Proceedings of the ASME 2010 3rd Joint US-European Fluids Engineering Summer Meeting and 8th International Conference on Nanochannels, Microchannels, and Minichannels
FEDSM-ICNMM2010
August 1-5, 2010, Montreal, Canada

FEDSM-ICNMM2010-31278

WAKE-INDUCED VIBRATION OF A PAIR OF CIRCULAR CYLINDERS AND ITS DEPENDENCY ON REYNOLDS NUMBER

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ABSTRACT

This paper investigates the wake-induced vibration (WIV) of the downstream cylinder of a pair as far as its dependency of Reynolds number is concerned. Experiments have been conducted in a circulating water channel with a rigid cylinder elastically mounted to respond with oscillations in the cross-flow direction. Various sets of coil springs were employed to vary the reduced velocity of the system maintaining constant the Reynolds number. Experiments performed with a cylinder mounted without springs provided the idealised case of reduced velocity equal to infinity. We conclude that the amplitude of the WIV response has a strong dependency on Reynolds number even within the small range between $Re = 2 \times 10^3$ *and* 2.5×10^4*. If the reduced velocity parameter is isolated — by making it equal to infinity, for instance — the Re-dependency still dominates over the behaviour of the response.*

INTRODUCTION

Wake-induced vibration (WIV) is a fluid-elastic mechanism able to excite into oscillatory motion a bluff body immersed in a wake generated from another body positioned upstream. An arrangement of a pair of cylinders is shown in Fig. 1 and our interest is in vibrations around the aligned configuration $y_0 = 0$. The upstream cylinder is exposed to a free stream with velocity U, but the downstream body is immersed in a disturbed flow region created by the wake of the upstream cylinder. Reynolds number in the present work is always based on the velocity approaching the upstream body. Vortices shed from the first body will not only pass by or impinge on the downstream cylinder, but will also interfere with vortex shedding from the downstream cylinder. Hence, if the downstream cylinder is mounted on an elastic base the response of the body will be influenced by the wake coming from the upstream body.

Previous works have found that the typical WIV response is characterised by an asymptotic build up of amplitude with increasing reduced velocity [1, 2, 3, 4]. In [5] we have investigated the origin of the fluid force involved in the excitation of the second cylinder. We concluded that WIV is indeed a wake-dependent type of flow-induced vibration (FIV), yet we found that it is the unsteadiness of the wake that plays a role in the WIV mechanism and not simply the displacement of a steady flow field. We have suggested that the WIV mechanism is sustained by unsteady vortex-structure interactions that input energy into the system as the downstream cylinder oscillates across the upstream wake.

We have shown that, for larger separations, the upstream static body sheds vortices as an isolated cylinder while the downstream elastic body responds with oscillations in a different frequency. For higher reduced velocities the upstream shedding frequency (f_s) can be many times the oscillation frequency (f), and yet the body will respond with severe vibrations. WIV is not a resonant phenomenon. Coherent vortices impinging on the second cylinder and merging with its own vortices induce fluctuations in lift that are not synchronised with the motion. While typical vortex-induced vibration (VIV) of a single cylinder finds its maximum amplitude of vibration at the resonance $f_s = f_0$ (f_0

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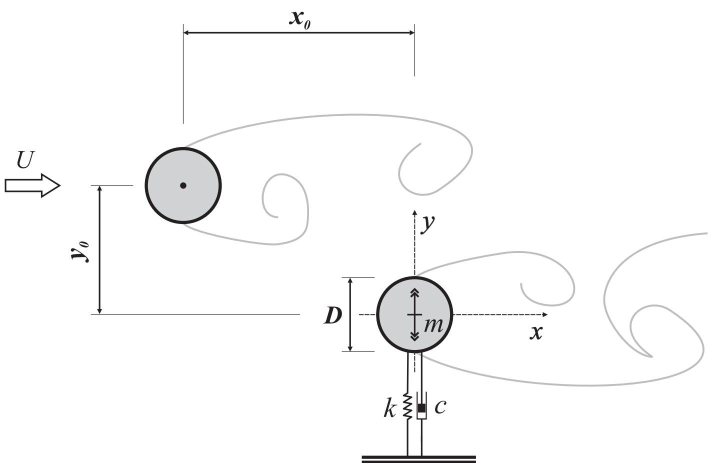

FIGURE 1. Tandem arrangement of a pair of cylinders. The downstream cylinder is elastically mounted to allow oscillations in the cross-flow direction only.

