AEROELASTIC INSTABILITY OF BENDING-TORSION WINGS CONTAINING A MASS SUBJECTED TO FOLLOWER FORCE

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ABSTRACT

The aeroelastic modeling and flutter characteristics of a wing/store under follower force is considered. The aeroelastic governing equations and boundary conditions are determined via Hamilton's variational principle. In order to exactly consider the spanwise location and properties of the attached store and follower force the generalized function theory is used. Also, unsteady aerodynamic pressure loadings are considered. The result partial differential equations are transformed into a set of eigenvalue equations through the extended Galerkin's approach. Numerical simulation highlighting the effects of the follower force and store parameters, such as mass ratio and attached locations, on the flutter speed are presented. the numerical results for the clean wing subjected to a follower force are validated with the published results and excellent agreement is observed.

1. INTRODUCTION

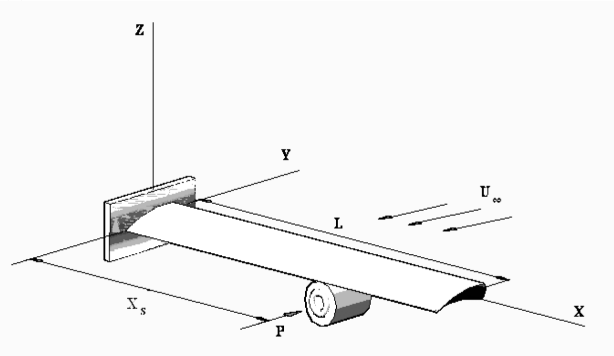
It is well known that aeroelastic instability may be induced in elastic systems because of non-conservative forces. The high-thrust engine mounted on the aircraft wing is a good example for acting neoconservative follower forces on the airplane structure. One of the most dramatic aeroelastic phenomenons is flutter, a dynamic instability which often leads to terrible structural failure in airplane components. By the fact that nowadays the importance of weight saving in flight vehicles increases the structural flexibility of the aircrafts, the effects of the thrust and store mass on the flutter speed may be very important.

The problem of a cantilever wing/store excited by a transverse follower force has not received much attention in the literature. Much of previous efforts have been made to simulate aeroelasticity of the wing have consider uniform straight wings. Como (1966) analyzed the stability of bending-torsional equilibrium of a cantilevered beam subjected at its end section to a lateral follower force. Restricting the location of the force and rigid body to the free end, Feldt and Herrmann (1974) investigated the flutter instability of a wing subjected to the transverse follower force in the presence of airflow. Aeroelastic stability of a swept wing with tip weights for an unrestrained vehicle is considered by Lottati (1987). The equations for a cantilevered thin beam are derived by Dowell et al (1974).They examined the possibility of controlling through feedback a thin cantilevered beam subjected to a nonconservative follower force. Gern and Librescu (1998) have made some efforts to show effects of externally mounted store on static and dynamic aeroelasticity of advanced swept cantilevered wings. Hodges et al (2002) show the effects of lateral follower force on flutter of cantilevered wings. However, in their work they did not take into account the concentrated weight effects. Some studies in recent years dealt with nonlinear flutter analysis of wing/store configurations without follower forces. In these works nonlinear structural and/or dynamical terms are investigated such as work done by Thompson and Strganac (2005). Most recently, an experimental model with a wing/store configuration with and without freeplay has been designed by Tang and Dowell (2006) for the study of flutter and LCO. According to the best of the authors' knowledge, in the available literature, aeroelastic analysis of the wings containing a mass subjected to the follower force have not yet been presented. It is the purpose of the present investigation to combine the follower force and the store effects, studied separately before.

