wake were captured simultaneously using two hotwires, allowing the estimate of *St* and correlation between vortices separated from the two cylinders.

2. EXPERIMENTAL DETAILS

Experiments were conducted in a closed-circuit wind tunnel with a 2.2-m-long test section of 0.3 m in width and 1.2 m in height. Two square cylinders of d = 42 mm, made of brass, spanned horizontally the test section width. The free-stream velocity, U_∞, was 17 m/s, resulting in a Re $(= U_\infty d / \nu)$ of 4.7×10^4, where ν is the kinematic viscosity of air. The flow was uniform within the central cross sectional area of 0.24 m × 0.95 m in the test section, and the longitudinal turbulence intensity was less than 0.5% in the absence of the cylinders. More details of the tunnel were given in Alam et al. (2005). The cylinder blockage and aspect ratios, i.e. the length-to-width ratio, in the test section were 3.5% and 7, respectively. No corrections were made to compensate the blockage effect. Fluid forces were measured over a small spanwise length (=1.07*d*) of the cylinders using load cells. The details of the load cells and fluid force measurement have been in Alam et al (2002). Two single hotwires, placed at 3*d* downstream and 1.5*d* laterally from the centers of the cylinders (Fig. 1), were used to measure the frequencies of vortex shedding from the cylinders.

3. RESULTS

3.1. Time-averaged forces

Figure 2 shows C_D of the individual cylinders/wakes and total time-averaged drag $C_{D(total)}$ (= $C_{D(cyl.1)}$ + $C_{D(cyl.2)}$) of the two cylinders. For the sake of simplicity to discuss, Cylinders 1 and 2 will be supposed to be associated with narrow and wide wakes, respectively (see Fig. 1). Note that the C_D for the individual wakes was determined from the measurement of C_D of the individual cylinders and/or by employing a conditional sampling technique when a signal consists of the two modes. The details of the technique have been given in Alam et al (2003). It is clear from the figure that the two cylinders experience two different magnitudes of C_D for *T/d* = 1.1 ~ 3.0. However, the difference in C_D between the cylinders is the greatest at and around *T/d* = 2.

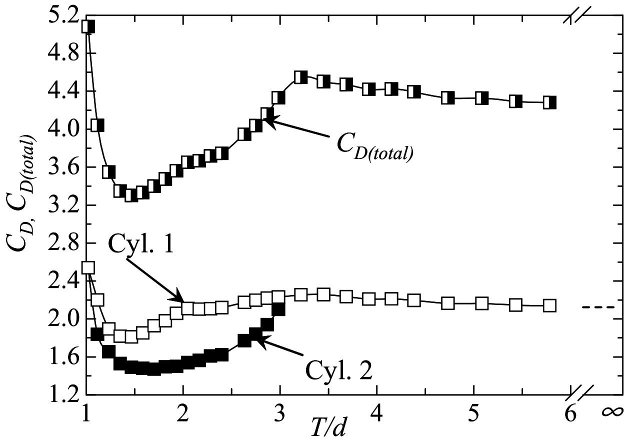

Figure 2: *Dependence on T/d of time-averaged drag coefficient,* C_D *of the individual cylinders, and total time-averaged drag coefficient,* $C_{D(total)}$. *– – –, single isolated cylinder.*

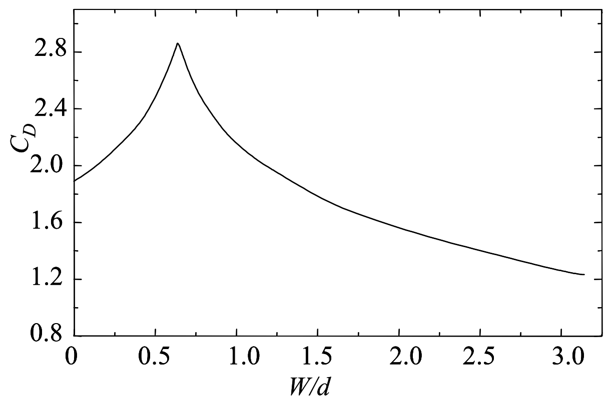

Figure 3: *Dependence of* C_D *on W/d of a rectangular cylinder (Courchesne and Laneville, 1982). Re =* 6×10^4.