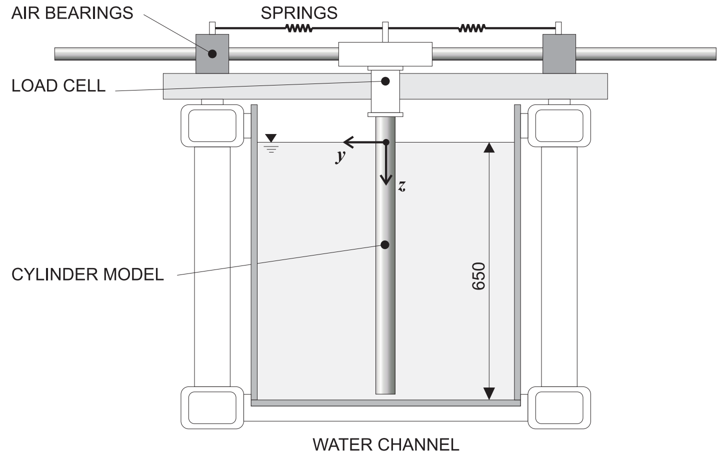

FIGURE 2. Schematic representation of the 1-dof rig holding the downstream cylinder. The free stream flows out of the page in the x-axis direction.

is the natural frequency of the structure), WIV keeps increasing $\hat{y}/D$ even when f_s is much higher than f_0.

The present experimental investigation is concerned with the Re-dependency of the cross-flow response of a rigid circular cylinder positioned at $x_0/D = 4.0$ downstream of an identical static cylinder (where x_0 is the streamwise separation measured from centre to centre). Experiments were performed in a recirculating water channel with controlled flow speed U. The downstream cylinder was elastically mounted on a system in which the structural stiffness (spring constant k and, consequently, structural natural frequency f_0) could be varied continuously. Individual control of U and k provided that both Reynolds number (ranging between $Re = 2\times10^3$ and 2.5×10^4) and reduced velocity (between $U/Df_0 = 2.0$ and 40) could be varied independently. In order to investigate the importance of Reynolds number over the WIV response we have performed experiments at constant Re varying reduced velocity by changing the spring stiffness (k) of the system.

EXPERIMENTAL SET-UP

The experimental set-up employed in the present study is described in more details in [5]. Experiments were performed in a recirculating water channel with a test section 0.6m wide, 0.7m deep and 8.0m long. Flow speed U was continuously variable up to 0.6m/s and free stream turbulence intensity was $(3.1\pm0.7)\%$ on average. The actual flow quality was proved to be adequate to perform our FIV tests. This was validated with a good agreement between our preliminary VIV results and other experiments presented in the literature [4, 6].

Two circular cylinders were made from a 50mm diameter acrylic tube with an aspect ratio of 13. Cylinders were hollow and filled with air in order to keep the mass as low as possible. It was judged preferable not to install end plates on the cylinder in order not to increase the fluid damping in the system; instead it was chosen to have the models terminating as close as possible to the glass floor of the test section. The upstream cylinder was rigidly attached to the structure of the channel preventing displacements in any direction, while the downstream cylinder was fixed from its upper end to an 1-dof elastic mounting.

The initial separation in the streamwise and cross-flow directions between cylinders (x_0 and y_0 in Fig. 1) could be varied by changing the position of the upstream model. Figure 2 shows a schematic representation of the 1-dof rig on which the downstream cylinder was mounted. Both models were aligned in the vertical direction passing through the free water surface down to the full depth of the section. The support system was firmly installed on the channel structure and the sliding cylindrical guides were free to move in the transverse direction defined by the y-axis.

A pair of sliding guides made out of a carbon fibre tube with a smooth finish ran through air bearings spanning the width of the section. All moving parts of the elastic base contributed to the effective mass oscillating along with the cylinder resulting in a mass ratio of $m^* = 2.6$ (calculated as the total mass divided by the mass of displaced water). A pair of coil springs connecting the moving base to the fixed supports provided the restoration force of the system; several spring sets were employed in order to vary the structural stiffness k. A particular condition has been prepared to represent an idealised system with no structural stiffness. This was obtained by removing the springs and letting the system to respond with no restoration force, thus making $k = 0$. By carrying out free decay tests in air it was also possible to estimate the natural frequency f_0 for each pair of springs and the overall structural damping of the system in $\zeta = 0.7\%$ (calculated as a percentage of the critical damping). Therefore, the product $m^*\zeta = 0.018$ for the majority of the experiments.

A load cell was attached between the model and the moving table to measure instantaneous and time-averaged hydrodynamic forces acting on the cylinder. An optical positioning sensor was installed to measure the y-displacement of the cylinder without introducing extra friction to damp the oscillations.

WAKE-INDUCED VIBRATION

Figure 3 presents the WIV response of the downstream cylinder of a pair mounted with springs at $x_0/D = 4.0$. The same pair of springs was employed during the whole experiment and the velocity of the flow in the test section was varied in order to cover a large range of reduced velocity, therefore yielding $Re = 2000 - 25000$. (We shall analyse the case 'without springs' later in this paper.) The first graph plots displacement versus reduced velocity. $\hat{y}/D$ is the harmonic amplitude of displacement (the rms of the signal multiplied by $\sqrt{2}$) and gives a good idea of the average amplitude of vibration for many cycles of oscillation. The characteristic build-up of response for higher reduced velocities, reported in previous works [1, 7], is clearly observed and contrasts with the typical VIV response obtained for a single cylinder [4]. A discrete hump is found to occur at around $U/Df_0 = 5.0$ and corresponds to the local peak of VIV resonance; although this happens slightly later in the reduced velocity scale due to the shielding effect of the wake of the upstream cylinder that reaches the second cylinder. Beyond that, a branch of monotonically increasing amplitude starts to build-up with increasing reduced velocity.

