Proceedings of the ASME 2010 3rd Joint US-European Fluids Engineering Summer Meeting and 8th International Conference on Nanochannels, Microchannels, and Minichannels
FEDSM-ICNMM2010
August 1-5, 2010, Montreal, Canada

FEDSM-ICNMM2010-31170

FLUID-STRUCTURE INTERACTION WITH MEAN FLOW: OVER-SCATTERING AND UNSTABLE RESONANCE GROWTH

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ABSTRACT

It is known theoretically [1-3] that infinitely long fluid loaded plates in mean flow exhibit a range of unusual phenomena in the 'long time' limit. These include convective instability, absolute instability and negative energy waves which are destabilized by dissipation. However, structures are necessarily of finite length and may have discontinuities. Moreover, linear instability waves can only grow over a limited number of cycles before non-linear effects become dominant. We have undertaken an analytical and computational study to investigate the response of finite, discontinuous plates to ascertain if these unusual effects might be realized in practice.

Analytically, we take a "wave scattering" [2,4] --as opposed to a "modal superposition" [5] -- view of the fluttering plate problem. First, we solve for the scattering coefficients of localized plate discontinuities and identify a range of parameter space, well outside the convective instability regime, where over-scattering or amplified reflection/transmission occurs. These are scattering processes that draw energy from the mean flow into the plate. Next, we use the Wiener-Hopf technique to solve for the scattering coefficients from the leading and trailing edges of a baffled plate. Finally, we construct the response of a finite, baffled plate by a superposition of infinite plate propagating waves continuously scattering off the plate ends and solve for the unstable resonance frequencies and temporal growth rates for long plates.

We present a comparison between our computational results and the infinite plate theory. In particular, the resonance response of a moderately sized plate is shown to be in excellent agreement with our long plate analytical predictions.

INTRODUCTION

The interaction of a flexible panel with an ideal flow is a fundamental problem for many engineering systems and has received considerable attention in recent years. Several distinct approaches to the problem can be identified in the literature:

a) For walls of finite length, the Galerkin method [5,6] has been used to identify the stability boundaries and unstable mode shapes by expanding the wall response in terms of the orthogonal modes of the corresponding *in-vacuo* elastic plate. b) Infinitely long plates have been studying using a travelling wave approach [7,1] based on the stability properties of the dispersion equation of the coupled fluid-structure system. c) In another approach, direct computational simulation [8] of the unsteady fluid-loaded plate equations is undertaken. d) More recently, 'long plate asymptotics' have been used [3,2] to build the finite plate response using the propagating waves of the corresponding infinite system. e) Finally, a hybrid analytical/computation eigenvalue method has most recently been used [9] to directly determine the stability properties of inhomogeneous finite structures.

The aim of this conference paper is to report recent results obtained by applying the approach d) via a comparison with direct computational studies of the long time linear response in the manner of the approach c). This is accomplished by a discussion of 'over-scattering' of propagating waves by plate discontinuities inspired by the recent work of [4], which provides an insightful bridge between the concepts of local instability and global instability.

THE INFINITE FLOW-LOADED PLATE

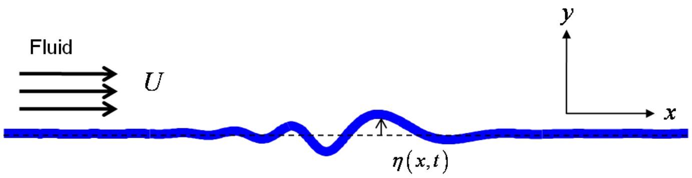

FIGURE 1: INFINITELY LONG FLUID-LOADED PLATE CONFIGURATION.