At *T/d* = 1.02 (gap = 1 mm), the same drag (C_D = 2.54) was measured for both cylinders. This drag is about 19% higher than that of a single isolated cylinder (C_D = 2.15). In fact, at this small *T/d*, the flow in the gap is negligible and the two cylinders behave like a unit, single rectangular cylinder whose width-to-height ratio *W/d* (an expression of afterbody length, where *W* is the cylinder width in the streamwise direction) is about 0.5. A question may arise, why does a cylinder at *T/d* = 1.02 experience a significantly higher drag than a single isolated cylinder though the two cylinders behaves like a single one? This is because C_D of a rectangular cylinder is highly dependent on *W/d*, and as *W/d* decreases from 1.0, C_D reaches a maximum value at the critical *W/d* = 0.6 (e.g., Courchesne and Laneville, 1982) and then decreases (Fig. 3). As seen in the Fig. 3, the C_D of a rectangular cylinder with 0.3 < *W/d* < 1.0 is higher than that with *W/d* = 1.0; specifically, C_D at *W/d* =

0.6 and 0.5 increases by 30% and 17%, respectively, than that at W/d = 1.0. In the literature, C_D of a rectangular cylinder with W/d = 0.5 was found to be 2.5 by Courchesne and Laneville (1982). This value agree well with the present C_D (= 2.54) at T/d = 1.02, proving the effect of the gap flow is definitely negligible to change the behaviors of the surrounding flow and hence C_D. Therefore, the higher value of C_D at T/d = 1.02 is due to the effective W/d close to 0.5. However, as T/d increases from 1.02 to 1.3 where two cylinders behave like a unit, single bluff body in terms of St (will be discussed later) and correspond to a change in effective W/d from 0.5 to 0.434, C_D drops drastically. This is because (i) the gap flow which behaves like a base bleed (e.g., Franssona et al, 2004) is now effective to decrease the magnitude of the base pressure and (ii) the effective W/d further decreases from 0.5.

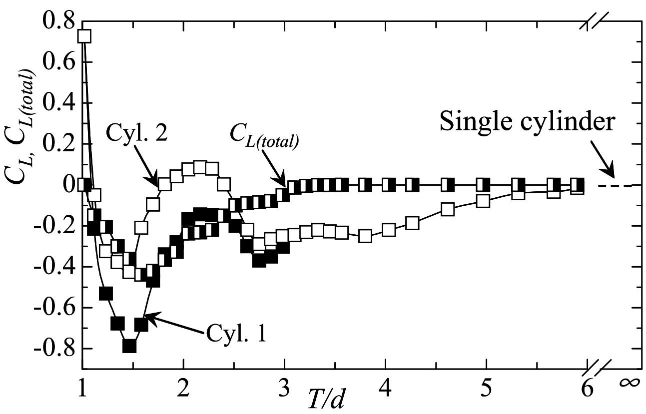

Figure 4: *Dependence on T/d of C_L and $C_{L(total)}$.*

$C_{D(total)}$ decreases as T/d increases from 1.02, and reaches a minimum value at T/d = 1.5. Further increasing T/d escalates $C_{D(total)}$ into a maximum at T/d = 3.2. The $C_{D(total)}$ at T/d =1.5 is about 24% lower than that when the cylinders are isolated.

