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ANALYSIS OF VIBARTING MICROPOLAR PLATE IN CONTACT WITH A FLUID

A. NAJAFI
Department of Mechanical engineering, Islamic Azad University, Shiraz Branch, Shiraz, Iran.
anajafi@shirazu.ac.ir

F. DANESHMAND
Department of Mechanical engineering, Shiraz University, Shiraz 71348-51154, Iran
Currently, Visiting Professor, Department of Mechanical Engineering, McGill University, Montreal,Quebec, Canada
farhang.daneshmand@mcgill.ca

S. R. Mohebpour
Mechanical Engineering Department, School of Engineering, Persian Gulf University, Bushehr, Iran
srmohebpour@yahoo.com

ABSTRACT

Micropolar theory constitutes extension of the classical field theories. It is based on the idea that every particles of the material can make both micro rotation and volumetric micro elongation in addition to the bulk deformation. Since this theory includes the effects of micro structure which could affect the overall behaviour of the medium, it reflects the physical realities much better than the classical theory for the engineering materials.

In the micropolar theory, the material points are considered to possess orientations. A material point carrying three rigid directors introduces one extra degree of freedom over the classical theory. This is because in micropolar continuum, a point is endowed with three rigid directors only. A material point is then equipped with the degrees of freedom for rigid rotations, in addition to the classical translational degrees of freedom. In fact, the micropolar covers the results of the classical continuum mechanics. The micropolar theory recently takes attentions in fluid mechanics and mathematicians and engineers are implementing this theory in various theoretical and practical applications.

In this paper the fluid-structure analysis of a vibrating micropolar plate in contact with a fluid is considered. The fluid is contained in a cube which all faces except for one of the lateral faces are rigid. The only non-rigid lateral face is made of a flexible micropolar plate and therefore, interacts with the fluid. An analytical approach is utilized to investigate the vibration characteristics of the aforementioned fluid-structure problem. The fluid is non-viscous and incompressible. Duplicate Chebyshev series, multiplied by boundary functions are used as admissible functions and the frequency equations of the micropolar plate are obtained by the use of Chebyshev-Ritz method.

Also the vibration analysis of the plates modeled by micropolar theory has been done. This analysis shows that some additional frequencies due to the micropolarity of the plate appears among the values of the frequencies obtained in the classical theory of elasticity, as expected. These new frequencies are called micro-rotational waves. We also observed that when the micropolar material constants vanish, these additional frequencies disappear and only the classical frequencies remain. Specially, we observed that these additional frequencies are more sensitive to the change of the micro elastic constants than the classical frequencies. The frequencies and mode shapes of the coupled fluid structure interaction problem are obtained in the present study based on the micropolar and classical modeling. The numerical results for the problem are compared with those obtained by the analytical method for their differences and to confirm the proposed method. The microrotatinal wave frequencies and mode shapes are also developed. The results show that the natural frequencies and mode shapes for the transverse vibrations of the problem are in good agreement with the classical one and our knowledge from the physical nature of the problem.

INTRODUCTION

In the classical theory of continuum mechanics, materials are assumed to be homogeneous. Nevertheless, some modern engineering structures have lots of defects with different sizes and forms which violate the assumption of continuity at micro scale. Such structures are made of materials possessing an internal structure. Polycrystalline materials, materials with fibrous or coarse grain structure come in this category [1]. The analysis of such materials requires incorporating the theory of oriented media.

Micropolare theory has been developed by Eringen [2−4] for elastic solids, fluid and further for nonlocal polar fields. Micropolar theory constitutes extension of the classical field theories concerned with the rotations, in microscopic space and short time scales. Mathematically, material particles are assumed to be geometrical points that possess physical and mathematical properties, e.g., mass, charge and rigid directors. The field equations constructed with this model are expected then to represent many new and wider classes of physical phenomena that fall outside the classical field theories.

In the micropolar theory, the material is endowed with microstructure, like atoms and molecules at microscopic scale. Homogenization of a basically heterogeneous material depends on scale of interest. When stress fluctuation is small enough compared to microstructure of the material, homogenization can be made without considering the detailed microstructure of the material. However, if it is not the case, the microstructure of material must be considered properly in a homogenized formulation [1, 2].

At each particle of a micropolar continuum, it is assumed that there is a microstructure which can rotate independently from the surrounding medium [5]. So every particle contains six degrees of freedom, three translational motions which are assigned to the macro-element and three rotational ones which are referred to the microstructure.

Due to theoretical and practical importance, many problems of waves and vibration of micropolar elasticity have been investigated by different researchers. A bending analysis of micropolar elastic beams using a 3-D finite element method has been developed by Huang et al. [6]. They first derived an analytical solution for straight beam problems based on the theory of material strength. Then, they applied a new 3-D finite element to solve both straight and curved beam problems.

Recently, a linear theory for the analysis of beams based on the micropolar continuum mechanics has been developed by Ramezani et al [5]. Power series expansions for the axial displacement and micro-rotation fields were assumed in their work. The governing equations were derived by integrating the momentum and moment of momentum equations in the micropolar continuum theory.

On the other hand, the fluid-structure problems have also been extensively contemplated by many researchers [7, 8]. In their works, the fluid is considered to be ideal and incompressible (compressible) with the Laplace (wave) as its governing equations, where the structure has had the variety of shapes and assumptions.

