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STUDY ON MODELING METHOD OF VORTEX SHEDDING SYNCHRONIZATION IN HEAT EXCHANGER TUBE BUNDLES

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ABSTRACT

Acoustic resonance may occur in heat exchangers such as gas heaters or boilers which contain tube bundles. The purpose of this study is to develop modeling method of vortex shedding synchronization because this is the most essential part of critical flow velocity prediction. Here, acoustic resonance level dependence of spatial correlation of vortex shedding is expressed by coherence function between wake-oscillator behaviors in any two locations in the cavity. The feedback effect in synchronization of vortex shedding is represented by resonant level dependence of the wake-oscillator phase fluctuation range. This method gives the result that when resonance level increases, synchronization level in the tube bundles also increases, which seems to be a reasonable conclusion. Experimental method to identify the undefined parameters in the proposed method is also mentioned.

1. INTRODUCTION

Acoustic resonance may occur in heat exchangers such as gas heaters or boilers which contain tube bundles. High level noise generated by resonance has concerned heat exchanger designers for a long time. Key research issues include prediction of resonance and estimation of the critical gas flow velocity for a given tube bundle configuration and acoustic space geometry in ducts. In order to clarify these issues, many studies have been conducted. Since the early 1950's there have been many research articles on resonance phenomena in heat exchangers. The work of Y. N. Chen[1)] in 1968 is well-known as a breakthrough in this field. He identified the relationship between resonant noise and Karman vortex shedding, and proposed parameters which can be used to predict potential resonances at the design stage[2)]. Following his work, the research works have been done which include resonance prediction methods at the design stage, countermeasures for existing resonance cases, mechanism of resonance generation with a focus on feedback phenomena et. al. and they are mentioned in the review papers by Weaver[3)], Paidoussis[4)], Blevins[5)], and Eisinger[6)].

As to the acoustic resonance prediction methods, they are classified into two kinds; empirical ones and feed-back model based ones. Empirical design rules has been proposed based on accumulated experimental or in-situ plant data on the critical flow velocity for the variety of tube bundle configurations. Representative works have been done by Grotz[7)], Y. N. Chen[1)], Fitzpatrick[8)], Ziada[9)], Blevins[10)] and Eisinger[11)] et. al. As to the latter methods[12)~ 15)], there are many issues to be resolved: most important one is how to include vortex shedding synchronization effect which increases as resonance amplitude increases.

In this study, in order to contribute the study of prediction method of resonance attack, modeling method of this phenomenon is proposed. Here, interaction between vortex shedding synchronization and acoustic resonance is modeled simply based on the knowledge of past researches. The proposed interaction model here is based on the literature of Blevins[16)]. The model of tube vibration induced by vortex shedding is mentioned there using wake oscillator model and one of its advantages might be in the compatibility with the experiment. It seems that the effects of tube vibration on vortex shedding behavior are similar to those of acoustical oscillation. Based on this hypothesis, vibration movement may be replaced by acoustic particle movement. Another improvement is the introduction of statistical modeling of wake oscillator. Here, the randomness of vortex shedding is explicitly modeled by probability density function of the phase of the oscillator and this function depends on the level of acoustic resonance.

The vortex/acoustics interaction model here is based on these ideas and the formulas are derived by deterministic expression in Chapter 3. Next, in chapter 4, in order to express the randomness of vortex shedding, statistical treatment are essential and it is introduced in the interaction model.