The variations of C_L of the two individual cylinders and total lift $C_{L(total)}$ with T/d are shown in Fig. 4. As reference to Fig. 1, repulsive (outward directed) and attractive (inward directed) lift forces are considered as positive and negative, respectively. For 1.2 ≤ T/d < 5, both cylinders experience a negative C_L except at T/d = 1.9 ~ 2.3 for Cylinder 2. C_L at T/d = 1.02 for the either cylinder was found to be the same and highly positive (≈ 0.73), implying that the pressure on the outer side surfaces of the two cylinders is more negative than that on the inner side surfaces. Because of almost no flow in the gap, the magnitude of pressure on the inner side surfaces is lower than that at the other side surfaces. When T/d increases from 1.02, the gap flow resembles a jet which causes highly negative pressure on the inner side surfaces, inducing negative C_L on the either cylinder. Possibly, the gap flow at T/d = 1.5 is most effective as jet, causing a maximum magnitude of negative C_L (Fig. 4) and a minimum $C_{D(total)}$ (Fig. 2). As T/d increases from 1.5, the magnitude of C_L decreases due to the flow in the gap growing to a flow similar to $T/d = \infty$. There may be some change in the flow structure for T/d around 2.2 that initiates C_L to increase in magnitude up to T/d = 2.7. When T/d is in the range of 2.7 ~ 4, C_L remains almost constant (≈ -0.28). It is interesting to find why C_L remains negative with a significant value for T/d = 2.7 ~ 4 though at least at T/d = 3 ~ 4 the flow structure behind each of the two cylinders is very similar to that behind an isolated single cylinder in terms of both C_D and St. Furthermore, it will be discussed later that an anti-phase vortex shedding from the cylinders will grow from T/d = 2.2 to 2.7 and dominantly prevails at T/d = 2.7 ~ 4. The growing anti-phase vortex shedding is linked to the increase in the magnitude of C_L at T/d = 2.2 ~ 2.7. In the literature, when two outer shear layers of the two cylinders shed vortices with a relative phase of 0° or 180°, the shedding is termed as anti-phase or in-phase vortex shedding, respectively, as presented in Fig. 5. The negative C_L at T/d = 2.7 ~ 4 is attributed to the domination of anti-phase vortex shedding from the cylinders for the T/d range. As such, in anti-phase vortex structure from two side-by-side cylinders, the gap flow sheds two opposite sign vortices simultaneously (Fig. 5a). Being very close to each other, the two opposite sign vortices are coupled and their velocity increases significantly (Couder and Basdevant, 1986; Alam et al, 2005). The increased velocity of the vortices causes an increase in flow velocity in the gap, hence a contribution to the negative C_L. Similar observation was made for two staggered circular cylinders by Alam et al. (2005).

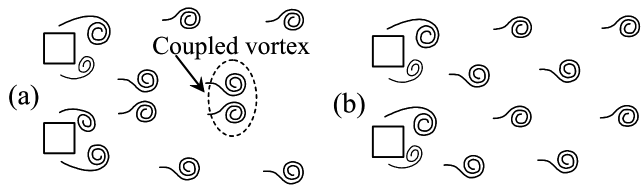

Figure 5: *Configurations of vortex shedding from the two cylinders: (a) anti-phase, (b) in-phase.*

The distribution of $C_{L(total)}$ (= $C_{L(cyl.1)}$-$C_{L(cyl.2)}$, i.e., where upward lift force is considered as positive, Fig. 1) indicates that the combined lift force is negative for T/d = 1.1 ~ 3.0 (biased flow regime). In other words, in the biased flow regime, the combined lift force is downward (narrow-wake cylinder to wide-wake cylinder). The maximum magnitude of $C_{L(total)}$ is 0.44 at T/d = 1.6. It is interesting that, in the biased flow regime, if one considers the two cylinders as a connected body, it is symmetric with respect to the incident flow. However, a lift force is still generated. Such lift could be used effectively in engineering field. For example, two masts placed on or beneath a boat or ship may move the boat or ship perpendicular to natural air flow or water current without expending any fuel energy. Previously a rotating mast of large diameter in ship, which needs extra fuel energy to rotate, was practically used to generate lift (known as the magnus effect) in a natural air flow. At T/d = 1.02 $C_{L(total)}$ is zero, indicating that the gap flow is negligible, and the two cylinders behave like a single body.

3.2. Fluctuating forces

The distributions C_D' and C_L' are illustrated in Fig. 6. C_D' at T/d = 1.02 is 0.69, which is about 2.37 times larger than that of a single isolated cylinder. Such a high magnitude of C_D' could be attributed to three reasons: (i) the gap flow is negligible and the two cylinders behave like a single bluff body, (ii) the rolling positions of the shear layers are close to the base surface as W/d for this T/d is close to the critical W/d (Bearman and Trueman 1972), and (iii) C_D' is measured for only half (one cylinder) of the total (two cylinders) body. In the biased flow regime (T/d = 1.1 ~ 3), Cylinder 1 experiences a higher C_D' and C_L' than Cylinder 2. C_D' and C_L' of both cylinders are small at 1.3 < T/d < 2.2, suggesting that the gap flow acts as an effective base bleed, hence weakens the vortex shedding from the outer sides of the cylinders at this T/d range. The difference between C_D' or C_L' of the two cylinders over the range of 1.3 < T/d < 2.2 is very small. As T/d increases from 2.2 to 2.7 C_L' increases sharply and the difference in C_L' between the two cylinders is significant. The magnitude of C_L' for the two cylinders is large for T/d = 2.7 ~ 4.6 due to presence of anti-phase vortex shedding.