2. GOVERNING EQUATIONS

The cantilever wing containing a mass subjected to a lateral follower force as shown in Fig.1.a is considered. The wing typical section is represented in Fig.1.b where y_S , z_S are the distances between the center of gravity of the store and the elastic axis of the wing. The wing is thinking as a thin beam and the structural model is used which incorporates bending-torsion flexibility. The store inertia as well as the follower force is accounts for in deriving the governing equations. These equations of motion are valid for long, straight, homogeneous, isotropic wings undergoing moderate displacements. Because of the wing flexibility two coordinate systems have been used here. As shown in Fig. 1 the orthogonal axes X,Y,Z are fixed on airplane base body in which the X axis lies in the spanwise direction. The other coordinate system xyz has been fixed on deformed wing.

(a)



(b)

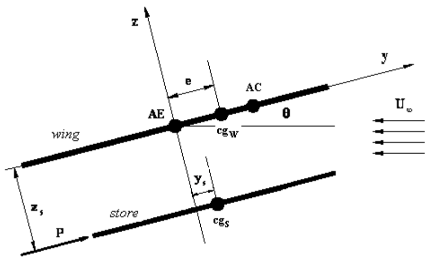


Figure 1: *The geometry and nomenclature of a wing/store configuration under follower force.*

After wing deformation the shear center of cross-section located at x is displaced an amount of u_o in x direction, v_o in y direction and w in z direction. Additionally the angle of twist of the cross-section changes to θ about the x axis. The coordinate transformation should be used between these two coordinates to derive governing equations. Assuming that the wing can be represented by a cantilever beam, the boundary conditions are as follows: at $x=0$, that is at the root of wing, deflection and slope are both zero (clamped end); at $x=l$, that is at the wing tip, moment and shear forces are both zero (free end). By using these boundary conditions, the aeroelastic governing equations will be solved. The equations of motion and boundary conditions are derived using Hamilton's variational principle that may be expressed as:

$$\int_{t_1}^{t_2} [\delta U - \delta T_w - \delta T_s - \delta W]\,dt = 0 \tag{1}$$

where U and T are strain energy and kinetic energy and W is the work done by non-conservative forces. The indices w and s identify the wing and externally mounted store, respectively. The first variation of kinetic energy of the wing is:

$$\delta T_W = \int_0^l \left\{\left(-m\ddot{w} - me\ddot{\theta}\right)\delta w + \left(-mk_m{}^2\ddot{\theta} - me\ddot{w}\right)\delta\theta\right\}dx \tag{2}$$

The first variation of the store kinetic energy also can be derived as:

$$\begin{aligned}\delta T_S = \iint_A \int_0^l\ & m_S\{z_S{}^2\ddot{w}''\delta w - (z_S{}^2 + y_S{}^2)\ddot{\theta}\delta\theta \\ & - \ddot{w}\delta w - y_S\ddot{w}\delta\theta - y_S\ddot{\theta}\delta w\}\delta(x - x_S)\,dxdA\end{aligned} \tag{3}$$

The strain energy is considered next. The first variation of the strain energy is:

$$\begin{aligned}\delta U = \int_0^l\ & \{[-GJ\theta'' + P(x_S - x)H(x_S - x)w'']\delta\theta + \\ & [EI_{y'}w'''' + P(x_S - x)H(x_S - x)\theta'' - 2P\theta' H(x_S - x)]\delta w\}dx\end{aligned} \tag{4}$$

where H and δ are Heaviside and Dirac delta functions, respectively. The virtual work of non-conservative forces acting on the wing may be expressed as:

$$\begin{aligned}\delta W = \int_0^l\ & \{[P\theta\delta(x - x_S) + \overline{L}]\delta w + \\ & [(Py_S\theta - Pz_S)\delta(x - x_S) + \overline{M}]\delta\theta\}dx\end{aligned} \tag{5}$$