In the present work, fluid-structure interaction problems having microstructure are modeled by the microstretch theory. In this work, an analytical formulation for vibration analysis of a micropolar plate in contact with fluid and based on the micropolar continuum mechanics is developed. We follow Eringen's method for constructing the micropolar plate theory [2].

We study the coupled problem to obtain natural frequencies of the fluid-structure problem. The fluid is considered to be ideal and incompressible. In this work we utilize the Chebyshev polynomials because of their simplicity for computations and coding and also their high accuracy and numerical reliability [8-10].

NOMENCLATURE

$\alpha,\ \beta,\ \gamma,\kappa$: Micropolar Elasticity Constants
u, v, w: Displacements in x, y, z directions
G: Shear Modulus
$\rho,\ \rho j$: Mass density and the micro-inertia
ψ_k : Microrotation
F, $\boldsymbol{L_l}$: External stress and couple stress
$\psi_{1mn}, \psi_{2mn}, w_{mn}$: Series Expansion terms related to displacements
Φ, φ: Deformation potential
T_L^*: Reference kinetic energy of the fluid
T_S^*: Reference kinetic energy of the structure
T_S: Kinetic energy of the structure
V_S : Maximum Reference potential energy of the structure
ω : Natural frequency of the structure vibration
Ω : Natural frequency of the fluid-structure vibration

MATHEMATICAL MODEL

As stated in the previous section, there are instances in which the assumption of material homogeneity is inadequate: either the size of the loaded structure is very small and comparable to the length scale of its constituent material microstructure or the length scale of the heterogeneity with the material structure is significantly larger than microscopic. Many nano-devices fall into the first category, whereas materials such as ceramics, cement, rock, soil, bone and short fiber and particulate-reinforced composites may be referred to as the second category. Micropolar theory is an alternative theory describing the behaviour of heterogeneous materials. The mathematical foundation of theory of micropolar continuum mechanics has been developed through the works of Eringen and his coworkers. In this section, we present some basic relations of the micropolar elasticity needed for our derivation in the next sections.

The vibration analysis of a micropolar plate in contact with a fluid is considered. The fluid is contained in a cube with all faces except one of the lateral face are rigid. The only non-rigid lateral face is made of a flexible micropolar plate and therefore, interacts with the fluid. The problem is shown in Fig. 1.

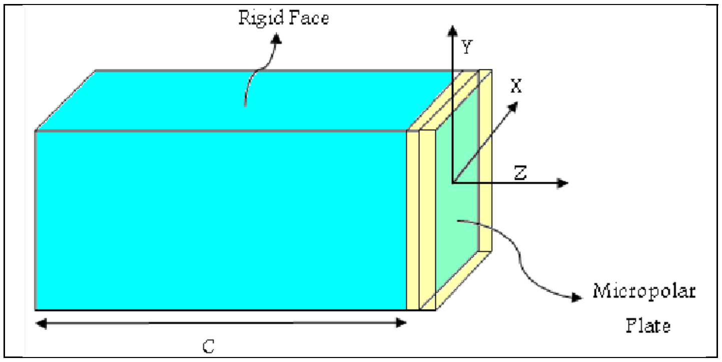

Fig. 1.

Structure Domain

As mentioned earlier, the micropolar elasticity defines more degrees of freedom. We consider the linear isotropic plate of lowest-order. The governing equations of the Three-dimensional micropolar elasticity are as follows [2, 4],

$$(\mu+\kappa)\nabla^2 u + (\lambda+\mu)\frac{\partial}{\partial x}(u_{,x}+v_{,y}+w_{,z}) - \kappa(\zeta_{y,z}-\zeta_{z,y}) = \rho u_{,tt} \tag{1}$$

$$(\mu+\kappa)\nabla^2 v + (\lambda+\mu)\frac{\partial}{\partial y}(u_{,x}+v_{,y}+w_{,z}) - \kappa(\zeta_{z,x}-\zeta_{x,z}) = \rho v_{,tt} \tag{2}$$

$$(\mu+\kappa)\nabla^2 w + (\lambda+\mu)\frac{\partial}{\partial z}(u_{,x}+v_{,y}+w_{,z}) - \kappa(\zeta_{x,y}-\zeta_{y,x}) = \rho w_{,tt} \tag{3}$$

$$\gamma\nabla^2\zeta_x - 2\kappa\zeta_x + (\alpha+\beta)\frac{\partial}{\partial x}(\zeta_{x,x}+\zeta_{y,y}+\zeta_{z,z}) - \kappa(v_{,z}-w_{,y}) = \rho j\zeta_{x,tt} \tag{4}$$

$$\gamma\nabla^2\zeta_y - 2\kappa\zeta_y + (\alpha+\beta)\frac{\partial}{\partial y}(\zeta_{x,x}+\zeta_{y,y}+\zeta_{z,z}) - \kappa(w_{,x}-u_{,z}) = \rho j\zeta_{y,tt} \tag{5}$$

$$\gamma\nabla^2\zeta_z - 2\kappa\zeta_z + (\alpha+\beta)\frac{\partial}{\partial z}(\zeta_{x,x}+\zeta_{y,y}+\zeta_{z,z}) - \kappa(u_{,y}-v_{,x}) = \rho j\zeta_{z,tt} \tag{6}$$

where ρ is the mass density, j is the micro-inertia, λ, μ, κ, α, β and γ are material constants. u, v, w and ζ_x, ζ_y, ζ_z are displacement and micro-rotation component, respectively.