The bottom graph of Fig. 3 shows the dominant frequency of oscillation. During the beginning of the VIV regime the frequency curve follows closely the $St = 0.2$ line until $f = f_0$, but later departs from it to follow the lock-in behaviour observed for a single cylinder within the synchronisation regime. But where the typical VIV regime would have finished for a single cylinder, say for $U/Df_0 > 15$, the f curve remains on the same trend as before, which is distinctively lower than $St = 0.2$. This is the first evidence that there must be a fluid force with a lower frequency that dominates the excitation — lower than the vortex shedding frequency of both cylinders.

Looking again at the response curve in Fig. 3 it is quite apparent that three different regimes can be identified by different inclinations of the displacement curve: (i) a VIV resonance hump (upper branch) around $U/Df_0 = 5$; (ii) a combined VIV (lower branch) and WIV regimes roughly in the range $U/Df_0 = 5 - 17$; and (iii) a WIV regime for $U/Df_0 > 17$. The WIV response of the downstream cylinder of a pair is distinctively different from the VIV response of a single cylinder. Although some aspects are common to both types of FIV, especially those related to the overlap of VIV regime in the WIV response, others are very different.

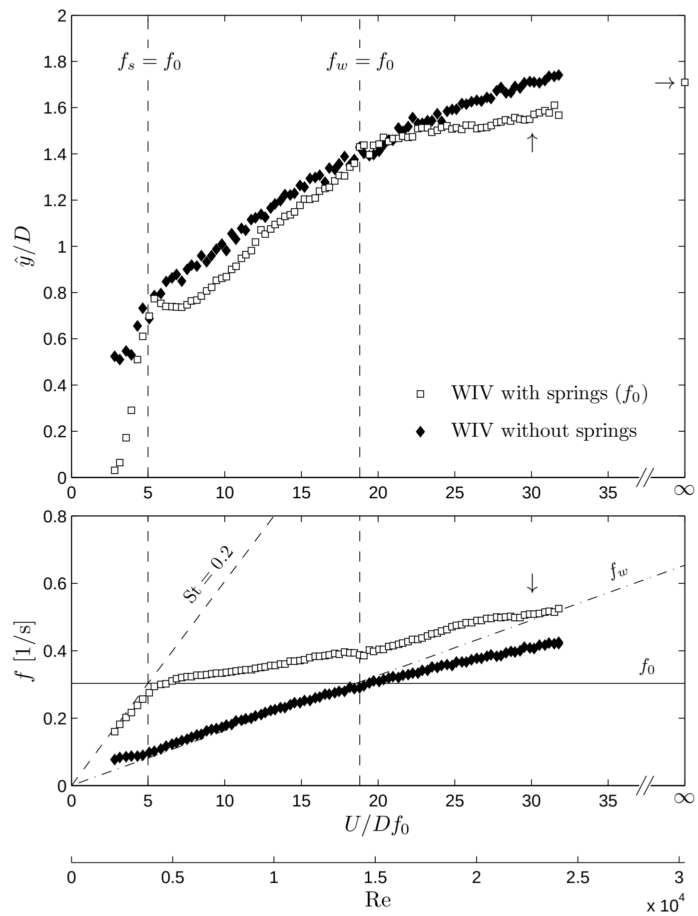

FIGURE 3. WIV response of a downstream cylinder mounted with and without springs at $x_0/D = 4.0$. Top: displacement; bottom: dominant frequency of oscillation.

The low frequency of response observed for high reduced velocities is not directly associated with the vortex shedding mechanism of either cylinder.

Figure 3 also presents the WIV response for a downstream cylinder mounted without springs. Both curves were obtained for the same variation of the flow speed; therefore both data sets share the same Reynolds number scale. But because the system without springs has no inherent f_0 it does not make sense to plot this curve with a reduced velocity axis. In fact, by making $f_0 = 0$ we are effectively making $U/Df_0 = \infty$ for all points of the response without springs; the variation of flow speed can only be represented by Re in this case.