Substituting the expressions for kinetic energy, strain energy and non-conservative works in Eq.(1) the aeroelastic governing equations are obtained as:

$$\begin{aligned}& m\ddot{w} + me\ddot{\theta} + EI_{y'}w'''' + \sum_{i=1}^{n} \{P_i(x_{si} - x)H(x_{si} - x)\theta'' \\ & - 2P_i\theta' H(x_{si} - x) + [M_{si}\ddot{w} + M_{si}y_{si}\ddot{\theta} - M_{si}z_{si}{}^2\ddot{w}'' \\ & - P_i\theta]\delta(x_{si} - x)\} = \overline{L}\end{aligned} \tag{6}$$

$$\begin{aligned}& mk_m^2\ddot{\theta} + me\ddot{w} - GJ\theta'' + \sum_{i=1}^{n} \{P_i(x_{si} - x) \\ & H(x_{si} - x)w'' + [M_{si}y_{si}\ddot{w} + M_{si}(z_{si}^2 + y_{si}^2)\ddot{\theta} \\ & + P_i z_{si} - P_i y_{si}\theta]\delta(x_{si} - x)\} = \overline{M}\end{aligned} \tag{7}$$

In these equations the Heaviside and Dirac delta functions are used in order to exactly consider the location and properties of the lateral follower force and the attached store, respectively and the index s identifies the affiliation of the respective quantity to the external store. It is important to note in these equations it is assumed that follower force acts on the store and apply exactly in the chordwise direction of the wing. $\overline{L}$ and $\overline{M}$ are unsteady aerodynamic lift and moment as below:

$$\overline{L} = -\pi\rho b^3\omega^2\left[l_w\overline{w}/b + l_\theta\overline{\theta}\right] \tag{8}$$

$$\overline{M} = \pi\rho b^4\omega^2\left[m_w\overline{w}/b + m_\theta\overline{\theta}\right] \tag{9}$$

In Eqs.(8,9) l_w, l_θ, m_w and m_θ are the aerodynamic coefficients as displayed by Hodges and Alvin Pierce (2002) and ρ is the air density.

3. METHOD OF SOLUTION

Due to intricacy of the aeroelastic governing equations, the solution is searched by using an approximate solution procedure. To this end, w, θ are represented by means of series of trial functions $\varphi_{\mathbf{i}}$, that should satisfy the boundary conditions, multiplied by time dependent generalized coordinates, $\mathbf{q_i}$. Consequently, the displacement quantities are expressed as:

$$w = \varphi_{\mathbf{1}}{}^{\mathbf{T}}\mathbf{q_1} \qquad , \theta = \varphi_2{}^{\mathbf{T}}\mathbf{q}_2 \tag{10}$$

Due to the complex boundary conditions and complex couplings involved in the above equations, it is difficult to generate proper comparison functions that fulfill all the geometric and natural boundary conditions. Therefore, in order to solve the above equations in a general way, the extended Galerkin's method is used. The underlying idea of this method is to select weight functions that need only fulfill the geometric boundary conditions, while the effects of the natural boundary conditions are kept in the governing equations. When the linear combination of these weight functions is capable to satisfying the natural boundary conditions, the convergence rate is usually excellent.

By substituting Eqs.(8-10) in Eqs. (6,7) and applying the Galerkin procedure on the governing equations and by using orthogonal properties in the required integrations the following set of ordinary differential equations are obtained:

$$[\mathbf{M}]\ddot{\mathbf{q}} + [\mathbf{K}]\mathbf{q} = \mathbf{Q}_{n.c} \tag{11}$$

Herein, [**M**], [**K**] and $\mathbf{Q_{n.c}}$ denote the mass matrix, stiffness matrix and nonconservative load vector respectively, while $\mathbf{q}$ is the overall vector of generalized coordinates. This representation finally leads to a complex eigenvalue problem expressed in matrix form as

$$\mathbf{A} - \omega^2\mathbf{B} = 0 \tag{12}$$

where A denotes the (real) stiffness matrix of the wing and B is the (complex) matrix representing the inertia terms of wing and external store as well as the complex aerodynamic parameters. The real part of the complex valued quantity ω represents the circular frequency of the oscillation, whereas its imaginary part constitutes the damping factor. The implemented solution methodology is based upon the inversion of the complex matrix $\mathbf{B}$ and subsequent calculation of complex eigenvalues and eigenvectors of the obtained system matrix $\mathbf{AB}^{-1}$. The flutter speed is calculated in a converging iteration process rendering zero the imaginary (damping) part of the complex eigenvalues.