$$k = \mu+\kappa \tag{7- a}$$
$$k_1 = \lambda+2\mu+\kappa \tag{7- b}$$
$$k_2 = \alpha+\beta+\gamma \tag{7- c}$$
$$k_3 = \alpha+\beta \tag{7- d}$$
$$k_4 = \lambda+\mu \tag{7- e}$$

For attaining the governing equations of the micropolar plate from three-dimensional micropolar elasticity, various methods have been used [2]. Some authors have used the perturbation method, when some others, have used the asymptotic analysis [11]. Based on the method used in [1], the governing equations of the lowest-order micropolar plate are obtained by some integration in the thickness direction of the micropolar media. The result is as follows

$$\left(G+\frac{\kappa}{2}\right)\nabla^2 w + \kappa\varepsilon_{kl3}\psi_{l,k} + \rho_s(F-\ddot{w}) = 0 \tag{8}$$

$$(\alpha+\beta)\psi_{k,lk} + \gamma\psi_{l,kk} - \kappa\varepsilon_{kl3}w_{,k} - 2\kappa\psi_l + \rho_s(L_l - j\ddot{\psi}_l) = 0 \tag{9}$$

where ρ_s is the mass density, j is the micro-inertia, G is the shear modulus. κ, α, β and γ are and micropolar material constants. w and ψ_l are displacement and micro-rotation component, respectively.

We consider the simply supported problem in which the boundary conditions are as follows

$$\text{on } x = -1, 1:\ \ w = 0,\ \psi_1 = \psi_2 = 0 \tag{10}$$
$$\text{on } y = -1, 1, :\ \ w = 0,\ \psi_1 = \psi_2 = 0 \tag{11}$$

Fluid Domain

For fluid domain, we have the following equations [7, 8]:

$$\nabla^2\Phi = 0 \tag{12}$$

where, Φ is the deformation potential. The related boundary conditions are as follows (Fig. 1)

$$\text{on } x = 0, a:\ \ \partial\Phi/\partial n = 0 \tag{13}$$
$$\text{on } y = 0, b:\ \ \partial\Phi/\partial n = 0 \tag{14}$$
$$\text{on } z = -c:\ \ \partial\Phi/\partial n = 0 \tag{15}$$
$$\text{on } z = 0:\ \ \partial\Phi/\partial n = w(x, y, 0, t) \tag{16}$$

Using separations of variable in fluid domains, yields the following results

$$\varphi = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} E_{mn}\cos(m\pi x)\cos(n\pi y)\cosh(\Lambda_{mn}(z+c)) \tag{17}$$

where

$$\Lambda_{mn} = [(m\pi)^2 + (n\pi)^2]^{1/2} \tag{18- a}$$
$$\Phi = \varphi(x, y, z)e^{i(\omega t+\theta)}; \tag{18- b}$$

Natural Frequency and Mode Shapes of the Free Vibrations of the Structure

For the natural frequency and mode shapes of the free vibrations of the structure, we have

$$\left(G+\frac{\kappa}{2}\right)\nabla^2 w + \kappa\varepsilon_{kl3}\phi_{l,k} = \rho_s\ddot{w} \tag{19}$$
$$(\alpha+\beta)\psi_{k,lk} + \gamma\psi_{l,kk} - \kappa\varepsilon_{kl3}w_{,k} - 2\kappa\psi_l = \rho_s j\ddot{\psi}_l \tag{20}$$

As shown in Fig. 2, the boundary conditions are

$$\text{on } x = -1, 1:\ \ w = 0,\ \psi_1 = \psi_2 = 0 \tag{21}$$
$$\text{on } y = -1, 1, :\ \ w = 0,\ \psi_1 = \psi_2 = 0 \tag{22}$$

The natural frequencies and mode shapes of the free vibrating micropolar plate, the following results can be obtained after performing some mathematical operations,

$$\begin{Bmatrix} w \\ \psi_1 \\ \psi_2 \end{Bmatrix} = (1-x^2)(1-y^2)\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\begin{Bmatrix} w_{mn} \\ \psi_{1mn} \\ \psi_{2mn} \end{Bmatrix} e^{i(\omega t+\theta)} \tag{23}$$

where

$$w_{mn} = A_{mn}P_m(x)P_n(y) \tag{24}$$
$$\psi_{1mn} = B_{mn}P_m(x)P_n(y) \tag{25}$$
$$\psi_{2mn} = C_{mn}P_m(x)P_n(y) \tag{26}$$

$P_m(x)$ is the m^{th} Chebyshev polynomial and is defined as $P_m(x) = \cos((m-1)\mathrm{Arccos}(x))$.

Also, the terms $(\mathbf{1}-\boldsymbol{x}^{2})$ and $(\mathbf{1}-\boldsymbol{y}^{2})$ are the boundary functions to meet the necessary condition for admissibility of the functions.