The linearized, non-dimensional equations for a line driven fluid-loaded plate with tension and spring foundation in mean (irrotational, incompressible) flow (FIGURE 1), are [1,2]:

$$
\begin{aligned}
&\left[\frac{\partial^4}{\partial x^4}-T\frac{\partial^2}{\partial x^2}+\lambda+\frac{\partial^2}{\partial t^2}\right]\eta(x,t)=-\alpha p(x,0,t)+F(t)\delta(x)\\
&\frac{\partial\phi(x,0,t)}{\partial y}=\left[\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right]\eta(x,t)\\
&p(x,y,t)=-\left[\frac{\partial}{\partial t}+U\frac{\partial}{\partial x}\right]\phi(x,y,t)\\
&\nabla^2\phi(x,y,t)=0\,,
\end{aligned}
\tag{1}
$$

where η is the plate displacement, p the fluid pressure and ϕ the fluid potential. Lengths have been non-dimensionalized by m/ρ_f, and time is non-dimensionalized by $m^{5/2}/\rho_f^2 B^{1/2}$, where m is the linear density of the plate, ρ_f is the fluid density and B is the plate bending stiffness. The parameter α equals 1 or 2 for one-sided or two-sided fluid loading respectively. T and λ are the non-dimensional tension and spring foundation constant, respectively. For the case where $T=\lambda=0$ and the bending stiffness is the only restoring force in the plate, the non-dimensional speed U is the sole parameter governing this system of equations.

Considering solutions of the form $\eta(x,t)=A\exp(ikx-i\omega t)$ the dispersion relation for waves on an infinite elastic plate is

$$
D(k,\omega)=\frac{\alpha(\omega-Uk)^2}{\gamma(k)}-k^4-Tk^2-\lambda+\omega^2=0\,,
\tag{2}
$$

where $\gamma(k)\equiv\sqrt{k^2}$ is defined as positive on the real k-axis.

The wave energy, which is the amount of work done to build up a wave from rest, is given by [10]

$$
E_w=|A|^2\frac{\omega}{4}\frac{\partial D}{\partial\omega}=|A|^2\frac{\omega^2}{4}\left[1+\frac{\alpha(\omega-Uk)}{\omega\gamma(k)}\right].
\tag{3}
$$

Positive energy waves (PEWs) have positive 'activation energy' (i.e., net energy required from an external agency to create a steady state wave from rest) and behave conventionally. Negative energy waves (NEWs) have negative 'activation energy' and their generation results in a net decrease in the energy of the fluid loaded plate system. NEWs are destabilized by damping and, more crucially, are responsible for the phenomenon of 'over-scattering' from plate discontinuities discussed in the next section.

The wave flux for a travelling wave is given by the product between the wave energy and the group velocity [1]:

$$
J_w=E_w c_g\,.
\tag{4}
$$

The wave impedance, defined as the power carried by a unit plate velocity amplitude, follows directly from the expression for the wave flux

$$
Z_w=\frac{J_w}{(-i\omega A)^2}=\frac{1}{8}\mathrm{Re}\left\{\left[1+\frac{\alpha(\omega-Uk)}{\omega\gamma(k)}\right]\frac{\partial\omega}{\partial k}\right\}.
\tag{5}
$$

For flow velocities larger than a critical value, $U>U_c$, the plate is absolutely unstable. For $U<U_c$, the *causal* response has three distinct frequency regimes [1] (FIGURE 2):

- $\omega>\omega_p$: Absolute stability: two conventional propagating waves (k_1^+ and k_1^-), and two conventional evanescent waves, (k_2^+ and k_2^-). Here, superfixes + and – refer to waves found downstream and upstream, respectively, of a point scatterer or drive point.
- $\omega_s<\omega<\omega_p$: Neutral stability: four propagating waves, two of which are PEWs (k_1^+ and k_1^-) and two, NEWs (k_2^+ and k_2^-).
- $\omega<\omega_s$: Convective instability: one exponentially increasing wave (k_1^+), one exponentially decaying wave (k_2^+), and two propagating waves, one of which is a PEW (k_1^-) and one, a NEW (k_2^-). The two downstream travelling wavenumbers k_1^+ and k_2^+ are complex conjugate pairs with an associated phase speed of U to leading order in ω.