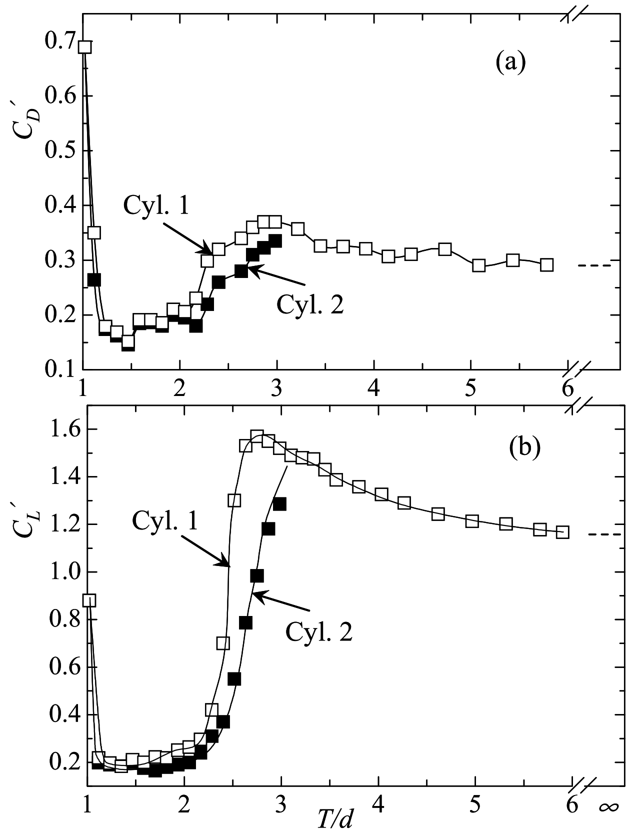

Figure 6: *Dependence on T/d of (a)* C_D'*, (b)* C_L'*. – – –, single isolated cylinder.*

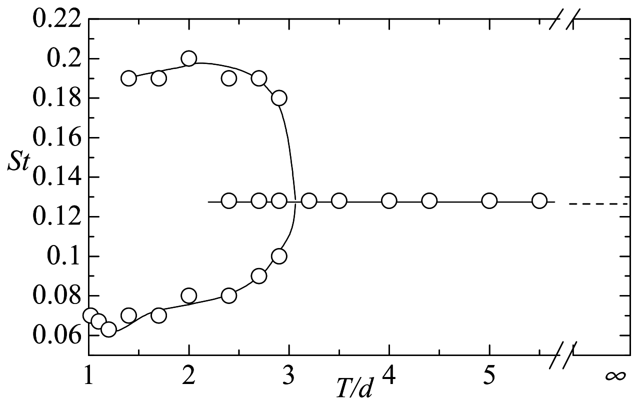

Figure 7: *Dependence of St on T/d. – – –, single isolated cylinder.*

3.3. Strouhal number

The dominant St is plotted as a function of T/d in Fig. 7. Apparently, both cylinders have the same St, 0.063 ~ 0.07, at T/d <1.3, implying that the two cylinders shed vortices at the same frequency. Considering the two cylinders as a whole body, the effective St based on the total height (T+d) for those T/d is estimated to be ≈0.14, which is very close to 0.138 for a rectangular cylinder with W/d = 0.5 (Norberg 1993). Note that a decreasing W/d from 1.0 will result in a slight increase in St (Nakaguchi et al, 1968). The observation suggests that at T/d < 1.3 only the outer shear layers dominantly shed vortices regarding the total height of the two cylinders, hence the two cylinders behave as a single body in terms of St. Though T/d < 1.3 is in the biased flow regime, the biased gap flow could not influence on the two outer-shear-layer shedding frequencies to be different. This is possibly due to a small gap width, hence a small amount of flow in the gap. For T/d > 3.0, both cylinders have the same St (= 0.128) which is equal to that of a single isolated cylinder, indicating that the two cylinders are separated sufficiently to shed vortices individually like single isolated cylinders. However, there are two values of St, a lower (0.07 ~ 0.08) and higher (0.19 ~ 0.20) at 1.3 < T/d < 2.2 and three values of St, the lower, higher and an intermediate (0.128) St at 2.2 < T/d < 3.0. The lower and the higher values of St correspond to the flow structures associated with wider and narrower wakes, respectively. As the intermediate St is the same as the single cylinder value, it is likely that either cylinder has a tendency to shed vortices like a single isolated cylinder. On the other hand, as the intermediate St is the same as the St for T/d > 3.0