The potential energy functional of the structure is

$$\boldsymbol{V}_s=\frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}[\boldsymbol{h}(\boldsymbol{\alpha}\boldsymbol{\psi}_{k,k}\,\boldsymbol{\psi}_{l,l}+\boldsymbol{\beta}\boldsymbol{\psi}_{k,l}\,\boldsymbol{\psi}_{l,k}+\boldsymbol{\gamma}\boldsymbol{\psi}_{k,l}\,\boldsymbol{\psi}_{k,l})+\boldsymbol{Ghw}_{,k}\,\boldsymbol{w}_{,k}+\mathbf{2}\boldsymbol{\kappa h}(\boldsymbol{r}_k-\boldsymbol{\psi}_k)(\boldsymbol{r}_k-\boldsymbol{\psi}_k)]\boldsymbol{dxdy} \quad (27)$$

and the corresponding Kinetic energy is

$$\boldsymbol{T}_s=\frac{\boldsymbol{h}}{\mathbf{2}}\int_{-1}^{1}\int_{-1}^{1}[\boldsymbol{\rho}_s\boldsymbol{j}\dot{\boldsymbol{\psi}}_k\dot{\boldsymbol{\psi}}_k+\boldsymbol{\rho}_s\dot{\boldsymbol{w}}^{2}]\boldsymbol{dxdy}$$

$$=\frac{\boldsymbol{h}}{2}\boldsymbol{\omega}^{2}\int_{-1}^{1}\int_{-1}^{1}[\boldsymbol{\rho}_s\boldsymbol{j}\boldsymbol{\psi}_k\boldsymbol{\psi}_k+\boldsymbol{\rho}_s\boldsymbol{w}^{2}]\boldsymbol{dxdy} \quad (28)$$

For more details see [2].

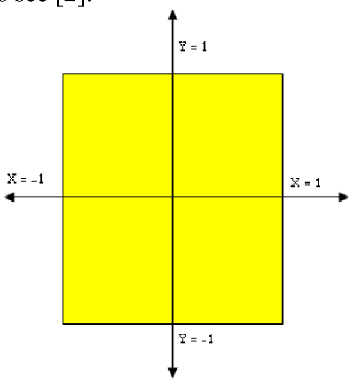

Fig. 2

Based on the above relations, one can have the reference kinetic energy of the structure as

$$\boldsymbol{T}_s^{*}=\frac{\boldsymbol{h}}{2}\int_{-1}^{1}\int_{-1}^{1}[\boldsymbol{\rho}_s\boldsymbol{j}\boldsymbol{\psi}_k\boldsymbol{\psi}_k+\boldsymbol{\rho}_s\boldsymbol{w}^{2}]\boldsymbol{dxdy} \quad (29)$$

The maximum reference potential energy of the structure can also be written as

$$\boldsymbol{V}_s=\frac{1}{2}\int_{-1}^{1}\int_{-1}^{1}[\boldsymbol{h}(\boldsymbol{\alpha}\boldsymbol{\psi}_{k,k}\,\boldsymbol{\psi}_{l,l}+\boldsymbol{\beta}\boldsymbol{\psi}_{k,l}\,\boldsymbol{\psi}_{l,k}+\boldsymbol{\gamma}\boldsymbol{\psi}_{k,l}\,\boldsymbol{\psi}_{k,l})+\boldsymbol{Ghw}_{,k}\,\boldsymbol{w}_{,k}+\mathbf{2}\boldsymbol{\kappa h}(\boldsymbol{r}_k-\boldsymbol{\psi}_k)(\boldsymbol{r}_k-\boldsymbol{\psi}_k)]\boldsymbol{dxdy} \quad (30)$$

Since we need to attain the natural frequencies and mode shapes of the free vibration of the structure, we define the following functional

$$\boldsymbol{\Pi}=\boldsymbol{V}_s-\boldsymbol{T}_s=\boldsymbol{V}_s-\boldsymbol{\omega}^{2}\boldsymbol{T}_s^{*} \quad (31)$$

By minimizing this functional with respect to A_{mn}, B_{mn}, C_{mn}, one can obtain the natural frequencies and natural modes of the free vibrations.

NUMERICAL RESULTS

In this paper we use the Chebyshev polynomials because of their simplicity for computations and coding and also their high accuracy and numerical reliability. First, we briefly introduce the Chebyshev polynomials and some of their properties. The first six terms of the Chebyshev polynomials are

$T_0=1;\ T_1=x;\ T_2=2x^2-1;\ T_3=4x^3-3x;$

$T_4=8x^4-8x^2+1;\ T_5=16x^5-20x^3+5x;$

These functions are mutually orthogonal in the interval [-1, 1] with the weighting function $(1-x^2)^{-1/2}$. The six polynomial graphs are shown in Fig. 3. Using Chebyshev-Ritz method, and approximating the number of series (we use 48 terms for all three variables), the problems turns to

$$\det\left(\begin{bmatrix}K_{ww} & K_{w\psi_1} & K_{w\psi_2}\\ K_{\psi_1 w} & K_{\psi_1\psi_1} & K_{\psi_1\psi_2}\\ K_{\psi_2 w} & K_{\psi_2\psi_1} & K_{\psi_2\psi_2}\end{bmatrix}_{48\times48}-\omega^2\begin{bmatrix}M_{ww} & 0 & 0\\ 0 & M_{\psi_1\psi_1} & 0\\ 0 & 0 & M_{\psi_2\psi_2}\end{bmatrix}_{48\times48}\right)=0 \quad (32)$$