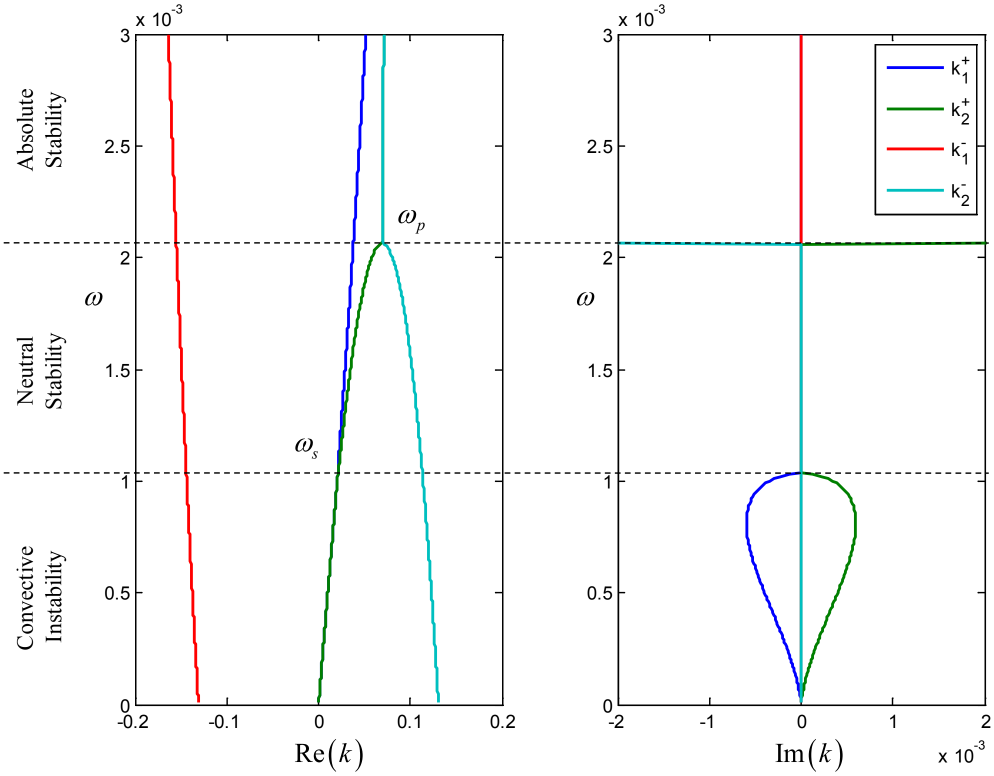

FIGURE 2: DISPERSION DIAGRAM FOR *U=0.05, T=0.8, λ=0.*

SCATTERING FROM PLATE DISCONTINUITIES

Structural discontinuities in the fluid loaded plate are modeled as multi-pole loads applied to the corresponding continuous plate [11],

$$\frac{\partial^4\eta}{\partial x^4} - T\frac{\partial^2\eta}{\partial x^2} + \lambda\eta + \frac{\partial^2\eta}{\partial t^2} + p = \sum_{m=0}^{3}\Delta f_m \delta^{(m)}(x-x_S), \qquad (6)$$

where the Δf_m's are the non-dimensional jump in shear force, moment, slope and displacement (*m=0-3,* respectively) at the discontinuity located at $x = x_s$.

The multi-pole strengths are determined by enforcing the structural edge conditions at the discontinuity. The scattered field is expressed as a sum over the one-sided multi-pole Greens functions of the fluid loaded plate

$$\eta^{\pm}_{scat}(x) = \Delta f_0 G_{\pm}(x-x_S) + \Delta f_1 G^{(1)}_{\pm}(x-x_S) + \Delta f_2 G^{(2)}_{\pm}(x-x_S) + \Delta f_3 G^{(3)}_{\pm}(x-x_S) \qquad (7)$$

where

$$G_{\pm}(x-x_S) = \frac{e^{ik_1^{\pm}(x-x_s)}}{D_{,k}(k_1^{\pm})} + \frac{e^{ik_2^{\pm}(x-x_s)}}{D_{,k}(k_2^{\pm})} + \int_0^{+\infty}\frac{e^{\mp v(x-x_s)}}{D(\pm iv)}dv + \int_0^{+\infty}\frac{e^{\mp v(x-x_s)}}{D(\mp iv)}dv \cdot \qquad (8)$$

and the dispersion function $D(k,\omega)$ is given in (2). In our notation for the one-sided greens functions, $G^{(j)}_{x+}$ signifies the jth differential with respect to the source variable x_s, the first derivative with respect to the receiver variable x defined over positive (+) $x - x_s$. The one-sided Greens functions are composed of both far field propagation terms generated by the roots of the dispersion equation and by near-field terms generated by the branch line integrals representing fluid loading.

