Proceedings of the ASME 2010 3rd Joint US-European Fluids Engineering Summer Meeting and 8th International Conference on Nanochannels, Microchannels, and Minichannels
FEDSM-ICNMM2010
August 1-5, 2010, Montreal, Canada

FEDSM-ICNMM2010-31120

SLOSHING IN A VERTICAL CIRCULAR CYLINDRICAL CONTAINER WITH A VERTICAL BAFFLE

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ABSTRACT

The linear problem of liquid sloshing in a cylindrical container with a vertical baffle is considered in the present paper. In this study, a theoretical oriented approach is developed for calculating the natural frequencies of liquid. The baffle is a thin-walled and open-ended cylindrical shell that is concentrically placed and partially submerged inside the container. The free surface of liquid is assumed to be perpendicular to axis of the container and is divided into two parts by the baffle. The method also captures the singular asymptotic behavior of the velocity potential at the sharp baffle edge. The liquid is assumed to be incompressible and inviscid and the method uses matched eigenfunction expansions and Galerkin expansions to derive unknown coefficients presented in the velocity potential series. A finite element analysis is also used to check the validity of the proposed method. The effects of some important parameters of system are also considered on the sloshing frequencies.

INTRODUCTION

The resonant frequencies of oscillation of liquid in a bounded container have been the subject of a number of studies over many years. In the present paper, a special class of this one that involves the introduction of a baffle (internal body) inside a container is analytically investigated. The internal body is partially or completely submerged inside the tank. The convex domain of the fluid changes to a non-convex domain because of the internal body, whereas the fluid domain is simply connected and continuous. There are axial and radial distances between the internal body and the container.

Some researches have been carried out on this class of problems. Evans and McIver (1987) explored the effect of introducing a vertical baffle into a rectangular container of water on fluid frequencies. The technique involved matching the appropriate eigenfunction expansions on either side of the baffle and the solution of the resulting integral equation for the horizontal fluid above or below the baffle. Watson and Evans (1991) extended this technique for a number of similar problems. Gavrilyuk et al. (2006) proposed fundamental solutions of the linearized problem on fluid sloshing in a vertical cylindrical container having a thin rigid-ring horizontal baffle. A pressure-based finite element technique is developed to analyze the slosh dynamics of a partially filled rigid container with bottom-mounted submerged components by Mitra and Sinhamahapatra (2007). Maleki and Ziyaeifar (2008) investigated the potential of baffles (horizontal ring and vertical blade baffles) in increasing the hydrodynamic damping of sloshing in circular-cylindrical storage tanks.

In the present study, we extend the analytical technique proposed in reference (Evans and McIver, 1987) and presented

an efficient analytical method for considering this class of problems having an open-ended and thin vertical baffle partially submerged inside the cylindrical tank capturing the singular asymptotic behavior of the velocity potential at the sharp baffle edge.

In the present paper, the eigenfunction expansion and the Galerkin method are used to derive the characteristic equation of the liquid frequencies. The baffle is a rigid, thin-walled and open-ended cylindrical shell and is concentrically and partially submerged inside the rigid container. The bottom plate of the container is assumed to be flat and rigid. The fluid is assumed to be incompressible and inviscid. The velocity potential is formulated in terms of eigenfunction expansions appropriate to three distinct fluid regions which can be matched across their common vertical boundary. The resulting equations can be solved by using the Galerkin method. The validity of the proposed theoretical method is verified by comparing the results with those obtained by a finite element model. In order to evaluate the dynamic characteristics of the system, the effects of radius of baffle are also investigated.

MATHEMATICAL MODELLING

Consider a thin-walled, rigid and cylindrical container of length L, radius a as shown in Fig. 1. The bottom plate of the container is considered to be flat and rigid. The radial, circumferential and axial coordinates in original coordinate system are denoted by r, θ and x, respectively. The container is partially filled with an inviscid and incompressible fluid of mass density ρ_L, with a free surface orthogonal to the vertical container axis. The free surface is at distance H from the bottom of the container. Another cylindrical shell is placed inside the container as internal body (Fig. 1). It is a thin-walled and open-ended rigid cylindrical shell of length h and radius b and is partially immersed in the container.

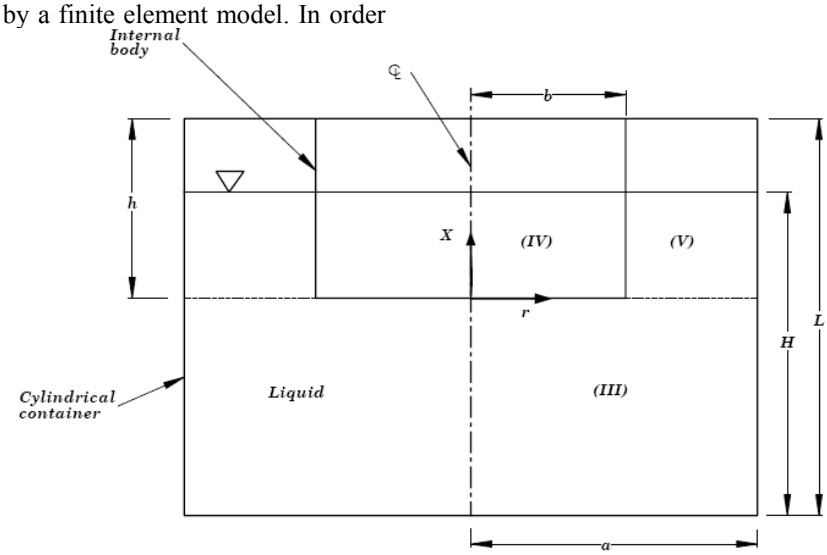

Fig. 1. The fluid domain of fluid divided into three parts (III), (IV) and (V).