2. NOMENCLATURE

・General

* : complex conjugate

$_{t,r,n}$: total, resonant and noise components
$\gamma_{x,y}(\omega)$: coherence spectrum of time-series data x and y
ω : angular frequency
ω_r : resonant angular frequency
$p(\theta)$: probability density function
sinc(x) : sinc function
D : tube diameter
E< > : ensemble average
$S_{x,x}(\omega)$: auto spectrum of time-series data x
$S_{x,y}(\omega)$: cross spectrum of time-series data x and y
U : uniform flow velocity

- **Sound Field**

ϕ_r, ϕ_n : deterministic phase component of acoustic particle velocity
θ_r, θ_n : statistical phase component of acoustic particle velocity
Θ_r, Θ_n : distribution range of θ_r, θ_n
c : sound speed
p : acoustic pressure
r : viscous damping factor in sound field
$\dot{y}_t(t) = \dot{y}_r(t) + \dot{y}_n(t)$: total of acoustic particle velocity
$\dot{y}_r(t) = \dot{y}_{r0} \cdot \cos(\omega t + \phi_r + \theta_r)$: resonant component of acoustic particle velocity
$\dot{y}_{r0}$: amplitude of resonant component of acoustic particle velocity, $\dot{y}_r(t)$
$\dot{y}_n(t) = \dot{y}_{n0} \cdot \cos(\omega t + \phi_n + \theta_n)$: noise component of acoustic particle velocity
$\dot{y}_{n0}$: amplitude of noise component of acoustic particle velocity, $\dot{y}_n(t)$
$\dot{y}_t(\omega), \dot{y}_r(\omega), \dot{y}_n(\omega)$: complex amplitude of acoustic particle velocity
M,C,K: mass, damping, stiffness matrices of sound field
$\mathbf{K_r}$: matrix which is composed of modal parameters of acoustic resonance modes
R : resonant index defined by equation (16)
$\mathbf{S}_y$: cross spectrum matrix of acoustic particle velocity
$\dot{\mathbf{Y}}_t, \dot{\mathbf{Y}}_r, \dot{\mathbf{Y}}_n$: vector of which elements are $\dot{y}_t(\omega), \dot{y}_r(\omega), \dot{y}_n(\omega)$
$\dot{Y}_r = \dot{y}_{r0} \cdot \exp(i\phi_r)$: deterministic part of resonant component of acoustic particle velocity
$\dot{Y}_n = \dot{y}_{n0} \cdot \exp(i\phi_n)$: deterministic part of noise component of acoustic particle velocity

- **Wake Oscillator**

α : parameter which shows the sensitivity of vortex shedding to acoustic noise level
$\phi_r = 0 \ or \ \pi$: phase of resonant acoustic velocity mode shape
ϕ_w : phase lag of wake to resonant acoustic velocity
θ_w : statistical phase component of wake velocity
Θ_w : distribution range of θ_w
ω_s : natural angular frequency of wake oscillator
a_i, a'_i : undecided constant
$\dot{w}(t) = \dot{w}_0 \cdot \cos(\omega t + \phi_r - \phi_w + \theta_w)$: wake velocity
$\dot{w}_r(t) = \dot{w}_0 \cdot \cos(\omega t + \phi_r - \phi_w)$: deterministic part of wake velocity.
$\dot{w}(\omega)$: complex amplitude of wake velocity $\dot{w}(t)$
$\dot{W}_r = \dot{w}_0 \cdot \exp\{i(\phi_r - \phi_w)\}$: deterministic part of complex amplitude of wake velocity

- **Interaction Force**

$f_v(t)$: fluid force on the tube of unit length generated by vortex shedding.
$f_v:(\omega)$: complex amplitude of $f_v(t)$.
$\bar{f}_v$: sound source vector composed of nodal forces which are reaction of fluid forces
$\mathbf{S}_{fv}(\omega)$: sound source cross spectrum matrix.

3 INTERACTION MODEL: DETERMINISTIC APPROACH

3.1 Mechanism Underlying Resonance

When the gas flow rate in the duct surpasses a critical level, generation of high level noise by acoustic resonance may occur. Fig. 1 shows the mechanism leading to acoustic resonance. As the gas flow rate increases, Karman vortex shedding frequency increases eventually reaching the natural frequency of an acoustic mode within the duct. The resulting resonance can potentially generate high levels of noise.