We present our results in terms of the power-normalized scattering coefficients at the plate discontinuity which represent the ratio of scattered to incident wave-powers. They are related to the wave amplitude through the wave impedances of the scattered and incident waves defined in (5) as

$$\overline{S}^{nm} = \frac{Z_w^n}{Z_w^m}\left|S^{nm}\right|^2, \tag{9}$$

where, m and n represent the incident and scattered waves respectively.

The total energy is conserved during the scattering process and the net power scattered is equal to the incident power, as long as the proper sign of the wave energy is retained in the wave impedance. As a result, an incident positive energy wave may 'over-scatter' into another positive energy wave with larger power if one or more negative energy waves are also generated during the scattering process.

EXAMPLE: BREAK IN PLATE

A break or "closed crack" in the plate at $x=0$ is produced by imposing a zero moment and zero shear force structural condition to both sides of a joint. It is equivalent to applying quadrupole and octupole loads with the following strengths:

$$\Delta f_2 = -\frac{k_{inc}^2\left(G_{xxx+}^{(3)} - G_{xxx-}^{(3)}\right) - ik_{inc}^3\left(G_{xx+}^{(3)} + G_{xx-}^{(3)}\right)}{\left(G_{xx+}^{(3)} - G_{xx-}^{(3)}\right)\left(G_{xxx+}^{(2)} - G_{xxx-}^{(2)}\right) - \left(G_{xx+}^{(2)} - G_{xx-}^{(2)}\right)\left(G_{xxx+}^{(3)} - G_{xxx-}^{(3)}\right)}$$
$$\Delta f_3 = 2\frac{k_{inc}^2\left(G_{xx+}^{(3)} - G_{xx-}^{(3)}\right) - ik_{inc}^3\left(G_{xx+}^{(2)} + G_{xx-}^{(2)}\right)}{\left(G_{xx+}^{(3)} - G_{xx-}^{(3)}\right)\left(G_{xxx+}^{(2)} - G_{xxx-}^{(2)}\right) - \left(G_{xx+}^{(2)} - G_{xx-}^{(2)}\right)\left(G_{xxx+}^{(3)} - G_{xxx-}^{(3)}\right)} \tag{10}$$

where k_{inc} is the incident wavenumber.

As shown in FIGURE 3, this type of discontinuity is highly transmissive for waves incident from the upstream (e.g. wavenumber k_1^+). This can be seen directly in the spatial response in FIGURE 3b (note how the total deflection in $x<0$ is only slightly distorted by the small-amplitude reflected waves), and can also be seen in the corresponding FIGURE 3a, where the power-normalized transmission coefficient for k_1^+ is very close to unity, while those for the other waves are at least 10 dB lower.

The wave impedance of a downstream propagating wave is dominated by the fluid based fluxes in the frequency range under consideration. As a result, the structural break in the plate does not present a significant impedance change and the incident wave is transmitted virtually intact.

In contrast, waves incident from downstream are over-transmitted and propagate downstream (e.g. wavenumber k_1^-) at significantly amplified levels over the range of frequencies that support negative energy waves ($\omega < \omega_p$). This can be seen by the significantly larger wave amplitudes in the transmitted region $x<0$ (FIGURE 3b), and indeed over-reflection into $x>0$ is also visible. This corresponds to the power-normalized reflection and transmission coefficients in the corresponding FIGURE 3a; for the frequency considered ($\omega = 0.001$) the transmitted wave k_1^+ and the reflected wave k_1^- have amplitudes significantly greater than unity.