4. NUMERICAL RESULTS

As stated in the previous section, the solution to this aeroelastic problem through extended Galerkin method is sought by using a numerical integration scheme. The effects of the store location and the follower force value on the flutter speed of cantilever wings are simulated.

Pertinent data, for the particular wing-weight combination used here are the same as those utilized by Harry et al (1949) and are considered in table.1. The store mass is equal to $1.578\,Kg$ and its mass moment of inertia is $0.0185\,Kg$.

Wing parameters		Dimension
Length	1.2192	m
Semi-chord	0.1016	m
Bending rigidity	403.76	$N-m^2$
Torsional rigidity	198.58	$N-m^2$
Mass per unit length	1.2942	kg/m
Moment of inertia	0.0036	$kg-m$
Spanwise elastic axis	43.7%	m
Center of gravity of wing	45.4%	m
Air density	1.224	kg/m^3

Table 1: characteristics of the wing

Dimensionless parameters are used in the numerical simulation is:

$$\lambda = \frac{EI}{GJ} \;,\; p = \frac{Pl^2}{\sqrt{GJEI}} \;,\; M_S^* = \frac{M_S}{ml} \;,\; v = \frac{U_\infty}{b\omega_\theta}$$
$$X_S = x_s/l,\; Y_S = y_s/b.$$

Where M_S is the store mass, l is the wing length, m is the mass per unit length of the wing, b is the wing semi-chord, U_∞ is the air speed and ω_θ is the first uncoupled torsional frequency. Here, the vertical distance between the center of gravity of the store and the elastic axis of the wing is equal zero. For model validation purpose the results for the wing without store are compared in Fig. 2 with Hodges et al (2002) and good agreement is reported. To this end the same wing characteristics as them are chose. In this figure flutter boundary for $\lambda = 10$ is illustrated. It is seen that there is a continue decrease in flutter speed accompanying the increase in the follower force. This can be explained as the addition of the follower force destabilizes the wing and leads to instability at lower speeds. Figure 3 shows a parametric study investigating the effect of λ on the flutter boundary of the clean wing. It can be seen that there is a continuous decrease in the magnitude of thrust required for instability with increase in airspeed. This is happened because the destabilizing effect of the aerodynamic forces added to the system. This leads to instability at lower levels of the follower force. It is seen that the stability region is quite different for lower values of λ as compared to the higher ones. This comes from the interactions between the thrust and aeroelastic destabilization mechanism.

Effect of store mass on flutter boundary of the wing/store is illustrated in Fig.4. The store is assumed to place at the tip and exactly on the elastic axis of the wing. It can be seen that the stability region of the wing is limited when the store is attached to it. This is almost independent of the mass ratio parameter M_S^*. For low values of air speed, the flutter speed increases, but, when the air speed is increased furthers the mode of instability changes from a dominant follower force mode to dominant aeroelastic instability.

The frequency and damping of the clean wing affected by a tip follower force are sketched in Fig. 5 for $M_S^* = 0$. It can be seen from Fig.5 (a) that the flutter is occurred due to intersection of the first bending mode with the second bending mode. It is clear from the Fig.5 (b) that the corresponding damping for this point is zero. Fig.6 shows the frequency and damping for the same configuration of the wing with $M_S^* = 1$. It can be understood from this figure as the store mass becomes greater, the intersection point corresponding to the flutter condition moves to left. This means that flutter occurs in smaller values of the follower force and the stability region is limited.