Where

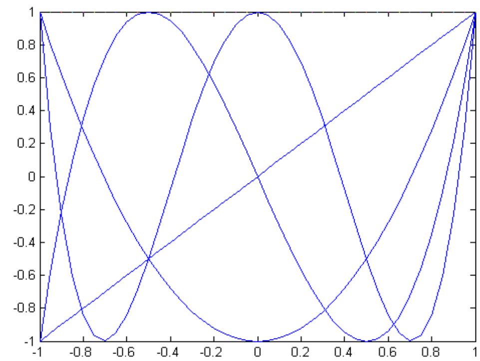

Fig. 3. Chebyshev polynomials

$$K_{ww}=\left[\frac{\partial^2\boldsymbol{V}_s}{\partial(A_{mn})^2}\right]_{16\times16} \quad (33)$$

$$K_{w\psi_1}=K_{\psi_1 w}=\left[\frac{\partial^2\boldsymbol{V}_s}{\partial A_{mn}\partial B_{mn}}\right]_{16\times16} \quad (34)$$

$$K_{w\psi_2}=K_{\psi_2 w}=\left[\frac{\partial^2\boldsymbol{V}_s}{\partial A_{mn}\partial C_{mn}}\right]_{16\times16} \quad (35)$$

$$K_{\psi_1\psi_1}=\left[\frac{\partial^2\boldsymbol{V}_s}{\partial(B_{mn})^2}\right]_{16\times16} \quad (36)$$

$$K_{\psi_2\psi_2}=\left[\frac{\partial^2\boldsymbol{V}_s}{\partial(C_{mn})^2}\right]_{16\times16} \quad (37)$$

$$M_{ww}=\left[\frac{\partial^2\boldsymbol{T}_s^{*}}{\partial(A_{mn})^2}\right]_{16\times16} \quad (38)$$

$$M_{\psi_1\psi_1}=\left[\frac{\partial^2\boldsymbol{T}_s^{*}}{\partial(B_{mn})^2}\right]_{16\times16} \quad (39)$$

$$M_{\psi_2\psi_2}=\left[\frac{\partial^2\boldsymbol{T}_s^{*}}{\partial(C_{mn})^2}\right]_{16\times16} \quad (40)$$

To our knowledge, there is no other result found for the frequencies of free vibration of a plate modeled by microstretch theory in the literature. Therefore we must compare our results with only the results obtained in the classical theory by taking all microstretch material constants zero. The numerical results from the present study for dry-structure are compared with those obtained from analytical solution [12] in Table 1.

Table.1 Frequencies of vibration for dry-structure, classic theory, ω (s^{-1})

Mode No.	Present Study	Analytical Results [12]	Error (%)
1	162.150	162.150	0
2	256.441	256.381	0.02
3	256.441	256.381	0.02
4	324.395	324.300	0.03
5	363.215	362.578	0.2

The numerical results for frequencies of transverse vibration of dry structure obtained by using the micropolar theory are given in Table 2 and are compared with the results of analytical method. The different parameters for the micropolar theory are also presented in this table.

Table 2. Results for frequency of transverse vibration for dry structure, micropolar theory, ω (s^{-1})

Mode No.	Micropolar Theory (Present Study)	Analytical Results [12]
1	162.150	162.150
2	256.441	256.381
3	256.441	256.381
4	324.395	324.300
5	363.215	362.578

As can be seen, even though in our computations the micropolar frequencies are a bit less than the classical frequencies, the micropolar frequencies are very close to the classical one (with two decimal digit accuracy). The main reason is that the micropolar theory admits the rigid body rotation for micro-elements. It should also be noted that some additional frequencies are observed due to microstructure of the plate among the values of the frequencies obtained from classical theory of the elasticity. The results for the micro-rotational wave frequencies are presented in Table 3. It should be noted that these additional frequencies disappear when all micropolar parameters are taken as zero.
As mentioned before, some additional are observed from the micromotion assumptions among the values of the frequencies obtained from the classical theory of elasticity. We observed that these additional frequencies disappear while the all microstretch constants are taken as zero. We also observed that these additional frequencies are more sensitive to the change of the micro elastic constants than the classical frequencies.

Table 3. Results for the Micro-rotaional wave Frequency, Dry Structure, ω (s^{-1})

$\alpha = 0.1236,\ \beta = 0.01585,\ \gamma = 0.05966,$ $j = 0.325 \times 10^{-7}, \kappa = 0.1316, h = 0.002,$ $G = 26.64 \times 10^{5}, E = 70.85 \times 10^{5}, \nu = 0.33$	
Mode No.	Micro-rotational Wave Frequency
1	95.206
2	173.905
3	191.051
4	198.261
5	258.348

It is in accordance with the results reported by [4]. The values of the additional frequencies increase by the change of the micro constants and then considerable amount of additional frequencies move out among the classical frequencies under consideration. We think that this observation may guide us to identify the microstretch material constants for different materials for future works. The shapes for the first four mode shapes of the micropolar structure are shown in Figs. 4-7. The mode shapes corresponding to the micro-rotational frequencies are also presented in Figs. 8-11.