The power sum of all the waves scattered is represented by the dashed line in the FIGURE 3a and is identically equal to 1 (0dB). The generation of negative energy waves during the scattering process has therefore resulted in the 'over-scattering' of positive energy waves.

FINITE PLATES AND UNSTABLE RESONANCE GROWTH

LINEAR COMPUTATIONAL STUDIES OF FINITE PLATES IN A BAFFLE

The discretization scheme for our computational studies consists of Finite Differences to 4th order in space for the structure, constant Boundary Elements for the fluid and an explicit 'leapfrog' time-stepping scheme consisting of 2nd order central differences in time [12].

We consider a baffled plate with $T=\lambda=0$ with clamped boundary conditions at both edges and apply an impulse consisting of a half-sine wave with frequency $\omega = \omega_p$ at the midpoint of the plate. For the example of a plate with half length $L=250$ shown in FIGURE 4, we observe the following:

- Downstream travelling disturbances grow in amplitude as they propagate and are convected at roughly the flow speed U.
- Upstream travelling disturbances are slower and are highly dispersive.
- Both upstream and downstream boundaries reflect incident disturbances with increased amplitudes.
- For large time, the initial impulse becomes highly dispersed and the plate response resembles a standing wave with exponentially increasing amplitude.

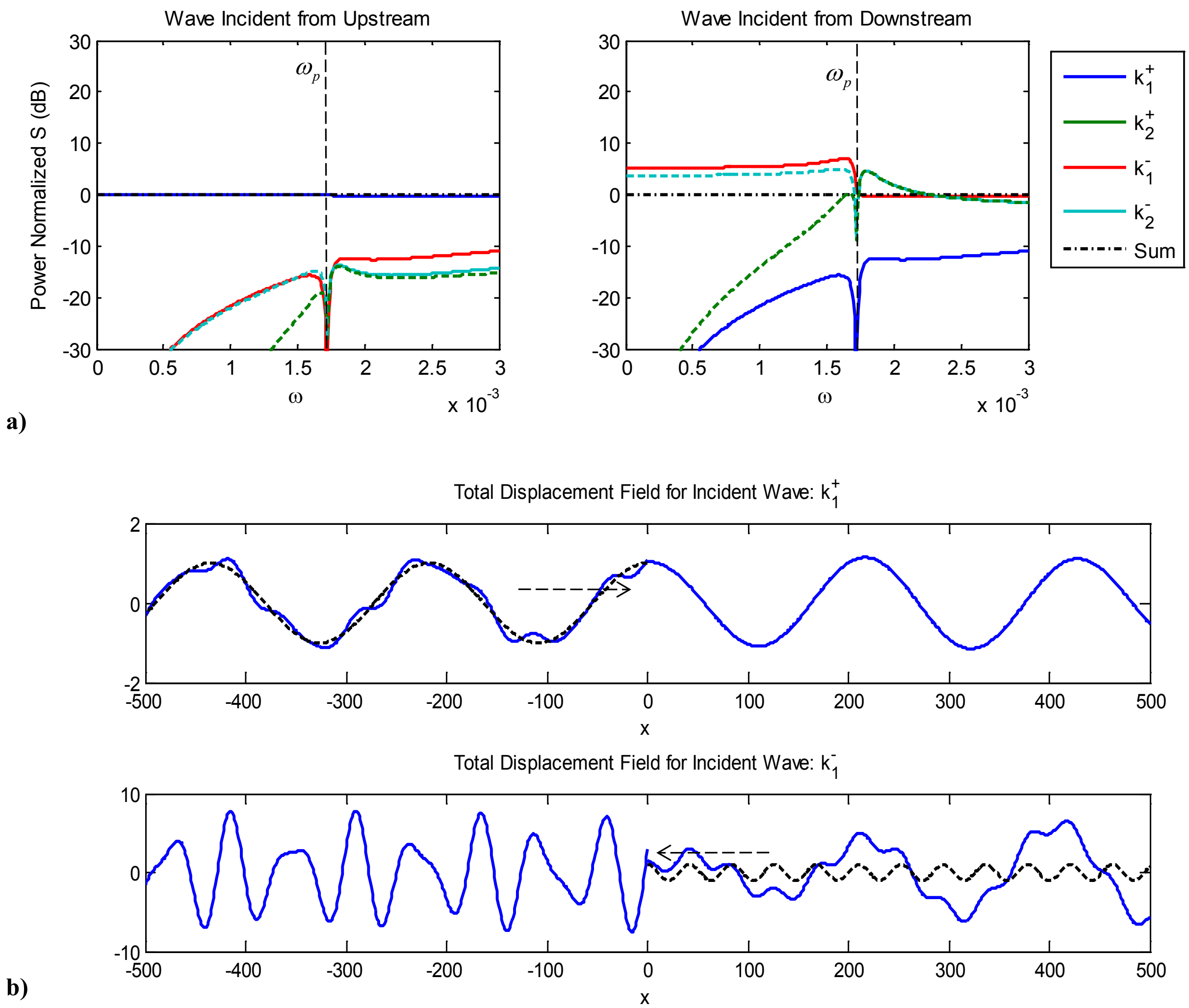

FIGURE 3: SCATTERING FROM A CLOSED CRACK ON INFINITE PLATE FOR *U=0.05, T=2U²*, *λ=0*. a) POWER NORMALIZED SCATTERING COEFFICIENTS, AND RESULTING b) SPATIAL RESPONSE FOR UNIT INCIDENT WAVES (DASHED LINE) WITH A TIME HARMONIC FREQUENCY OF ω=0.001 INCIDENT FROM THE UPSTREAM AND DOWNSTREAM.