It was interesting to observe that a cylinder without springs was able to sustain oscillations, but most surprisingly the amplitude of response was remarkably similar to the case with springs. As far as the amplitude of response is concerned, it appears that

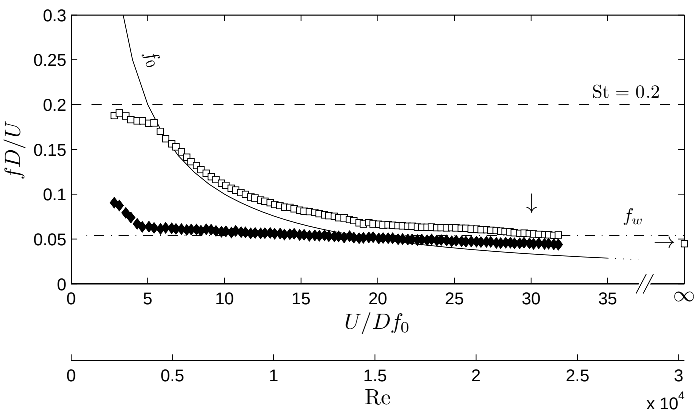

FIGURE 4. Non-dimensionalised dominant frequency of oscillation of a downstream cylinder mounted with and without springs. Please refer to Fig. 3.

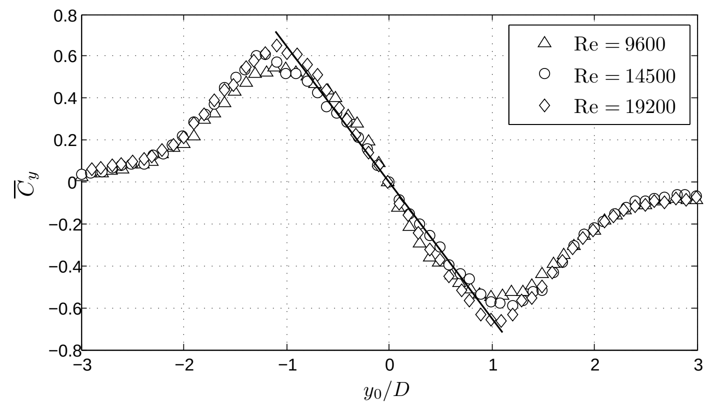

FIGURE 5. Steady fluid forces on a static downstream cylinder at $x_0/D = 4.0$ and various staggered positions.

the absence of springs is insignificant for the WIV mechanism. As expected, the local peak of VIV around $U/Df_0 = 5.0$ disappeared once the resonance $f_s = f_0$ was eliminated by removing the springs. But the overall response for both cases, with and without springs, is notably similar. The fact that $\hat{y}/D$ increases with flow speed is not an effect of reduced velocity. In other words, the increase in WIV response observed for a cylinder without springs cannot be related to any structural stiffness; instead, it seems that the response reveals some dependency simply on Reynolds number. Since both curves are essentially very similar, we suggest that an independency of response from reduced velocity and a dependency on Re might as well be occurring for the cylinder mounted with springs.

Let us turn now to the frequency of response presented in the bottom graph of Fig. 3. Since f_0 is not defined for the case without springs, we can only compare both curves if they are plotted in dimensional form (1/s). In contrast with the frequency response with springs the case without springs shows no effect of VIV synchronisation — that is obvious since there is no f_0 for it to be synchronised with — but follows an almost straight line as the flow speed is increased. In fact, we note that it follows very closely a dash-dotted line marked as f_w, which we shall explain later. Another way to analyse this result is to create a non-dimensional parameter fD/U, a type of Strouhal number, plotted in Fig. 4. This way, the $St = 0.2$ line presented in Fig. 3 becomes a constant in Fig. 4 and all the data is distorted to incorporate the effect of U varying in both axes. We shall return to this graph after some analytical modelling that will follow in the next sections.

THE WAKE STIFFNESS CONCEPT

If the downstream cylinder is dislocated from the centreline of the wake it will experience a steady lift force even if the bodies are held static in the flow [8, 2]. Zdravkovich [1] presents a map of steady fluid forces acting on a cylinder across the wake for separations as large as $x_0/D = 5.0$. His results, which are in agreement with many other maps in the literature, clearly show that the steady lift always points towards the centreline of the wake, i.e. as restoring the staggered downstream cylinder back to the tandem configuration. The steady lift is zero on the centreline of the wake, increases as the second cylinder is displaced towards the wake interference boundary and is reduced as the body is positioned farther out of the wake. Keeping the downstream cylinder at $x_0/D = 4.0$ and traversing it in fine steps across the wake we built the quasi-steady behaviour of lift ($\overline{C}_y$) versus lateral spacing presented in Fig. 5. Once more it shows that the steady lift acting on the downstream cylinder points towards the centreline of the wake for all y_0/D separations.