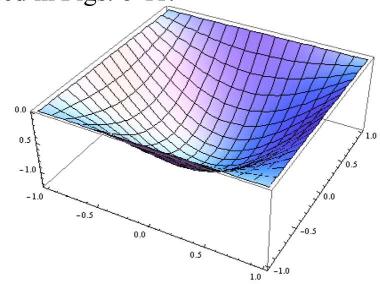

Fig. 4. First Mode Shape of the Micropolar Structure

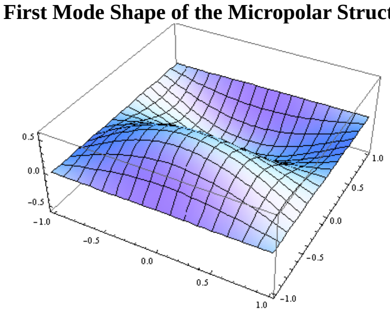

Fig. 5. Second Mode Shape of the Micropolar Structure

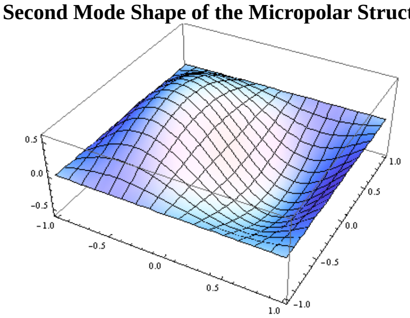

Fig. 6. Third Mode Shape of the Micropolar Structure

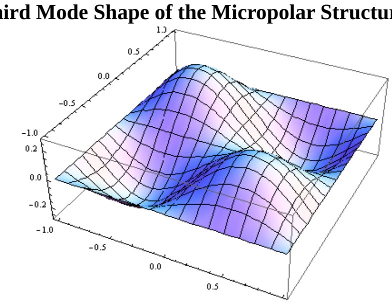

Fig. 7. Fourth Mode Shape of the Micropolar Structure

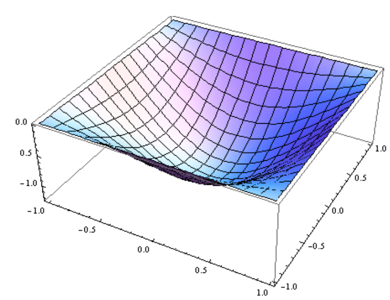

Fig. 8. First Mode Shape of the Micropolar Structure for $\psi_{1,2}$

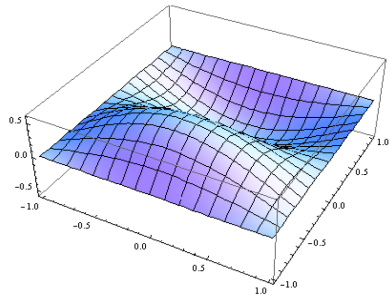

Fig. 9. Second Mode Shape of the Micropolar Structure for $\psi_{1,2}$

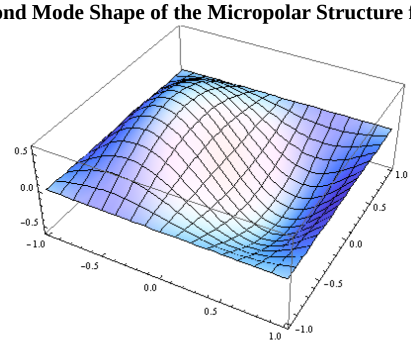

Fig. 10. Third Mode Shape of the Micropolar Structure for $\psi_{1,2}$

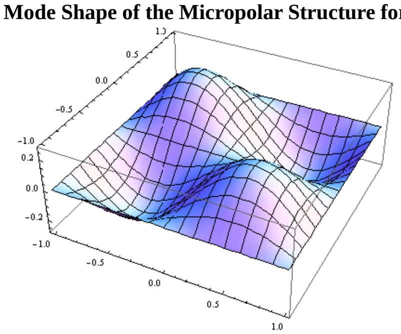

Fig. 11. Fourth Mode Shape of the Micropolar structure for $\psi_{1,2}$

FLUID-STRUCTURE INTERACTION

The coupling between the fluid and structure occurs in a boundary condition at $z = 0$. As we know this coupling has the following form

$$\text{on } z = 0:\ \partial\Phi/\partial n = w(x,y,t) \tag{41}$$

As mentioned earlier, we have the following relations for the fluid domain

$$\varphi(x,y,z) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} E_{mn}\cos(m\pi x)\cos(n\pi y)\cosh[\Lambda_{mn}(z+c)] \tag{42}$$

and for the structure, we have

$$\left(G+\frac{\kappa}{2}\right)\nabla^2 w + \kappa\varepsilon_{kl3}\phi_{l,k} + P = \rho\ddot{w} \tag{43}$$

$$(\alpha+\beta)\psi_{k,lk} + \gamma\psi_{l,kk} - \kappa\varepsilon_{kl3}w_{,k} - 2\kappa\psi_l += \rho j\ddot{\psi}_l \tag{44}$$