and noted that the intermediate *St* peak in power spectra becomes smaller with decreasing *T/d* from 3.0, it seems that the flow structure that appears for *T/d*>3.0 continues to appear intermittently at 2.2 < *T/d* < 3.0. In other words, the 2.2 < *T/d* < 3.0 could be termed as a transition regime of two-frequency (low and high) flow to the same-frequency (*T/d* > 3) flow.

3.4. Co-spectrum between u_1 and u_2

Figure 8 presents the co-spectrum between u_1 and u_2 for various separations. The figure reveals some interesting information on the flow characteristics. (i) Negative peak at the *St* is observed over *T/d*=1.02 ~ 1.2 and the peak is the strongest at *T/d* = 1.02, implying that the outer shear layers of the two cylinders shed vortices in alternating fashion for these *T/d*; however, their alternating shedding tendency and/or strength of vortices are strong at *T/d* = 1.02. This is another proof on the behalf of being two cylinders united. (ii) At 1.3 < *T/d* < 2.2, no dominant peak is observed, suggesting that the two outer shear layers always shed vortices at the different frequencies. (iii) For *T/d* > 3.0 where the gap flow is non-biased, positive peak at f^* = 0.128 (=fd/U_∞, where *f* is Fourier frequency) indicates that the outer shear layers shed vortices in synchronization at *St* = 0.128 with ≈ 0° phase, i.e,. an anti-phase vortex shedding prevails behind the cylinders. (iv) As 2.2 < *T/d* < 3.0 is in the biased flow regime, the presence of positive peak indicates that the gap flow is slightly biased but intermittent synchronized vortex shedding from the two cylinders occurs at *St* = 0.128 along with low and high frequency modes (Fig. 7). (v) Negative small peak at the close proximity to the positive peak is self-evident for *T/d* ≥ 5.0, implying an existence of in-phase vortex structure along with anti-phase one. As the negative peak takes place just in the right side of the positive peak, it is likely that the *St* for in-phase vortex structure would be slightly higher than that for anti-phase structure.