We note that $\overline{C}_y$ acting towards the centreline has a rather good linear behaviour between $-1.0 < y_0/D < 1.0$ and does not vary with Re (at least within the range of the experiments). Of course nonlinearities appear for larger separations, but we can estimate the slope

$$\alpha_{\overline{C}_y} \equiv \left| \frac{\partial \overline{C}_y}{\partial (y_0/D)} \right|_{y_0/D=0} = 0.65 \qquad (1)$$

within 95% confidence around the centreline and inside the wake interference region. For convenience, we shall refer to this slope simply as $\alpha_{\overline{C}_y}$ from now on. In [5] we have suggested that such a strong steady lift is induced by the unsteady interaction of vortices present in the wake coming from the upstream cylinder. However, in the present work it suffices to know that this effect works as a restoring force towards the centreline — a type of fluid-dynamic "spring" — providing a flow-originated stiffness to the system due to the wake interference effect; hence such an effect will be referred to as *wake stiffness*. The equivalent spring constant that would generate such a flow effect is given by

$$k_w = \alpha_{\overline{C}_y} \frac{1}{2} \rho U^2; \tag{2}$$

thus an equivalent natural frequency f_w could also be associated with the wake stiffness and expressed by

$$f_w = \frac{1}{2\pi} \sqrt{\frac{k_w}{(m^* + C_a)\rho \frac{\pi D^2}{4}}} \tag{3}$$

(where C_a denotes the potential added mass coefficient).

Since wake stiffness is a fluid-dynamic force, its effect would be equivalent to a spring with a constant k_w that increases with U^2, hence the associated natural frequency f_w increases linearly with *Re*. In fact, the wake stiffness is so dominant that even if we remove the coil springs from the 1-dof rig employed in the experiments the downstream cylinder is able to respond with oscillatory movements sustained by this fluid-dynamic restoration as seen in Fig. 3.

If we spend some time modelling the WIV response of a cylinder without springs [5], i.e. making $k = 0$ in the equation of motion, we find that the amplitude of response

$$\frac{\hat{y}}{D} = \frac{1}{4\pi} \hat{C}_y \sin\phi \frac{U}{Df} \frac{\rho U D}{\mu} \frac{\mu}{c}. \tag{4}$$

is a function of non-dimensional groups that include the flow speed U, friction damping c, frequency of oscillation f and the phase angle ϕ between the excitation force $\hat{C}_y$ and the displacement. Note that neither the mass nor any stiffness comes into the equation, but the excitation is simply balancing the structural damping of the system. Knowing that μ is a physical property of the fluid and assuming that viscous damping c is only based on the friction of the air bearings, we conclude that μ/c does not vary with Reynolds number. We are left with three non-dimensional groups that might have some dependence on flow speed: (i) $C_y \sin\phi$ is associated with the excitation force and can be thought of as a constant in respect of *Re*[5]; (ii) U/Df represents the inverse of a non-dimensional frequency of oscillation; (iii) $\rho U D/\mu$ is the Reynolds number itself.

Substituting Eqn. 2 in Eqn. 3 and multiplying it by D/U results in a Strouhal-type non-dimensional parameter

$$\frac{f_w D}{U} = \frac{1}{2\pi} \sqrt{\frac{2}{\pi} \frac{\alpha_{\overline{C}_y}}{(m^* + C_a)}}. \tag{5}$$

We already know that $\alpha_{\overline{C}_y}$ is independent of *Re*. If it is true that the cylinder is oscillating with the characteristic frequency of wake stiffness ($f = f_w$ in Figs. 3 and 4), regarding that C_a cannot vary with *Re*, we conclude that $f_w D/U$ is a constant irrespective of *Re*.

Turning back to Fig. 3 we will note that f for a cylinder without springs presents a remarkable linear behaviour that grows with *Re*, which is represented by an almost constant curve far from $St = 0.2$ in Fig. 4. This suggests that there must be a fluid force with a characteristic frequency lower than f_s dominating the excitation. Note that this force cannot be related to f_0 because the system has no springs. Therefore we are left with the possibility that this restoration is indeed coming from the $\overline{C}_y$ field, hence it must be related to $\alpha_{\overline{C}_y}$. Now if we substitute $\alpha_{\overline{C}_y} = 0.65$, $m^* = 2.6$ and $C_a = 1.0$ in Eqn. 5 we find that $f_w D/U = 0.054$, which is represented by the f_w dot-dashed line in Figs. 3 and 4. The agreement between f_w and the WIV response without springs is remarkable. This is evidence that a cylinder without springs may as well be responding to the wake stiffness with $f = f_w$ for the whole range of *Re*.

If it is true that $f = f_w$, Eqn. 5 tells us that fD/U is also a constant and the cylinder indeed oscillates with f that increases linearly with *Re*. In Fig. 3 we note that f closely follows f_w up to around $Re = 1.5 \times 10^4$ when the response amplitude reaches about $\hat{y}/D = 1.4$. Beyond this point the amplitude grows towards values around $\hat{y}/D = 1.8$ meaning that the cylinder is oscillating further out of the wake interference region. From the $\overline{C}_y$ map for $x_0/D = 4.0$ (Fig. 5) we know that the steady lift grows linearly with lateral separation up to around $y_0/D = 1.0$. Farther than that non linear effects start to appear and the wake stiffness cannot be represented simply by the constant slope $\alpha_{\overline{C}_y}$ but should gradually be reduced. This is exactly what is observed as the frequency curve begins to depart from the f_w line as $\hat{y}/D$ increases. Of course some effect in reducing f must be coming from the fact that secondary effects in the effective added mass of fluid may be appearing as the cylinder moves in and out of the wake interference region. But even considering that the effective added mass is constant throughout *Re* the agreement is still very good.