Dynamic behaviour of the fluid-structure interaction

The inviscid, incompressible and irrotational fluid permits the introduction of velocity potential for the fluid motion. Assuming simple harmonic motion of radian frequency ω, the velocity potential can be expressed as

$$\tilde{\varphi}(r,\theta,x,t) = i\omega\,\varphi_{(s)}(r,\theta,x)\,e^{i\omega t}, \qquad i^2 = -1. \tag{1}$$

The radial, circumferential and axial coordinates are denoted by r, θ and X, respectively as shown in Fig. 1. This coordinate system can be changed to original coordinate system using $X = x - L + h$. In order to compute the time-independent velocity potential, $\varphi_{(s)}$, the fluid domain can be divided into three parts (*III*, *IV*, *V*) as shown in Fig. 1, (Evans and McIver, 1987; Askari and Daneshmand, 2009)

$$\begin{aligned} \mathrm{III} &= \{(r,\theta,x):\ 0 \le x \le L-h,\ \ r < a\}, \\ \mathrm{IV} &= \{(r,\theta,x):\ L-h \le x \le H,\ \ r < b\}, \\ V &= \{(r,\theta,x):\ L-h \le x \le H,\ \ b < r < a\} \end{aligned} \tag{2}$$

In order to find $\varphi_{(s)}^{III}$, we restrict our attention to region (*III*) satisfying conditions (3) to (5) in that region,

$$\nabla^2 \varphi_{(s)}^{III} = 0, \tag{3}$$

$$\frac{\partial \varphi_{(s)}^{III}}{\partial x} = 0, \quad r < a, \quad x = 0 \tag{4}$$

$$\frac{\partial \varphi_{(s)}^{III}}{\partial r} = 0, \quad r = a, \quad 0 \le x \le L-h \tag{5}$$

It will be convenient to define

$$\zeta : \{x = L-h,\ 0 < r < b\} \quad , \quad \zeta' : \{x = L-h,\ b < r < a\} \tag{6}$$

where ζ is boundary contact between the two fluid regions (*III*) and (*IV*), and ζ' is boundary contact between the two fluid regions (*IV*) and (*V*).

In the fluid region (*III*), for asymmetric modes (n>0), the method of separation of variables gives

$$\varphi_{(s)}^{III} = \cos(n\theta)\sum_{s=1}^{\infty} A_{ns} \mathrm{J}_n(\frac{\xi_{ns} r}{a}), \left[\cosh(\frac{\xi_{ns}(x-h_1)}{a}) + \tanh(\frac{\xi_{ns} h_1}{a})\sinh(\frac{\xi_{ns}(x-h_1)}{a})\right] \quad (7)$$

$$h_1 = L - h$$

and for axisymmetric modes (n=0), Eq. (7) is replaced by

$$\varphi_{(s)}^{III} = A_{00} + \sum_{s=1}^{\infty} A_{0s} \mathrm{J}_0(\frac{\xi_{0s} r}{a}) \left[\cosh(\frac{\xi_{0s}(x-h_1)}{a}) + \tanh(\frac{\xi_{0s} h_1}{a})\sinh(\frac{\xi_{0s}(x-h_1)}{a})\right] \quad (8)$$

where functions $\varphi_{(s)}^{III}$ satisfy conditions (4) and (5), ξ_{ns} are given by

$$J_n'(\xi_{ns}) = 0, \qquad s = 1,\ldots\infty \quad (9)$$

and the modified Bessel function of second order Y_n is discarded in order to avoid infinite velocities at r=0.

Now, we can restrict our attention to region (*IV*), and seek function $\varphi^{IV}(r,\theta,x)$, satisfying conditions (10) to (12) in that region.

$$\nabla^2 \varphi_{(s)}^{IV} = 0, \quad (10)$$

$$\frac{\partial \varphi_{(s)}^{IV}}{\partial r} = 0, \quad r = b, \quad L - h < x < H \quad (11)$$

on the free surface $x = H$, the sloshing condition is

$$\frac{\partial}{\partial x}\left(\varphi_{(s)}^{IV}\right) = \frac{\omega^2}{g}\varphi_{(s)}^{IV}, \quad r < b, x = H \quad (12)$$

In the fluid region (*IV*), for asymmetric modes (n>0), the method of separation of variables gives

$$\varphi_{(s)}^{IV} = \cos(n\theta)\sum_{s=1}^{\infty} \mathrm{J}_n(\frac{\xi_{ns} r}{b}) \left[B_{ns}\cosh(\frac{\xi_{ns}(x-h_1)}{b}) + C_{ns}\sinh(\frac{\xi_{ns}(x-h_1)}{b})\right] \quad (13)$$

and for axisymmetric modes (n=0), Eq. (13) is replaced by

$$\varphi_{(s)}^{IV} = \sum_{s=1}^{\infty} \mathrm{J}_0(\frac{\xi_{0s} r}{b}) \left[B_{0s}\cosh(\frac{\xi_{0s}(x-h_1)}{b}) + C_{0s}\sinh(\frac{\xi_{0s}(x-h_1)}{b})\right] + B_{00}(x-h_1) + C_{00} \quad (14)$$