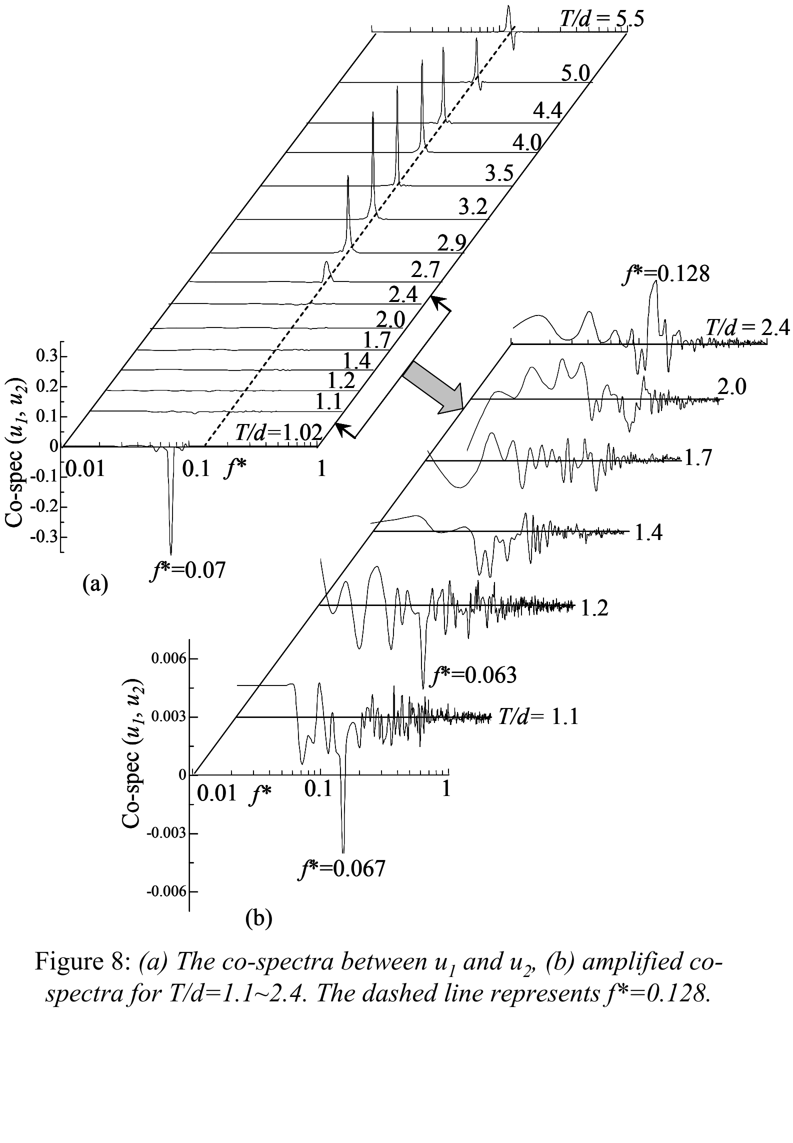

Figure 8: *(a) The co-spectra between u_1 and u_2, (b) amplified co-spectra for T/d=1.1~2.4. The dashed line represents f*=0.128.*

4. DISCUSSION AND CONCLUSIONS

The present investigation suggests four distinct flow regimes, i.e., A, B, C and D*,* depending on *T/d,* in the wake of two side-by-side square cylinders (Fig. 9). (i) In regime *A* or the single-body regime (*T/d* < 1.3), the two cylinders behave like a single body in terms of *St*, with vortices separated alternately from the outer sides of the cylinders. The gap flow between the cylinders has a negligibly small effect for *T/d* ≤ 1.02 but not for 1.02 < *T/d* <1.3. As such, the single-body regime *A* may be subdivided into the perfect single-body regime A_1 and single-body-like regime A_2. In A_1 (*T/d* ≤ 1.02), *St* and the total drag or lift are all the same as in a single cylinder wake; in A_2 (1.02 < *T/d* < 1.3), only *St* remains the same as in the single cylinder wake. In regime A_1 the forces on the two cylinders are the same. So is *St* associated with each cylinder. At *T/d* = 1.02, C_D and C_D' reach 1.19 and 2.37 times, respectively, of their counterpart in an isolated cylinder wake since *W/d* of the combined cylinders reaches a critical value. The repulsive C_L on the cylinders reaches 0.73 due to the stagnant fluid between the cylinders. (ii) In regime *B* or the two-frequency regime (1.3 < *T/d* < 2.2), the two cylinders shed vortices at distinct frequencies, the gap flow acting as an effective base bleed to weaken vortex shedding from the outer side of the cylinders, resulting in very small C_D, C_D', C_L' for both cylinders. (iii) In regime *C* or the transition regime (2.2 < *T/d* < 3), the two distinct frequencies of vortex shedding from the two cylinders may switch to one (the same as *St* in an isolated cylinder wake), as the case of coupled vortex shedding, or vice versa; the corresponding C_D, C_D', C_L' grow with increasing *T/d*. (iv) In regime *D* or

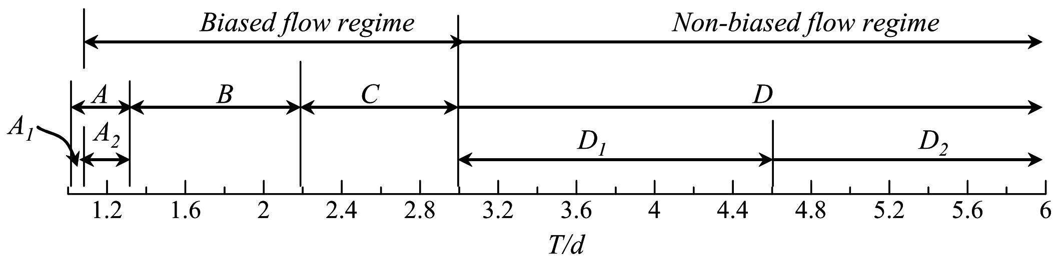

Figure 9: *Flow regimes depending on T/d. A: Single body regime,(A_1: perfectly single-body regime; A_2: single body-like regime. B: two-frequency regime. C: transition regime of two-frequency to coupled. D: coupled vortex-shedding regime (D_1: anti-phase dominant vortex shedding. D_2: anti-phase and in-phase vortex shedding regime).*