Turning back to Eqn. 4 we can now verify that μ/c, U/Df and $\hat{C}_y \sin\phi$ are approximately invariant with *Re*, leaving only the Reynolds number term itself on the right-hand side of the equation. As a result it is evident from this analysis that $\hat{y}/D$ is linearly dependent on *Re* and the WIV response should increase with flow speed up to a critical amplitude. Once the cylinder starts to be displaced out of the wake interference region nonlinear effects become important limiting the response to an asymptotic value. Secondary effects may be acting on U/Df and $\hat{C}_y \sin\phi$ conferring on the response the curved shape presented in Fig. 3. The analysis developed above is in good agreement with displacement curves presented for both cases with and without springs. Therefore we conclude the mechanism that is building up the amplitude of vibration in WIV is definitely not a consequence of reduced velocity but a direct effect of Reynolds number.

Picking a displacement point from the curve without springs at an arbitrary value of $Re = 2.3\times10^4$ (represented by a vertical arrow in Fig. 3) we are able to estimate the limiting value the response is asymptotically approaching as $U/Df_0 \to \infty$ for that specific *Re*. Of course this is the data point from the curve without springs immediately above the vertical arrow, but it can also be represented on the right-hand side axis for $U/Df_0 = \infty$. Such a strong *Re* dependency turned out to be a rather unexpected result. It took us some time to comprehend how a fluid-elastic system could show considerably high variations in such a short *Re* range. But if we consider that our system actually possesses a fluid-dynamic spring that increases stiffness with U^2 (Eqn. 2) we are left with the only conclusion that $\hat{y}/D$ must indeed vary with flow speed.

VIV AND WIV RESONANCES

If the wake-stiffness is dominant over the vortex-impulse term it is straightforward to predict that the cylinder should respond with $f = f_w$ and not $f = f_s$. As we have seen so far $f_w D/U$ does not vary with flow speed, thus f_w increases linearly with *Re*. Since f_0 is a constant defined by the springs, there must be a critical point where the wake stiffness has the same intensity as the spring stiffness, i.e. $k_w = k$ and $f_w = f_0$. This occurs in Figs. 3 and 4 where f_w crosses the f_0 line at $Re = 1.4\times10^4$ (equivalent to $U/Df_0 = 18.8$ for the case with springs). We know the present set of coil springs provides the system with a stiffness of $k = 11.8$N/m. But considering the steady lift map with $\alpha_{\overline{C}_y} = 0.65$ in Eqn. 2 we see that the wake stiffness can reach values as high as $k_w = 34$N/m at the end of the *Re* range of the experiments. For the case with springs we find f following closer to the f_0 line during the range where VIV is relevant, with the lock-in peak occurring around the intersection of f with both f_0 and $St = 0.2$ lines. This first VIV resonance is marked by the vertical line $f_s = f_0$ in Figs. 3 and 4. At this point $k_w = 1.8$N/m is only 15% of k provided by the springs. As the flow speed is increased the VIV synchronisation tends to disappear as $St = 0.2$ moves away from f_0. At the same time the wake stiffness is also getting stronger until both k_w and k have the same value. As we saw, this occurs for $U/Df_0 = 18.8$ and is marked by the second WIV resonance line $f_w = f_0$, beyond which k_w is greater than k.

The two resonance lines divide the response for a cylinder with springs in three regimes that are best identified in Fig. 3. (i) Before $f_s = f_0$, when $St = 0.2$ is approaching f_0, the displacement resemble an initial branch of VIV and f follows the Strouhal line up to the resonance peak. (ii) The second regime, between $f_s = f_0$ and $f_w = f_0$, is marked by a steep slope in the displacement curve; f remains rather close to f_0 as the VIV synchronisation range gradually gives way to a wake stiffness that is growing stronger with *Re*. (iii) The third regime, beyond the second resonance $f_w = f_0$ is characterised by a change of slope in both the displacement and frequency curves. With $k_w > k$ the WIV response is established and dominates alone for the rest of the *Re* range.