The transition from transient to steady state behavior can be observed in the wavenumber spectrogram of FIGURE 5b. It consists of a plot of the magnitude of the spatial Fourier transform of the plate displacement at discrete time steps. The wavenumber spectrum at each time step has been normalized by its maximum value at that time step. The discrete spots in wavenumber at early time represent the scattering of the initial impulse at the plate boundaries. After about fifteen round-trips, the initial disturbances have sufficiently dispersed to allow for a more continuous, or steady state, wavenumber signature to emerge.

Immediately following the initial excitation, the disturbances travelling back and forth on the plate carry a range of wavenumbers and frequencies. However, each wavenumber-frequency pair falls on the dispersion curve for the equivalent infinite plate as shown in FIGURE 5a. The discrete data points in FIGURE 5a correspond to peaks in the spatial and temporal Fourier Transforms of the plate displacement field taken over the entire plate length and over successive time intervals corresponding to a single round-trip travel time of the pulse.

Each set of three circles of the same color represents the spectral content of the response over the same interval. For instance, the three data points in red correspond to the plate response taken over an interval of time corresponding to the first round trip performed by the impulse (i.e., for $0 < t < 1.6\times10^4$), the set of points in blue correspond to the second round trip, and so forth, culminating in the data points in magenta which correspond to an interval of response taken around the $t = 3\times10^5$ time mark.