Now, we can restrict our attention to region (*V*), and seek function $\varphi_{(s)}^{V}(r,\theta,x)$, satisfying conditions (15) to (17) in that region.

$$\nabla^2 \varphi_{(s)}^{V} = 0, \quad (15)$$

$$\frac{\partial \varphi_{(s)}^{V}}{\partial r} = 0, \quad r = a, \quad L - h < x < H \quad (16)$$

$$\frac{\partial \varphi_{(s)}^{V}}{\partial r} = 0, \quad r = b, \quad L - h < x < H \quad (17)$$

on the free surface $x = H$, the sloshing condition is

$$\frac{\partial}{\partial x}\left(\varphi_{(s)}^{V}\right) = \frac{\omega^2}{g}\varphi_{(s)}^{V}, \quad b < r < a, \quad x = H \quad (18)$$

In the fluid region (*V*), for asymmetric modes (n>0), the method of separation of variables gives

$$\varphi_{(s)}^{V} = \cos(n\theta)(\sum_{s=1}^{\infty}\left[D_{ns}\sinh(\eta_{ns}(x-h_1))\right] + \sum_{s=1}^{\infty}[E_{ns}\cosh(\eta_{ns}(x-h_1))])\left[\mathrm{J}_n(\eta_{ns} r) - \frac{\mathrm{J}_n'(\eta_{ns} a)}{\mathrm{Y}_n'(\eta_{ns} a)}\mathrm{Y}_n(\eta_{ns} r)\right] \quad (19)$$

and for axisymmetric modes (n=0), Eq. (19) is replaced by

$$\varphi_{(s)}^{V} = \sum_{s=1}^{\infty}\left[D_{0s}\sinh(\eta_{0s}(x-h_1)) + E_{ns}\cosh(\eta_{0s}(x-h_1))\right] \left[\mathrm{J}_0(\eta_{0s} r) - \frac{\mathrm{J}_0'(\eta_{0s} a)}{\mathrm{Y}_0'(\eta_{0s} a)}\mathrm{Y}_0(\eta_{0s} r)\right] + D_{00}(x-h_1) + E_{00} \quad (20)$$

where η_{ns} is given by

$$\mathrm{J}_n'(\eta_{ns} b) - \frac{\mathrm{J}_n'(\eta_{ns} a)}{\mathrm{Y}_n'(\eta_{ns} a)}\mathrm{Y}_n'(\eta_{ns} b) = 0, \quad s = 1,\ldots\infty \quad (21)$$

and Y_n is the modified Bessel function of the second kind of order *n,* and also J_n' and Y_n' indicate the derivatives of J_n and Y_n with respect to *r*, respectively. A_{ns}, B_{ns}, C_{ns}, D_{ns}, E_{ns}, $A_{00}, B_{00}, C_{00}, D_{00}$ and E_{00} are unknown coefficients depending on the integers *n* and *s* and have to be determined by the matching process. It is necessary to ensure that the potential and velocity of fluid are continuous along boundaries ζ, ζ', and the sloshing conditions (44) and (50) are satisfied on the free surface. These conditions are presented as follow

$$\frac{\partial \varphi_{(s)}^{III}}{\partial x} = \begin{cases} \dfrac{\partial \varphi_{(s)}^{IV}}{\partial x} & on\ \zeta \\ \dfrac{\partial \varphi_{(s)}^{V}}{\partial x} & on\ \zeta' \end{cases} \quad (22)$$

$$\varphi_{(s)}^{IV} = \varphi_{(s)}^{III} \qquad on\ \zeta \quad (23)$$

$$\varphi_{(s)}^{V} = \varphi_{(s)}^{III} \qquad on\ \zeta' \quad (24)$$

$$\frac{\partial}{\partial x}\left(\varphi_{(s)}^{IV}\right) = \frac{\omega^2}{g}\varphi_{(s)}^{IV}, \quad r < b, \quad x = H \quad (25)$$

$$\frac{\partial}{\partial x}\left(\varphi_{(s)}^{V}\right) = \frac{\omega^2}{g}\varphi_{(s)}^{V}, \quad b < r < a, \quad x = H \quad (26)$$

where Eq. (22) indicates that the axial velocities of the fluid are continuous along ζ and ζ', Eqs. (23) and (24) are the continuity condition for the velocity potentials along boundaries ζ and ζ'. Eqs. (25) and (26) are the sloshing conditions on the free surface.

Eq. (22) must be satisfied for all values $r \le a$ (on the boundary, $\zeta + \zeta'$). Multiplying this equation by $J_n(\frac{\xi_{nj} r}{a})\frac{r}{a^2}$ and integrating between 0 and *a*, one obtains

$$A_1\mathbf{A} = A_3\mathbf{C} + A_5\mathbf{E}$$

$$A_1(j,s) = \frac{\xi_{ns}}{a}\tanh(\frac{\xi_{ns}h_1}{a})\chi_{nj}\delta_{js},$$

$$A_3(j,s) = \frac{\xi_{ns}}{b}\int_0^b J_n(\frac{\xi_{ns}r}{b})J_n(\frac{\xi_{nj}r}{a})\frac{r}{a^2}\,\mathrm{d}r,$$