With the following boundary conditions

$$\text{on } z = 0:\ \partial\Phi/\partial n = w(x,y,t) \tag{45}$$

$$\text{on } x = -1,1:\ w = 0,\ \psi_1 = \psi_2 = 0 \tag{46}$$

$$\text{on } y = -1,1,:\ w = 0,\ \psi_1 = \psi_2 = 0 \tag{47}$$

Where, P is the hydrodynamic pressure due to interaction between fluid and structure. At this step, we use the Chebyshev polynomial to write the plate parameters.

$$(1-x^2)(1-y^2)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}P_m(x)P_n(y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Lambda_{mn}E_{mn}\cos(m\pi x)\cos(n\pi y)\sinh[\Lambda_{mn}c] \tag{48}$$

By some straightforward algebraic operations, one can obtain

$$E_{pq} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} csch(\Lambda_{pq}c)A_{mn}\int_{-1}^{1}\int_{-1}^{1}(1-x^2)(1-y^2)\cos(m\pi x)\cos(n\pi y)P_m(x)P_n(y)dxdy \tag{49}$$

Because of the orthonormality nature of the Chebyshev polynomials in [-1, 1], the above relations can easily be computed.

Chebyshev-Ritz Method

At this step, we are ready for finding natural frequencies of the couple problem. It is necessary to construct the Rayleigh quotient. For finding the Rayleigh quotient, the related reference energies are presented:

Fluid Reference Kinetic Energy

The reference kinetic energy of the fluid is due to fluid-structure interaction, therefore [7, 8],

$$T_L^* = \frac{1}{2}\rho_L\iint_S \varphi\frac{\partial\varphi}{\partial n}dS \tag{50}$$

Which yields the following relation for the fluid reference kinetic energy is as

$$T_L^* = \frac{1}{4}\rho_L\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} E_{mn}^2\Lambda_{mn}\sinh(2\Lambda_{mn}c) \tag{51}$$

Structure Reference Energy

The reference kinetic energy of the structure is as follows [2],

$$T_s^* = \frac{h}{2}\int_{-1}^{1}\int_{-1}^{1}[\rho_s j\psi_k\psi_k + \rho_s w^2]dxdy \tag{52}$$

Maximum Potential Energy of the Structure

The maximum potential energy of the structure is [2],

$$V_s = \frac{1}{2}h\int_{-1}^{1}\int_{-1}^{1}\{\alpha\psi_{l,l}\psi_{k,k} + \beta\psi_{k,l}\psi_{l,k} + \gamma\psi_{k,l}\psi_{k,l} + Gw_{,k}w_{,k} + 2\kappa(r_k-\psi_k)(r_k-\psi_k)\} \tag{53}$$

Hence, we are ready to find the natural frequencies of the coupled problem. Utilizing Rayleigh quotient,

$$\Omega^2 = \frac{V_s}{T_L^*+T_s^*} \tag{54}$$

and minimizing the following functional with respect to A_{mn}, B_{mn} and C_{mn} yields the desired result

$$\Pi = V_s - \Omega^2(T_L^*+T_s^*) \tag{39}$$

which results

$$\frac{\partial\Pi}{\partial A_{pq}} = 0 \tag{40}$$

$$\frac{\partial\Pi}{\partial B_{pq}} = 0 \tag{41}$$

$$\frac{\partial\Pi}{\partial C_{pq}} = 0 \tag{42}$$

Noting that E_{mn}'s are functions of A_{mn}, B_{mn}, C_{mn}; therefore we can compute the mode shapes and natural frequencies of the coupled problem.

NUMERICAL RESULTS

Using again Chebyshev-Ritz method, and approximating the number of series (we use 16 terms for fluid domain), the problems turns to

$$\det\left(\begin{bmatrix} K_{ww} & K_{w\psi_1} & K_{w\psi_2} \\ K_{\psi_1\psi_1} & K_{\psi_1\psi_1} & K_{\psi_1\psi_2} \\ K_{\psi_2 w} & K_{\psi_2\psi_1} & K_{\psi_2\psi_2} \end{bmatrix}_{48\times48} - \boldsymbol{\Omega}^2 \begin{bmatrix} M_{ww} & 0 & 0 \\ 0 & M_{\psi_1\psi_1} & 0 \\ 0 & 0 & M_{\psi_2\psi_2} \end{bmatrix}_{48\times48}\right) = 0 \quad (55)$$