Influence of the spanwise location of the store on flutter speed of the wing for selected values of the follower force is shown in Fig. 7. It is important to note that follower force acts on the store and apply exactly in the chordwise direction of the wing. In this case the mass ratio is $M_S^* = 1$. In the absence of the follower force the least value of the flutter speed takes place around $X_S = 0.6$. So one can say that this point is the critical location for store mounting. This behavior is also observed by Gern and Librescu (1998). Including the follower force it can be seen in this figure that increasing the distance of the store from the wing root will decrease the flutter speed. This is more apparent for greater values of the follower force. For $p = 4$ an unusual behavior is observed. This can be qualitatively explained as the increase of the destabilizing effect of the follower force leading to instability, even at zero air speed. Dimensionless flutter speed of the wing is sketched in Fig.8 versus the dimensionless spanwise location of the store for several values of the store mass ratio. The store is mounted on elastic axis and the dimensionless follower force is $p = 2$. Its clear from the figure that increasing the store mass decreases the flutter speed of the wing. It is important to notice that although results subjected to each mass ratio is unique, trend of results are the same for all cases.

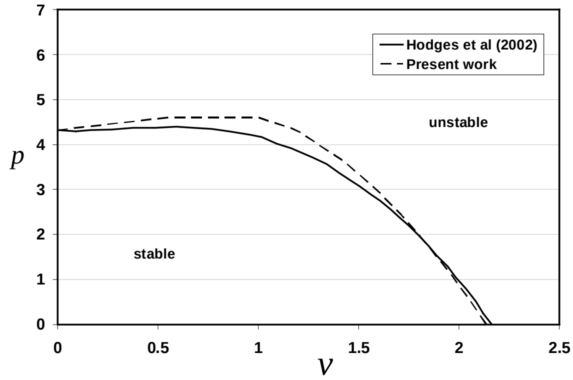


Figure 2*: Validation of flutter boundaries.*

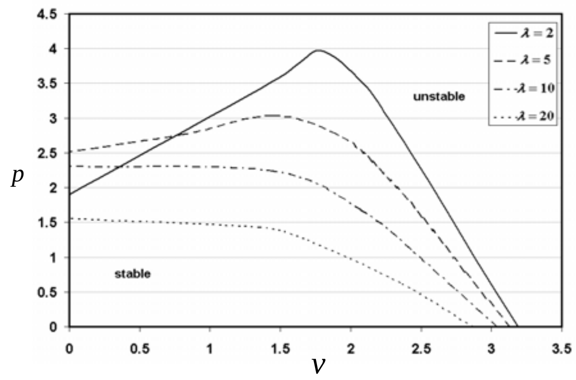


Figure 3*: Flutter boundaries for values of* λ with $X_S = 1$, $Y_S = 0$ *and* $M_S^* = 0$.

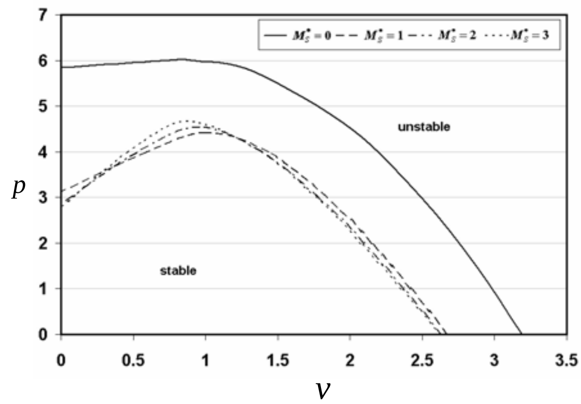


Figure 4*: Flutter boundaries for values of* M_S^* with $X_S = 0.8$ *and* $Y_S = 0$.