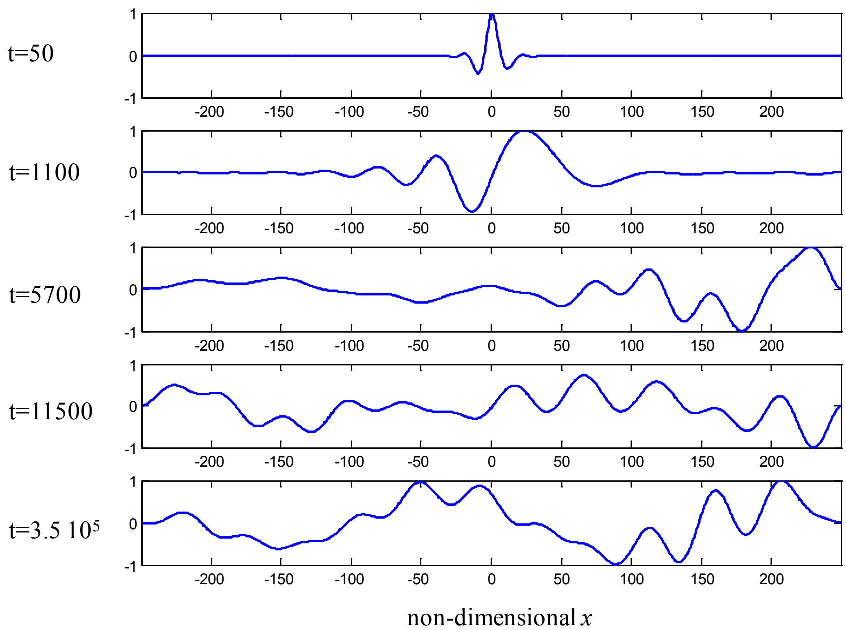

FIGURE 4: SNAPSHOTS OF NORMALIZED PLATE DISPLACEMENTS FOR *U=0.05, L=250, T=0, λ=0.*

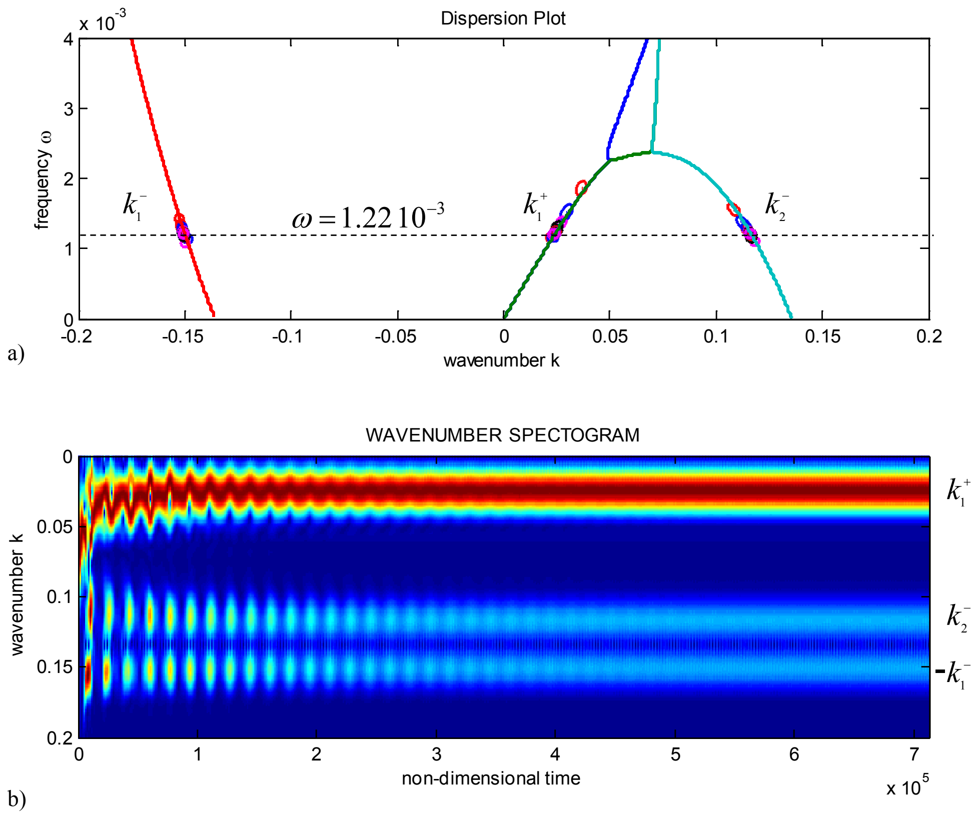

FIGURE 5: COMPARISON BETWEEN FINITE-PLATE COMPUTATIONAL RESULTS AND THE INFINITE PLATE THEORY FOR *U=0.05, T=λ=0* AND *L=250*. a) INFINITE PLATE DISPERSION PLOT OVERLAID WITH THE FREQUENCY-WAVENUMBER CONTENT OF THE FINITE PLATE RESPONSE AT DISCREET TIME INTERVALS. b) NORMALIZED WAVENUMBER SPECTROGRAM OF FINITE PLATE RESPONSE AS A FUNCTION OF TIME. DARKER HUES REPRESENT HIGHER SPECTRAL CONTENT.