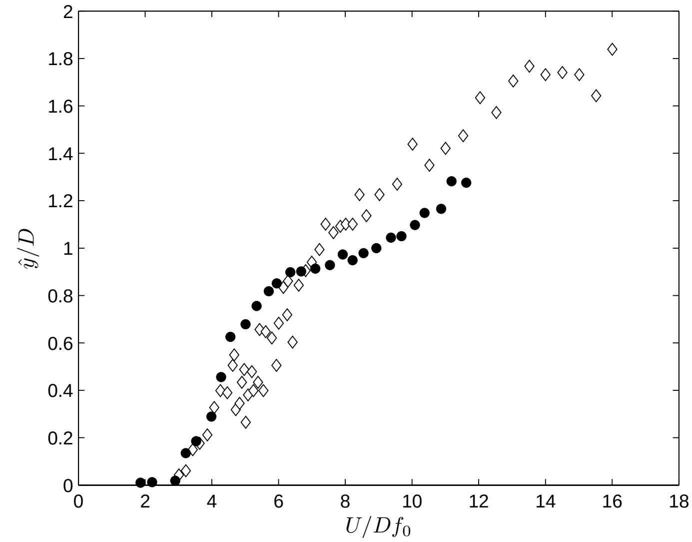

FIGURE 6. Response in the cross-flow direction of the downstream cylinder under WIV. ◇, $x_0/D = 4.75$, $m^* = 3.0$, $\zeta = 0.04$, $Re = 3\times10^4$ [3]; •, $x_0/D = 4.0$, $m^* = 1.9$, $\zeta = 0.007$, $Re = 3000-13000$ [4].

It works out as if the set of springs is important only in the first regimes before the $f_w = f_0$ resonance, but the structural stiffness (given by k) becomes less significant to the system as k_w gets relatively stronger. It appears that out of the resonances $f_s = f_0$ and $f_w = f_0$ the spring acts against the WIV excitation with the effect of reducing the amplitude of vibration. This idea is in agreement with the classical theory of linear oscillators; if the excitation force is out of the resonance of the system the response will not be as high as the resonance peak.

EXPERIMENTS AT CONSTANT REYNOLDS NUMBERS

At this point one may recall the results from [3], reproduced in Fig. 6, who measured the WIV response of a cylinder at $x_0/D = 4.75$ and constant $Re = 3\times10^4$. They achieved that by varying the spring stiffness of a force-feedback system. In spite of operating at a fixed Reynolds number, they were able to measure a build up of response that increased with reduced velocity. In principle, this seems to contradict our theory that the WIV response is not affected by reduced velocity.

Considering that their separation of $x_0/D = 4.75$ must provide a wake stiffness effect in the order of $\alpha_{\overline{C}_y} = 0.55$ (based on a map similar to Fig. 5 but for $x_0/D = 4.75$ [5, 1]), we can estimate that the critical reduced velocity at which the wake stiffness

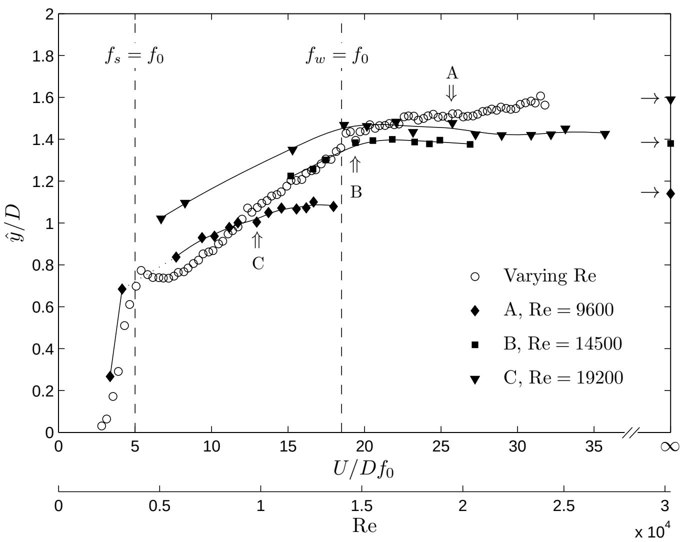

FIGURE 7. WIV response at constant Re for $x_0/D = 4.0$. Reduced velocity varied by changing the springs.

equals the spring stiffness ($k_w = k$) is as high as $U/Df_0 = 21$ (considering $C_a = 1.0$ and their value of $m^* = 3.0$). However, the maximum reduced velocity achieved in their experiment is only around 17. Hence the regime [3] observed was still between the resonances $f_s = f_0$ and $f_w = f_0$, a region where VIV still has some significance.

According to our theory, we would expect their results to reach an asymptotic value around $\hat{y}/D = 1.5$ for $Re = 3 \times 10^4$, what is in good agreement with their curve reproduced in the present work. Note, however, that [3] do not plot $\hat{y}/D$ but an average of the 10% highest peaks of displacement which can be considerably greater then the averaged $\hat{y}/D$ that we usually employ. The same observation is also true for the results obtained by [4] also presented in Fig. 6. Even though k was constant, they could not reach the regime above the WIV resonance $f_w = f_0$ due to a limitation in the maximum flow speed.

In order to verify this phenomenon, we have prepared a series of experiments for three constant Reynolds numbers at $x_0/D = 4.0$. The flow speed was fixed and reduced velocity was varied by changing the set of springs and, consequently, changing f_0. Fig. 7 presents the results compared to our reference WIV response of a cylinder with fixed springs and varying U/Df_0 by varying flow speed (the secondary axis of Re refers to this curve only).

Three vertical arrows, one for each Re curve, mark the condition where the stiffness of the varying spring matches the fixed spring k. Hence all data points to the right of these arrows have a spring that is softer than our reference curve (and stiffer to the left). None of the curves was able to span the three regimes defined by the resonance lines $f_s = f_0$ and $f_w = f_0$, but considering the results of all three curves we are able to understand the general behaviour of the response at a constant Re.