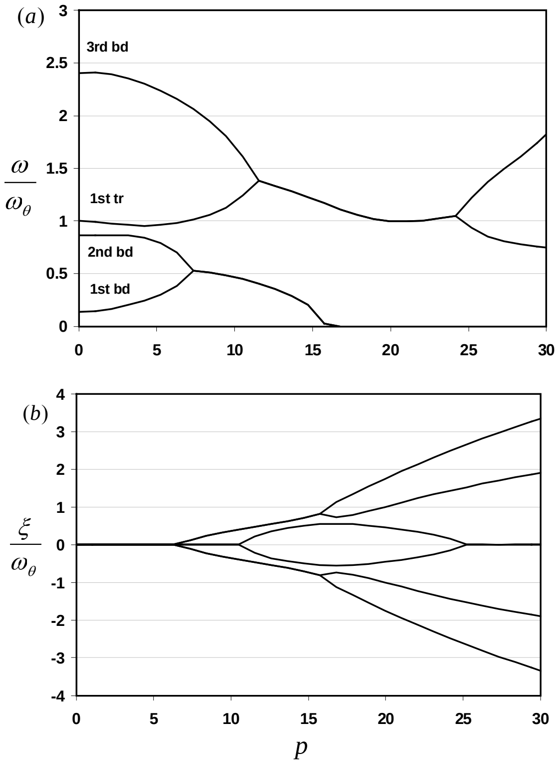


Figure 5: *Frequency and damping ratios vs follower force for* $X_S = 1$, $Y_S = 0$ *and* $M_S^* = 0$.

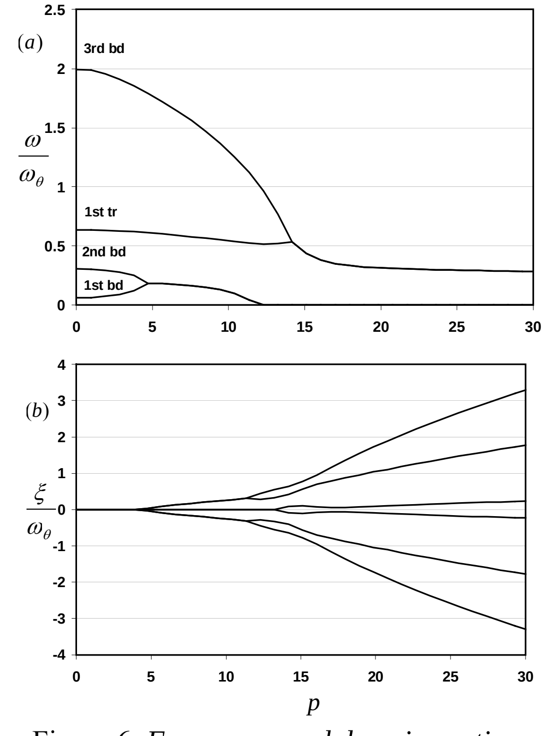


Figure 6: *Frequency and damping ratios vs follower force for* $X_S = 1$, $Y_S = 0$ *and* $M_S^* = 1$.

Also it can be observed that in presence of the follower force the critical location of the store, in the sense of dynamic stability, is the tip of the wing. This fact is independent of store mass. Influence of the chordwise location of the store on flutter speed of the wing for different values of store mass ratios is showed in Fig.9. The store is located at the tip of the wing and the dimensionless follower force is $p = 0.9$. It is observed that chordwise location of the store contributes different aeroelastic behavior for the wing with or without follower force. Sliding the store toward the front of the wing will increase the flutter speed in the case of the wing carrying the tip mass subjected to the follower force. This phenomenon is inverted in the absence of the follower force. Figure 10 shows the effect of non-dimensional chord-wise location of the store on the wing flutter speed for selected values of the follower force acting on the tip weight with $M_S^* = 0.5$. It shows that the flutter speed is increases by sliding the store toward the front of the wing. In addition, from this plot it is also possible to infer about the effects of the follower force on wing flutter speed. The result shows that a continue decrease of flutter speed accompanies an increase of the follower force, implying that the follower force decreases the flutter speed of the airplane.