the coupled vortex-shedding regime, both cylinders shed vortices at the same frequency as in an isolated cylinder wake and maintain a constant phase shift between their shedding. Depending on the phase shift, this regime could be further divided into two: regime D_1 or the anti-phase regime ($3 < T/d < 4.6$), where vortices separated from the two cylinders are anti-phased, and regime D_2 or the anti- and in-phased vortex shedding regime ($4.6 < T/d < 6$), where both anti- and in-phased vortex shedding occur.

5. ACKNOWLEDGMENTS

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6. REFERENCES

Agrawal, A., Djenidi, L. and Antonia, R.A., 2005, Investigation of flow around a pair of side-by-side square cylinders using the lattice Boltzmann method. Computers and Fluids, **35**, 1093-1107.

Alam, M. M., Sakamoto H., Zhou, Y., 2005, Determination of flow configurations and fluid forces acting on two staggered circular cylinders of equal diameter in cross-flow. *Journal of Fluids Structures*, **21**, 363-394.

Alam, M.M., Moriya, M., Sakamoto, H., 2003, Aerodynamic characteristics of two side-by-side circular cylinders and application of wavelet analysis on the switching phenomenon. *Journal Fluids and Structures*, **18**, 325-346.

Alam, M.M., Moriya, M., Takai, K., Sakamoto, H., 2002, Suppression of fluid forces acting on two square cylinders in a tandem arrangement by passive control of flow. *Journal Fluids and Structures*, **16**, 1073-1092.

Bearman, P.W.,Trueman, D.M., 1972, An investigation of the flow around rectangular cylinders. *Aeronautical Quarterly* **XXIII, 3**, 229-237.

Couder, Y., Basdevant, C.B., 1986, Experimental and numerical study of vortex couples in two-dimensional flows. *Journal of Fluid Mechanics*, **173**, 225-251.

Courchesne, J., Laneville, A., 1982, A experimental evaluation of drag coefficient for two-dimensional rectangular cylinders exposed to grid turbulence. *ASME Journal of Fluids Engineering*, **104**, 523-528.

Franssona, J.H.M., Koniecznyb, P., Alfredsson, P. H., 2004, Flow around a porous cylinder subject to continuous suction or blowing. *Journal of Fluids and Structures*, **198**, 1031-1048.

Kim, H.J., Durbin, P.A., 1988, Investigation of the flow between a pair of circular cylinders in the flopping regime. *Journal of Fluid Mechanics*, **196**, 431-448.

Kolář, V., Lyn, D.A., Rodi, W., 1997, Ensemble-averaged measurements in the turbulent near wake of two side-by-side square cylinders, *Journal of Fluid Mechanics*, **346**, 201-237.

Nakaguchi, H., Hasimoto, K., Muto, S., 1968, An experimental study of aerodynamic drag on rectangular cylinders. *Journal Japan Society Aeronautical Space Science*, **16**, 1-5.

Norberg, C., 1993, Flow around rectangular cylinders: pressure forces and wake frequencies. *Journal of Wind Engineering and Industrial Aerodynamics*, **49**, 187-196.

Sakanoto, H., Haniu, H., 1988, Aerodynamic forces acting on two square prisms placed vertically in a turbulent boundary layer. *Journal of Wind Engineering & Industrial Aerodynamics*, **31**, 41-66.

Zdravkovich, M.M., 1987, The effects of interference between circular cylinders in cross flow. *Journal of Fluids and Structures*, **1**, 239-261.

Zhou, Y., Wang, Z.J., Xu, S.J., Jin, W., 2001, Free vibrations of two side-by-side cylinders in a cross-flow. *Journal of Fluid Mechanics*, **443**, 197-229.

Zhou, Y., Zhang, H. J. & Yiu, M.W.: 2002 The Turbulent Wake of Two Side-by-Side Circular Cylinders, *J. Fluid Mech*. **458**, 303-332.