$$A_5(j,s) = \eta_{ns}\int_b^a\left(J_n(\eta_{ns}r) - \frac{J_n'(\eta_{ns}a)}{Y_n'(\eta_{ns}a)}Y_n(\eta_{ns}r)\right)J_n(\frac{\xi_{nj}r}{a})\frac{r}{a^2}\,\mathrm{d}r, \tag{27}$$

and for axisymmetric modes (*n*=0), the following equation must also be added:

$$B_{00} = -D_{00} - \mu_1\mathbf{E}$$

$$\mu_1(s) = 2\eta_{ns}\int_b^a\left(J_n(\eta_{ns}r) - \frac{J_n'(\eta_{ns}a)}{Y_n'(\eta_{ns}a)}Y_n(\eta_{ns}r)\right)\frac{r}{a^2}\,\mathrm{d}r \tag{28}$$

where

$$\chi_{nj} = \int_0^a J_n^2(\frac{\xi_{nj}r}{a})\frac{r}{a^2}\,\mathrm{d}r = \int_0^b J_n^2(\frac{\xi_{nj}r}{b})\frac{r}{b^2}\,\mathrm{d}r \tag{29}$$

Eq. (23) must be satisfied for all values $r \le b$ (on the boundary ζ). If one multiplies this equation by $J_n(\frac{\xi_{nj}r}{b})\frac{r}{b^2}$ and then integrates between *0* and *b*, one obtains

$$D_1\mathbf{B} = D_3\mathbf{A}$$

$$D_1(j,s) = \chi_{nj}\delta_{js},$$

$$D_3(j,s) = \int_0^b J_n(\frac{\xi_{ns}r}{a})J_n(\frac{\xi_{nj}r}{b})\frac{r}{b^2}\,\mathrm{d}r \tag{30}$$

and for axisymmetric modes (*n*=0), the following equation must also be added:

$$C_{00} = A_{00} + \mu_2\mathbf{A}$$

$$\mu_2(s) = 2\int_0^b J_0(\frac{\xi_{0s}r}{a})\frac{r}{b^2}\,\mathrm{d}r \tag{31}$$

Eq. (24) must be satisfied for all values $b < r \le a$ (on the free surface in region *V*). If one multiplies this equation by $\alpha_{nj} = \left(\mathrm{J}_n(\eta_{nj}r) - \frac{\mathrm{J}_n'(\eta_{nj}a)}{\mathrm{Y}_n'(\eta_{nj}a)}\mathrm{Y}_n(\eta_{nj}r)\right)$ and then integrates between *b* and *a*, one obtains

$$E_1\mathbf{D} = E_3\mathbf{A}$$

$$E_1(j,s) = \int_b^a \alpha_{ns}\alpha_{nj}\,\mathrm{d}r, \quad E_3(j,s) = \int_b^a J_n(\frac{\xi_{ns}r}{a})\alpha_{nj}\,\mathrm{d}r \tag{32}$$

and for axisymmetric modes (*n*=0), the following equation must also be added:

$$E_{00} = A_{00} + \mu_3\mathbf{A} - \mu_4\mathbf{D},$$

$$\mu_3(s) = \frac{2}{a^2 - b^2}\int_b^a J_0(\frac{\xi_{0s}r}{a})r\,\mathrm{d}r, \quad \mu_4(s) = \int_b^a \alpha_{0s}r\,\mathrm{d}r \tag{33}$$

Eq. (25) must be satisfied for all values $r \le b$ (on the free surface in region *IV*). Multiplying this equation by $\mathrm{J}_n(\frac{\xi_{nj}r}{b})\frac{r}{b^2}$ and integrating between 0 and *b*, one obtains

$$B_2\mathbf{B} + B_3\mathbf{C} = \frac{\omega^2}{g}(B_4\mathbf{B} + B_5\mathbf{C}),$$

$$B_2(j,s) = \frac{\xi_{ns}}{b}\sinh(\frac{\xi_{ns}}{b}(H - h_1))\chi_{nj}\delta_{js}$$

$$B_3(j,s) = \frac{\xi_{ns}}{b}\cosh(\frac{\xi_{ns}}{b}(H - h_1))\chi_{nj}\delta_{js} \tag{34}$$

$$B_4(j,s) = \cosh(\frac{\xi_{ns}}{b}(H - h_1))\chi_{nj}\delta_{js}$$

$$B_2(j,s) = \sinh(\frac{\xi_{ns}}{b}(H - h_1))\chi_{nj}\delta_{js}$$

and for axisymmetric modes (*n*=0), the following equation must also be added:

$$B_{00} = \frac{\omega^2}{g}\left(C_{00} + B_{00}(H - h_1)\right); \tag{35}$$

Eq. (26) must be satisfied for all values $b < r \le a$ (on the free surface in region *V*). Multiplying this equation by $\mathrm{J}_n(\eta_{nj}r)r$ and integrating between b and *a*, one obtains

$$C_2\mathbf{D} + C_3\mathbf{E} = \frac{\omega^2}{g}(C_4\mathbf{D} + C_5\mathbf{E}),$$

$$C_2(j,s) = \eta_{ns}\sinh(\eta_{ns}(H - h_1))\int_b^a \alpha_{ns}\mathrm{J}_n(\eta_{nj}r)r\,\mathrm{d}r,$$

$$C_3(j,s) = \eta_{ns}\cosh(\eta_{ns}(H - h_1))\int_b^a \alpha_{ns}\mathrm{J}_n(\eta_{nj}r)r\,\mathrm{d}r, \tag{36}$$