Within three to four round trips, the linear response asymptotes to a single frequency and single temporal growth rate for all waves. Spatially, this 'steady state' resembles a standing wave pattern formed by the k_1^+, k_1^- and k_2^- waves. This complex resonance frequency is unrelated to the maximum growth frequency of the infinite plate and is a function of plate length. It corresponds exactly to the complex resonance frequency predicted by our infinite plate based analytics described in the next sub-section.

ANALYTICAL STUDY OF TEMPORALLY UNSTABLE RESONANCE IN LONG, BAFFLED PLATES

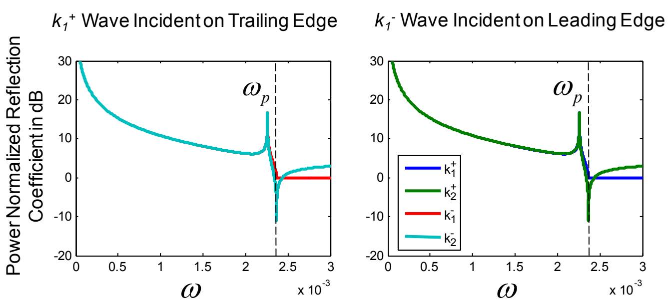

FIGURE 6: POWER NORMALIZED REFLECTION COEFFICIENTS FOR THE LEADING AND TRAILING EDGES OF A BAFFLED PLATE FOR *U=0.05, T=λ=0*.

Following the approach outlined in [2] and [4], we solve for the reflection matrix from the leading and trailing edges of a clamped-clamped plate in an infinite baffle using the Wiener-Hopf technique. We find that the plate over-reflects at both the leading and trailing edges for all frequencies, $\omega < \omega_p$, over which negative energy exists, irrespective of the presence of convectively unstable waves (FIGURE 6).

These reflection matrices, combined with the round trip propagation matrices of the plate, yield an equation for the linear response of a finite plate. The condition for resonance follows directly from the roots of the related eigenvalue equation. The approach is described in great detail in an earlier publication [2] by one of the co-authors and will not be repeated here.

We find that for a given plate length, there are a discrete set of complex frequencies that have positive imaginary parts and that result in temporally unstable resonance. These unstable resonances are limited to the frequency range over which negative energy waves exist, and over which the plate edges over-reflect.

For the particular plate length and configuration under consideration, there exist five resonance frequencies with positive imaginary parts as shown in the left hand table in FIGURE 7. The resonance frequency with the largest positive imaginary part is expected to dominate the response of a finite plate for large time. This is confirmed by our computational results as shown in the right hand table in FIGURE 7.

Analytical Predictions

	$Re(\omega_{res})$	$Im(\omega_{res})$
A	$2.40\ 10^{-3}$	$0.029\ 10^{-3}$
B	$2.02\ 10^{-3}$	$0.234\ 10^{-3}$
C	$1.61\ 10^{-3}$	$0.252\ 10^{-3}$
D	$1.22\ 10^{-3}$	$\mathbf{0.264\ 10^{-3}}$
E	$0.84\ 10^{-3}$	$0.245\ 10^{-3}$

Largest temporal growth rate

Computational Results

$Re(\omega_{res})$	$Im(\omega_{res})$
$1.22\ 10^{-3}$	$\mathbf{0.262\ 10^{-3}}$

FIGURE 7: COMPARISON BETWEEN ANALYTICAL PREDICTIONS AND COMPUTATIONAL RESULTS FOR MOST UNSTABLE RESONANCE FREQUENCY AND TEMPORAL GROWTH RATE FOR *U=0.05, T=λ=0* AND *L=250*.

CONCLUSIONS

The presence of negative energy waves (NEW) leads to over-scattering from discontinuities and edges and is responsible for unstable resonance growth in finite plates. The unstable behavior of long, finite plates can be captured by the use of infinite plate theory (propagation properties of waves), combined with scattering coefficients of edges obtained from semi-infinite theory.

Further work on the effects of nonlinear terms, and on a full range of edge conditions (including a free trailing edge with wake) is well underway.

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