Where

$$K_{ww} = \left[\frac{\partial^2 \boldsymbol{V_s}}{\partial (A_{mn})^2}\right]_{16\times16} \quad (56)$$

$$K_{w\psi_1} = K_{\psi_1 w} = \left[\frac{\partial^2 \boldsymbol{V_s}}{\partial A_{mn}\partial B_{mn}}\right]_{16\times16} \quad (57)$$

$$K_{w\psi_2} = K_{\psi_2 w} = \left[\frac{\partial^2 \boldsymbol{V_s}}{\partial A_{mn}\partial C_{mn}}\right]_{16\times16} \quad (58)$$

$$K_{\psi_1\psi_1} = \left[\frac{\partial^2 \boldsymbol{V_s}}{\partial (B_{mn})^2}\right]_{16\times16} \quad (59)$$

$$K_{\psi_2\psi_2} = \left[\frac{\partial^2 \boldsymbol{V_s}}{\partial (C_{mn})^2}\right]_{16\times16} \quad (60)$$

$$M_{ww} = \left[\frac{\partial^2 (\boldsymbol{T_L^*}+\boldsymbol{T_s^*})}{\partial (A_{mn})^2}\right]_{16\times16} \quad (61)$$

$$M_{\psi_1\psi_1} = \left[\frac{\partial^2 (\boldsymbol{T_L^*}+\boldsymbol{T_s^*})}{\partial (B_{mn})^2}\right]_{16\times16} \quad (62)$$

$$M_{\psi_2\psi_2} = \left[\frac{\partial^2 (\boldsymbol{T_L^*}+\boldsymbol{T_s^*})}{\partial (C_{mn})^2}\right]_{16\times16} \quad (63)$$

The numerical results from the present study for wet-micropolar structure are given in Table 4 and are compared with those obtained for the dry micropolar structure.

Table 4. Results for FSI Frequency ($\boldsymbol{\Omega}$) in (s^{-1})

$\alpha = 0.1236$, $\beta = 0.01585$, $\gamma = 0.05966$, $j = 0.325\times10^{-7}$, $\kappa = 0.1316$, $h = 0.002$, $G = 26.64\times10^5$, $E = 70.85\times10^5$, $\nu = 0.33$		
Mode No.	Wet Frequencies	Dry Frequencies
1	32.913	162.150
2	59.664	256.441
3	64.679	256.441
4	93.057	324.395
5	170.788	363.215

Table 5. Results for FSI Frequency of Micropolar and Classic Structures($\boldsymbol{\Omega}$) in (s^{-1})

Mode No.	Micropolar Theory (Present Study)	Classical Theory (Present Study)
1	32.913	32.913
2	59.664	59.664
3	64.679	64.679
4	93.057	93.057
5	170.788	170.788
6	249.077	249.077
7	256.441	256.441

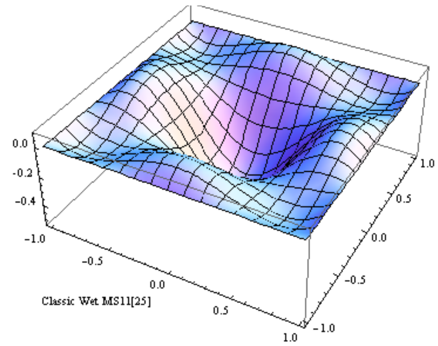

Fig. 12. First Mode Shape of the Micropolar Fluid-Structure

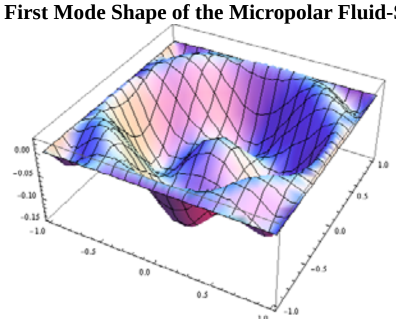

Fig. 13. Second Mode Shape of the Micropolar Fluid-Structure

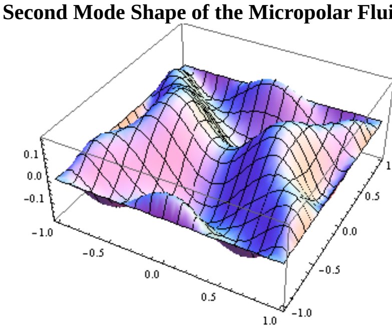

Fig. 14. Third Mode Shape of the Micropolar Fluid-Structure

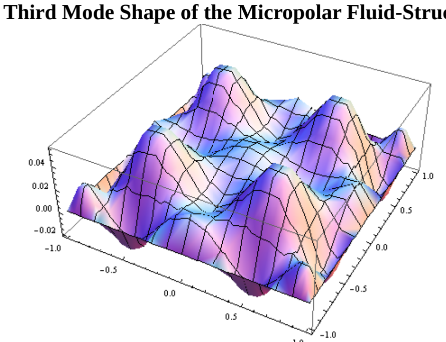

Fig. 15. Fourth Mode Shape of the Micropolar Fluid-Structure

The results for micromotions $\psi_{1,2}$ is same as the results for the structure without having contact with a fluid and therefore we omit them.

CONCLUSION

In this paper the coupled vibrations of the lowest-order micropolar plate in contact with the ideal incompressible fluid has been investigated. The method for finding the natural frequencies of the coupled vibrations relies on the Chebyshev-Ritz method.

This theory predicts the existence of micro-rotational waves which are not present in any of the known plate theories based on the classical continuum mechanics. Moreover, When all microstretch constants are taken as zero, these additional frequencies disappear and only classical frequencies. Excellent agreement between micropolar elasticity and classical elasticity has been found in finding natural frequencies of free vibrations. As expected, the fluid-structure frequencies have found to be less than the natural frequencies for the dry structure. It can be concluded that this observation may be used to determine microstretch material constants in future works by combining this analysis with some other assumptions for analysis of fluid structure interaction problems.

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