$$C_4(j,s) = \cosh(\eta_{ns}(H - h_1))\int_b^a \alpha_{ns}\mathrm{J}_n(\eta_{nj}r)r\,\mathrm{d}r,$$

$$C_5(j,s) = \sinh(\eta_{ns}(H - h_1))\int_b^a \alpha_{ns}\mathrm{J}_n(\eta_{jn}r)r\,\mathrm{d}r,$$

and for axisymmetric modes (*n*=0), the following equation must also be added:

$$\mu_8\mathbf{D} + \mu_9\mathbf{E} + D_{00}(a^2 - b^2)/2 =$$

$$\frac{\omega^2}{g}\left\{(H - h_1)\left(\frac{a^2 - b^2}{2}\right)D_{00} + E_{00}\left(\frac{a^2 - b^2}{2}\right) + \mu_{10}\mathbf{D} + \mu_{11}\mathbf{E}\right\}$$

$$\mu_8(s) = \eta_{ns}\sinh(\eta_{ns}(H - h_1))\int_b^a \alpha_{ns}r\,\mathrm{d}r,$$

$$\mu_9(s) = \eta_{ns}\cosh(\eta_{ns}(H - h_1))\int_b^a \alpha_{ns}r\,\mathrm{d}r, \tag{37}$$

$$\mu_{10}(s) = \cosh(\eta_{ns}(H - h_1))\int_b^a \alpha_{ns}r\,\mathrm{d}r,$$

$$\mu_{11}(s) = \sinh(\eta_{ns}(H - h_1))\int_b^a \alpha_{ns}r\,\mathrm{d}r,$$

Eq. (34) by using Eq. (30) is rewritten with respect to variables ($\mathbf{A}$, $\mathbf{C}$, A_{00}, D_{00}) as follows

$$B_2 D_1^{-1} D_3 \mathbf{A} + B_3 \mathbf{C} = \frac{\omega^2}{g}\left(B_4 D_1^{-1} D_3 \mathbf{A} + B_5 \mathbf{C}\right) \tag{38}$$

and for axisymmetric modes (n=0), the following equation must also be added:

$$\begin{aligned} &-D_{00} - \mu_1 A_5^{-1} A_1 \mathbf{A} + \mu_1 A_5^{-1} A_3 \mathbf{C} = \\ &\frac{\omega^2}{g}\Big[A_{00} + \left(\mu_2 - (H-h_1)\mu_1 A_5^{-1} A_1\right)\mathbf{A} - D_{00}(H-h_1) \\ &\quad + (H-h_1)\mu_1 A_5^{-1} A_3 \mathbf{C}\Big] \end{aligned} \tag{39}$$

Eq. (36) by using Eqs. (27) and (32) is rewritten with respect variables ($\mathbf{A}$, $\mathbf{C}$, A_{00}, D_{00}) as follows

$$\begin{aligned} &\left(C_2 E_1^{-1} E_3 + C_3 A_5^{-1} A_1\right)\mathbf{A} - C_3 A_5^{-1} A_3 \mathbf{C} = \\ &\frac{\omega^2}{g}\left[\left(C_4 E_1^{-1} E_3 + C_5 A_5^{-1} A_1\right)\mathbf{A} - C_5 A_5^{-1} A_3 \mathbf{C}\right] \end{aligned} \tag{40}$$

and for axisymmetric modes (n=0), the following equation must also be added:

$$\begin{aligned} &\left(\mu_8 E_1^{-1} E_3 + \mu_9 A_5^{-1} A_1\right)\mathbf{A} - \mu_9 A_5^{-1} A_3 \mathbf{C} + D_{00}(a^2 - b^2)/2 = \\ &\frac{\omega^2}{g}\Bigg\{(H-h_1)\left(\frac{a^2-b^2}{2}\right)D_{00} + A_{00}\left(\frac{a^2-b^2}{2}\right) + \\ &\left[\left(\frac{a^2-b^2}{2}\right)\left(\mu_3 - \mu_4 E_1^{-1} E_3\right) + \mu_{10} E_1^{-1} E_3 + \mu_{11} A_5^{-1} A_1\right]\mathbf{A} - \mu_{11} A_5^{-1} A_3 \mathbf{C}\Bigg\} \end{aligned} \tag{41}$$

The eigenvalue problem

For the numerical calculation of the sloshing natural frequencies, $\widetilde{N}$ terms in the expansion of $\varphi_{(s)}$ ($\widetilde{N}+1$ in the case of axisymmetric modes) is considered, where $\widetilde{N}$ is chosen large enough to give the required accuracy. So, all the above relations are given by finite summations. In above section, all of fluid equations associated with the sloshing of the fluid were rewritten with respect to variables ($\mathbf{A}, \mathbf{C}, A_{00}, D_{00}$).It is convenient to introduce a vectorial notation. The vector $\mathbf{q}$ of the parameters of the Ritz expansion is defined by

$$\mathbf{q} = \begin{Bmatrix} \mathbf{A} \\ \mathbf{C} \end{Bmatrix}, \quad \mathbf{A} = \begin{Bmatrix} A_{n1} \\ \vdots \\ A_{n\widetilde{N}} \end{Bmatrix}, \quad \mathbf{C} = \begin{Bmatrix} C_{n1} \\ \vdots \\ C_{n\widetilde{N}} \end{Bmatrix} \tag{42}$$