The curve for $Re = 9600$ does not have enough data points to reveal a local peak of VIV at $f_s = f_0$, but the majority of the points fall within the first regime between the resonances, where VIV is gradually losing its influence to WIV. In our experiment with varying Re we have noticed that the amplitude of response generally presents a positive slope in this first regime; this is verified now for a constant Re as well. As we have discussed above, [3] found increasing response also for a constant Re in this regime. Our data agrees with theirs in showing a build up of response between $f_s = f_0$ and $f_w = f_0$. Such an effect is also observed for our curve at $Re = 19200$.

Let us move on to the other curves at $Re = 14500$ and 19200 that are able to cross $f_w = f_0$ and enter the second regime where WIV dominates. Now that the wake stiffness is greater than the spring stiffness we see that the response is not influenced by reduced velocity anymore, but presents a rather constant level of amplitude for each fixed value of Re. Even if the reduced velocity is increased from 20 to 35 the amplitude of response seems not to be much affected and the data points appear to follow the same trend as long as Re is kept constant. Going back to the curve without springs in Fig. 3 we are able to find a displacement amplitude for each of our Re curves at $U/Df_0 = \infty$ towards which the data points should be converging. We note that they are slightly higher than the level of amplitude the curves are reaching beyond $f_w = f_0$, but we have to remember that we are still operating with springs, although soft one, that might be contributing to reduce the response away from the resonance lines.

While on one hand the VIV peak at $f_s = f_0$ seems to always reach $\hat{y}/D$ around 1.0 (for this value of $m^*\zeta$), the amplitude at the end of the first regime, at $f_w = f_0$, varies with the intensity of the wake stiffness effect. Because k_w increases with Re the amplitude at $f_w = f_0$ must also increase with Re. This level of amplitude is already very close to the asymptotic value predicted by the experiments without springs; hence, as the spring stiffness gets softer beyond $f_w = f_0$, we expect the curves to be converging towards the values plotted at $U/Df_0 = \infty$.

This series of experiments at constant Re proved that while the response below $f_w = f_0$ is dependent on both Re and reduced velocity, the response for $f_w > f_0$ is clearly governed by Re only. In other words, we conclude that in the first regime where VIV and WIV are competing (or cooperating) the response increases due to a combination of spring and wake-stiffness effects. Even with constant Re we note a build up of response while the ratio between k and k_w makes reduced velocity an important parameter. But once the wake stiffness becomes dominant over the springs the response takes no note of the structural stiffness and is only governed by wake stiffness. Now this second regime is clearly dominated by a Reynolds number effect.

CONCLUSION

We have shown by experiments and analytical modelling that the WIV response of the downstream cylinder increases with flow speed due to the wake stiffness effect as a function of Reynolds number. However, a simple model that did not account for nonlinear effects in the fluid force was not able to predict the correct level of amplitude. We found that the WIV response should converge to an asymptotic value that depends on *Re* but not on reduced velocity. As $\hat{y}/D$ is increased beyond a certain limit the cylinder starts to reach amplitudes out of the wake interference region. The wake stiffness effect cannot be represented by a linear spring anymore, but the overall stiffness tends to be reduced.

In our experiments we observed a gradual transition from an initial VIV regime to a dominating WIV regime as flow speed was increased. The boundaries between them were found to be related to two resonances: $f_s = f_0$ and $f_w = f_0$.

(i) The first regime has a clear VIV character, with a local peak of displacement occurring at $f_s = f_0$. The wake stiffness is still smaller than the spring stiffness, making U/Df_0 a significant parameter.

(ii) During the transition between both regimes we find an intermediate condition in which VIV is losing strength and WIV is taking control. Between the resonances $f_s = f_0$ and $f_w = f_0$ the response takes off from the VIV peak until it reaches a characteristic value at $f_w = f_0$ that is dependent on *Re*. During the transition, reduced velocity gradually loses its influence until the WIV response is only dominated by *Re* as it enters the second regime.

(iii) The second regime is characterised by an established WIV response that suffers no influence of VIV. Beyond $f_w = f_0$ the wake stiffness effect is dominant over the spring stiffness and reduced velocity becomes irrelevant. The amplitude of response is governed by *Re* and tends towards an asymptotic value estimated by experiments at $U/Df_0 = \infty$.

The total stiffness of the system is not only caused by either the wake stiffness (k_w) or the spring stiffness (k) alone, but it is a combinations of both. k is very relevant in the first regime, but k_w becomes dominant in the second. Nevertheless, both k and k_w contribute in parts to the characteristic displacement and frequency responses.

ACKNOWLEDGEMENTS

Authors wish to thank FAPESP and EPSRC for financial support. G.R.S. Assi is grateful to CAPES, Brazilian Ministry of Education, for his PhD scholarship.

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