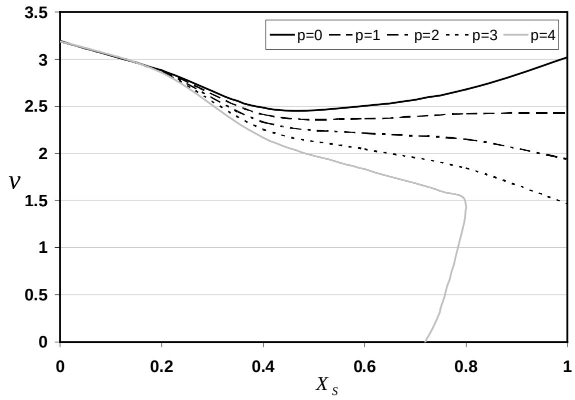


Figure 7*: Flutter speed vs spanwise position of follower force and store for* $Y_S = 0$ *and* $M_S^* = 1$.

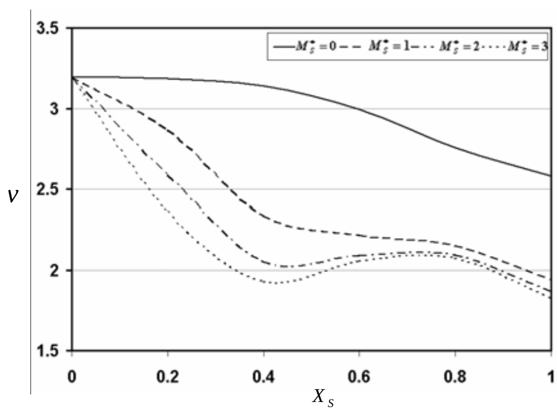


Figure 8*: Flutter speed vs spanwise position of follower force and store for* $Y_S = 0$ *and* $p = 2$.

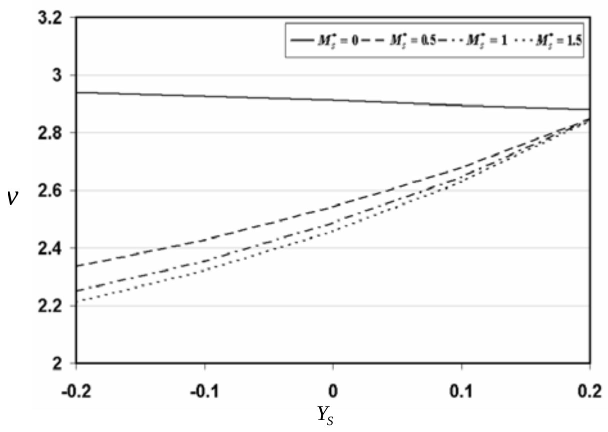


Figure 9*: Flutter speed vs chordwise position of follower force and store* for $X_S = 1$ *and* $p = 0.9$.

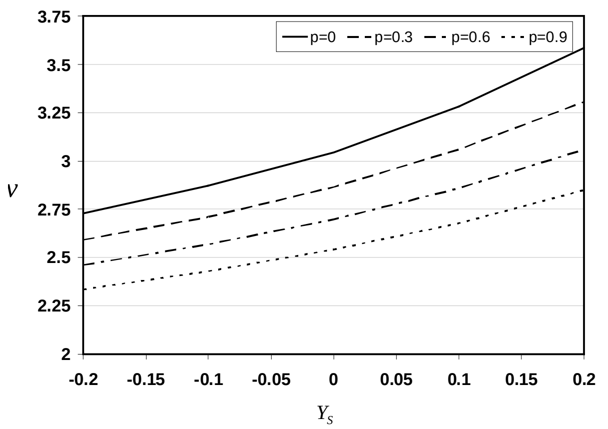


Figure 10*: Flutter speed vs chordwise position of follower force and store for* $X_S = 1$ *and* $M_S^* = 0.5$.

5. CONCLUSION

The complete aeroelastic equations for an isotropic aircraft wing carrying external store which subjected to follower force are formulated. The developed model is based on rigidity of the attached store. These equations are valid for long, straight, homogeneous wings undergoing moderate displacements. In order to exactly consider the spanwise location and properties of the attached store and follower force the Heaviside and Dirac delta functions are used. Results are indicative of the important influence of the store location and follower force on flutter speed of the wing/store. The chordwise and spanwise location of the store and the magnitude of the follower force affect the stability region of the wing dramatically.

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