where A is the vector of unknown coefficients addressed in Eq. (7) and C is the vector of unknown coefficients addressed in Eq. (13). For axisymmetric (n=0) modes, the coefficient A_{00} and D_{00} must be included in the vectors $\mathbf{q}$. The sloshing condition in region (IV), Eq. (38), is given by

$$\begin{aligned} &[[\mathbf{0}], [\mathbf{0}], [\mathbf{G}_1], [\mathbf{G}_2], [\mathbf{G}_3]]\mathbf{q} = \omega^2 [[\mathbf{0}], [\mathbf{0}], [\mathbf{0}], [\mathbf{G}_4], [\mathbf{G}_5]]\mathbf{q}, \\ &\mathbf{G}_1 = g\mathbf{B_1}, \quad \mathbf{G}_2 = g\left(\mathbf{B_2 D_1^{-1} D_3}\right), \quad \mathbf{G}_3 = g\mathbf{B_3}, \\ &\mathbf{G}_4 = \mathbf{B_4 D_1^{-1} D_3}, \quad \mathbf{G}_5 = \mathbf{B_5}. \end{aligned} \tag{43}$$

The sloshing condition in region (V), Eq. (40), is given by

$$\begin{aligned} &[[\mathbf{0}], [\mathbf{0}], [\mathbf{F}_1], [\mathbf{F}_2], [\mathbf{F}_3]]\mathbf{q} = \omega^2 [[\mathbf{0}], [\mathbf{0}], [\mathbf{0}], [\mathbf{F}_4], [\mathbf{F}_5]]\mathbf{q}, \\ &\mathbf{F}_1 = g\mathbf{C}_1, \quad \mathbf{F}_2 = g\left(\mathbf{C_2 E_1^{-1} E_3} + \mathbf{C_3 A_5^{-1} A_1}\right), \\ &\mathbf{F}_3 = -g\mathbf{C_3 A_5^{-1} A_3}, \\ &\mathbf{F}_4 = \left(\mathbf{C_4 E_1^{-1} E_3} + \mathbf{C_5 A_5^{-1} A_1}\right), \\ &\mathbf{F}_5 = -\mathbf{C_5 A_5^{-1} A_3}. \end{aligned} \tag{44}$$

where all of submatrices have dimension $\widetilde{N} \times \widetilde{N}$.

For axisymmetric modes, the dimension $\widetilde{N}$ of all the matrices must be changed into $\widetilde{N}+1$. The additional row of the matrices in the Eqs. (39) and (41) for n=0 and in regions (IV) and (V), respectively are as follow

$$\begin{aligned} &[\{\mathbf{0}\}, \{\mathbf{0}\}, \{\mathbf{Q}_1\}, \{\mathbf{Q}_2\}, \{\mathbf{Q}_3\}, \{Q_4\}, \{0\}]\begin{Bmatrix} \mathbf{q} \\ D_{00} \\ A_{00} \end{Bmatrix} = \\ &\omega^2 [\{\mathbf{0}\}, \{\mathbf{0}\}, \{\mathbf{0}\}, \{\mathbf{Q}_5\}, \{\mathbf{Q}_6\}, \{0\}, \{Q_7\}]\begin{Bmatrix} \mathbf{q} \\ D_{00} \\ A_{00} \end{Bmatrix}, \\ &\mathbf{Q}_1 = g\mu_5, \quad \mathbf{Q}_2 = -g\left(\boldsymbol{\mu}_1 \mathbf{A_5^{-1} A_1}\right), \quad \mathbf{Q}_3 = g\boldsymbol{\mu}_1 \mathbf{A_5^{-1} A_3}, \\ &\mathbf{Q}_4 = -g, \quad \mathbf{Q}_5 = \left(\mu_2 - (H-h_1)\boldsymbol{\mu}_1 \mathbf{A_5^{-1} A_1}\right), \\ &\mathbf{Q}_6 = (H-h_1)\boldsymbol{\mu}_1 \mathbf{A_5^{-1} A_3}, \quad Q_7 = 1. \end{aligned} \tag{45}$$

$$\begin{aligned} &[\{\mathbf{0}\}, \{\mathbf{0}\}, \{\mathbf{Z}_1\}, \{\mathbf{Z}_2\}, \{\mathbf{Z}_3\}, \{Z_4\}, \{0\}]\begin{Bmatrix} \mathbf{q} \\ D_{00} \\ A_{00} \end{Bmatrix} = \\ &\omega^2 [\{\mathbf{0}\}, \{\mathbf{0}\}, \{\mathbf{0}\}, \{\mathbf{Z}_5\}, \{\mathbf{Z}_6\}, \{Z_7\}, \{Z_8\}]\begin{Bmatrix} \mathbf{q} \\ D_{00} \\ A_{00} \end{Bmatrix}, \\ &\mathbf{Z}_1 = g(\boldsymbol{\mu}_6 + \boldsymbol{\mu}_7), \quad \mathbf{Z}_2 = g\left(\boldsymbol{\mu}_8 \mathbf{E_1^{-1} E_3} + \boldsymbol{\mu}_9 \mathbf{A_5^{-1} A_1}\right), \\ &\mathbf{Z}_3 = -g\boldsymbol{\mu}_9 \mathbf{A_5^{-1} A_3}, \quad \mathbf{Z}_4 = g(a^2 - b^2)/2, \\ &\mathbf{Z}_5 = \left[\left(\frac{a^2-b^2}{2}\right)\left(\boldsymbol{\mu}_3 - \boldsymbol{\mu}_4 \mathbf{E_1^{-1} E_3}\right) + \boldsymbol{\mu}_{10} \mathbf{E_1^{-1} E_3} + \boldsymbol{\mu}_{11} \mathbf{A_5^{-1} A_1}\right], \\ &\mathbf{Z}_6 = -\boldsymbol{\mu}_{11} \mathbf{A_5^{-1} A_3}, \\ &\mathbf{Z}_7 = (H-h_1)\left(\frac{a^2-b^2}{2}\right), \quad Z_8 = \left(\frac{a^2-b^2}{2}\right) \end{aligned} \tag{46}$$

where ω is the circular frequency of the liquid sloshing.

RESULTS AND DISCUSSION

Based on the preceding analysis, the eigenvalue problems, Eq. (43) and (44), are solved at the same time to find the sloshing natural frequencies. A finite element analysis is also used to check the validity of the present method for the partially water-filled cylindrical container with internal body. Fluid element is defined by four nodes having three degrees of

freedom at each node: three translations in each direction (Zienkiewicz, 1977).

The velocities of the fluid nodes along the wet surface of the container are zero. The radial velocities of the fluid nodes along the wet surface of the internal body are zero. In calculation of natural frequencies of the system, the system of equations is condensed down to those interest DOFs by Guyan reduction (Guyan, 1965).

The following material properties are used: the fluid is water with mass density ρ_L=1000 kg/m^3. This container is partially filled to H=0.4 m and its dimensions are a=0.2 m, b=0.5a, L=3a, h=0.5L.

To check the convergence of the present method, a partially water-filled container with a cylindrical shell as a baffle is analyzed. Table 1 shows the convergence of present method with number of nodal diameters n=1 and 2 for different numbers of term $\widetilde{N}$ used in the series expansions. From the frequencies presented in this table, we conclude that 8 terms in the series expansions are adequate for convergence.

To validate the present theoretical method, the natural frequencies (Hz) of the sloshing modes for the partially water-filled rigid container are compared with those obtained from the finite element analysis, in Table 2. It is seen that the present results are very close to those obtained from finite element analysis.

Convergence and Validation study

Table 1.

Convergence study of natural frequencies (Hz) of the first three sloshing

Number of terms	n=2			n=1		
	1st mode	2nd mode	3rd mode	1st mode	2nd mode	3rd mode
2	1.8424	2.7582	2.9632	1.3888	2.1645	2.8580
4	1.8409	2.7568	2.9626	1.3848	2.1587	2.8566
6	1.8404	2.7566	2.9626	1.3835	2.1574	2.8565
7	1.8404	2.7565	2.9626	1.3836	2.1571	2.8564
8	1.8404	2.7565	2.9626	1.3834	2.1570	2.8564

Table 2.

Sloshing frequencies (Hz) of a partially fluid-filled cylindrical container with internal body

Mode No.	b=0.1		b=0.14	
m=1/n	A	B	A	B
1	1.3852	1.3368	1.3804	1.3731
2	1.8409	1.8303	1.7606	1.7547
3	2.2201	2.2214	2.1097	2.1125
4	2.5362	2.5381	2.4226	2.4255
5	2.8064	2.8075	2.7026	2.7041
6	3.0449	3.0533	2.9557	2.9564

A: flexible container; B: rigid container

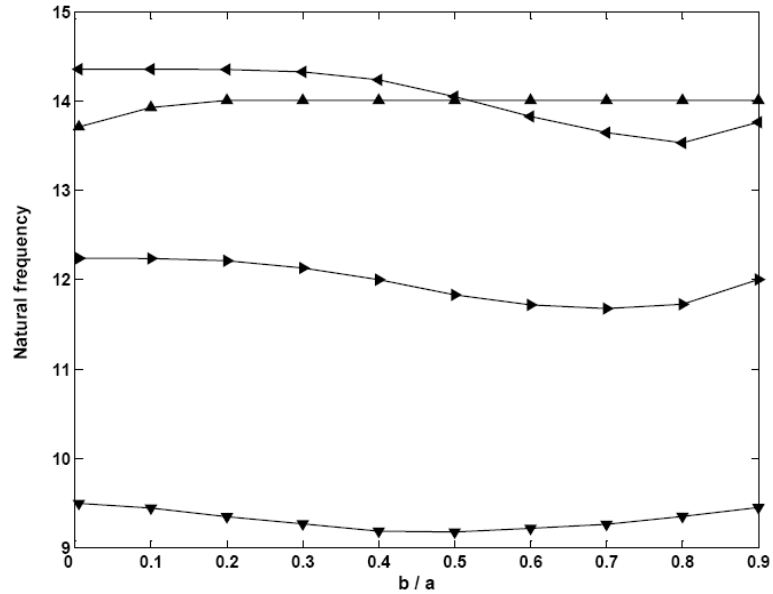

Fig. 2. Effect of radius ratio (*b*/*a*) on the first natural frequencies (rad/s) of the fluid for various numbers of nodal diameters (*h*=0.25 m). (▲, *n*=0; ▼, *n*=1; ▶, *n*=2; ◀, *n*=3)

Effect of internal body radius on sloshing frequencies

Effect of the internal body radius on sloshing frequencies (rad/s) for various numbers of nodal diameters are considered and shown in Figs. 2, 3 and 4. It can be observed from these figures that the variation trend for nodal diameters *n*=1, 2 and 3 are approximately similar but for nodal diameter *n*=0, it is different from others. The internal body radius has a small effect on the first natural frequency as seen in Fig. 2 when compared with the second and third natural frequencies shown in Figs. 3 and 4, respectively. Comparing with the natural frequency for the first radius ratio, the maximum increase about 30% and the maximum decrease about 25% can be observed in Figs. 3 and 4. As shown in Fig. 3, the local maximum points are occurred in radius ratios 0.2, 0.3, 0.4 and 0.5 for the nodal diameters *n*=0, 1, 2 and 3, respectively. The local region around the radius ration 0.5 in Fig. 4 should also be noted where it seems to be a local minimum for the third natural frequency.

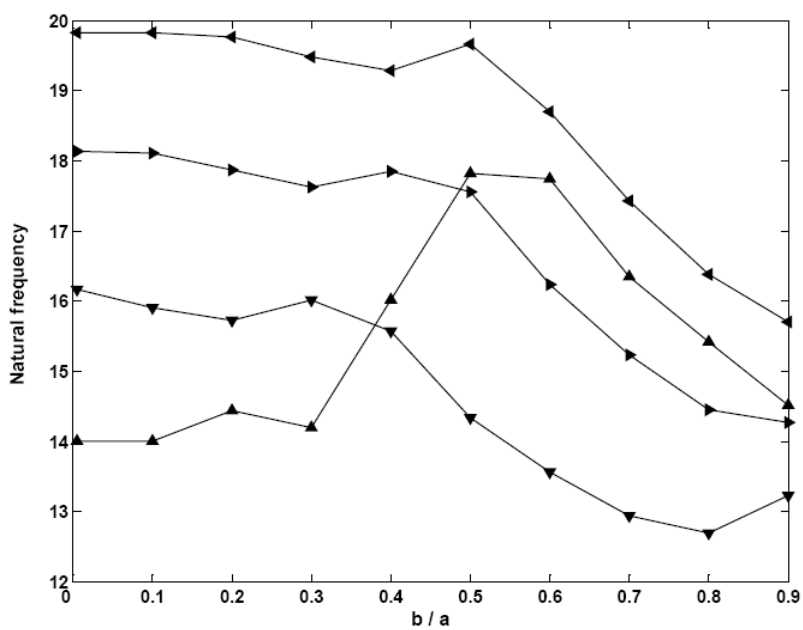

Fig. 3. Effect of radius ratio (*b*/*a*) on the second natural frequencies (rad/s) of the fluid for various numbers of nodal diameters (*h*=0.25 m). (▲, *n*=0; ▼, *n*=1; ▶, *n*=2; ◀, *n*=3).

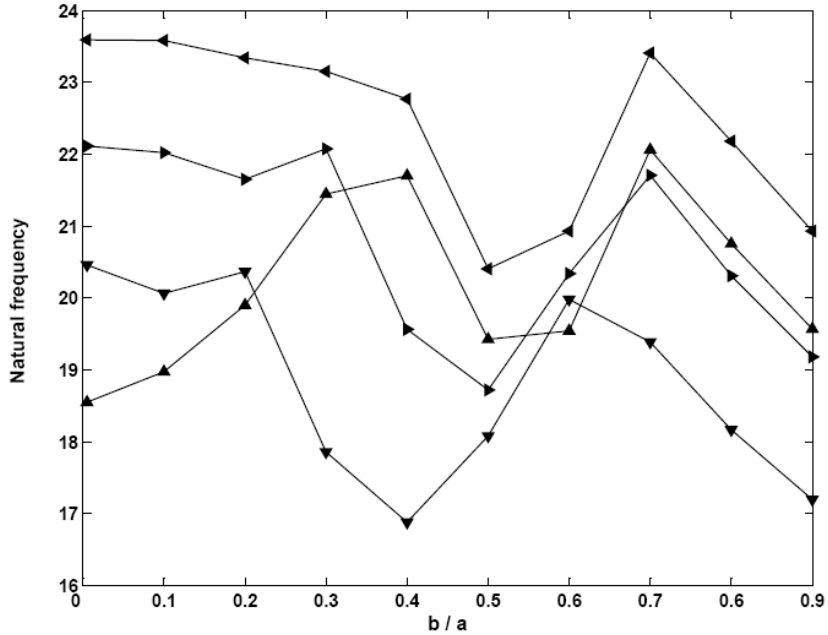

Fig. 4. Effect of radius ratio (*b*/*a*) on the third natural frequencies (rad/s) of the fluid for various numbers of nodal diameters (*h*=0.25 m). (—▲—, *n*=0; —▼—, *n*=1; —▶—, *n*=2; —◀—, *n*=3)

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CONCLUSION

Pursuing an analytically oriented method to consider sloshing phenomena happened in a partially liquid-filled cylindrical container with a cylindrical internal body, we developed an efficient approach that captures the analytical features of the velocity potential in a non-convex, continuous, and simply connected fluid domain. It was verified that this theoretical approach can predict the dynamic characteristics of sloshing liquid inside a cylindrical container having a baffle excellently. Another important advantage of this method is the possibility of testing and analyzing different sizes of internal bodies within the container making the necessary physical and engineering conclusions. Moreover, effects of the baffle's radius on the sloshing frequencies of the system were also considered. It was found that the increasing of baffle's radius changed the sloshing frequencies. Effects of the baffle's radius on the liquid natural frequencies in the partially fluid-filled container vary with numbers of nodal diameters and axial modes.

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