The acoustic resonance phenomenon is classified as a self-excited oscillation induced by the vortex/acoustics interaction. Fig. 2 shows the basic mechanism. In the stable state, the acoustic pressure is low and has random characteristics in time and space. In this state, the vortex shedding frequency is locally unique but is not synchronized in space as shown in Fig. 2 (a). On the other hand, under the resonance conditions, as is shown in Fig. 2 (b), the acoustic mode generates high level synchronized pressure fluctuations. Therefore acoustic resonance affects the vortex shedding mechanism in two ways; first, the vortex strength is increased. Second, spatial correlation of vortex shedding (region of vortex shedding synchronization) is expanded in three dimensions. These two feedback mechanisms are incorporated in the proposed interaction model by simple method, as shown in the following chapter.

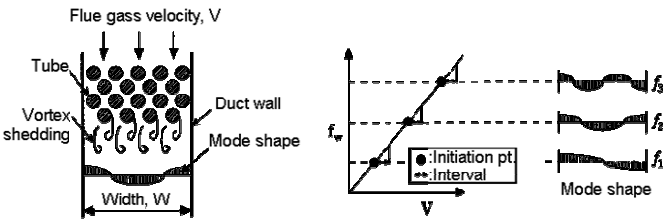

Fig.1 Overview of acoustic resonance

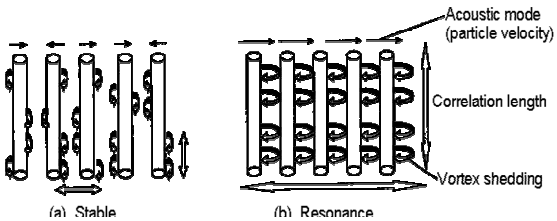

Fig.2 View showing vortex/sound interaction

3.2 Outline of Proposed Modeling Method

Along with the Blevins's literature[16], vortex shedding is described by wake oscillator model shown in Fig. 3. Fig. 4 shows the block diagram of feedback between vortex shedding and sound field expressed by acoustic particle velocity. This chapter mentions the deterministic description of this diagram. Here, as simplicity, acoustic modes perpendicular to flow-direction are treated. The same approach may be applied to the acoustic modes which are dominant in flow-direction. The term 'fluid force' referred to as f_v means the force on the tube and the tube applies the reaction force, '$-f_v$' to the sound field. Hereafter f_v is supposed to be the force on the tube of unit length, so f_v have the unit 'N/m'.

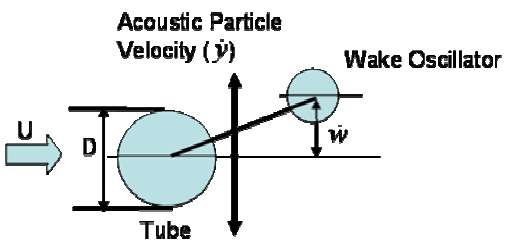

Fig.3 Wake oscillator model

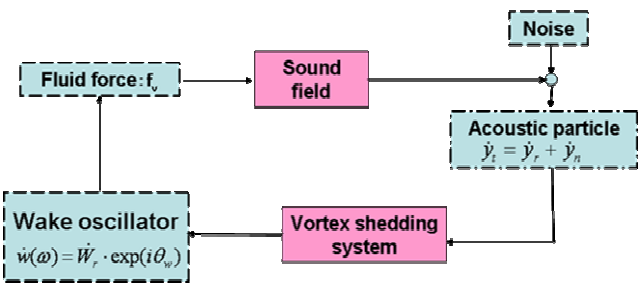

Fig. 4 Block Diagram of Vortex-Sound Interaction

3.3 Governing Equation of Sound Field and Vortex Shedding

(1) Sound Field

Most popular field variable is acoustic pressure, but in order to describe vortex/sound interaction, acoustic particle velocity is more suitable than acoustic pressure. And adopting discrete form, governing equation becomes as follows:

$$\mathbf{M}\ddot{\mathbf{y}}_r + \mathbf{C}\dot{\mathbf{y}}_r + \mathbf{K}\mathbf{y}_r = \bar{\mathbf{f}}_v \qquad (1)$$

Here, suffix 'r' means resonant component of acoustic particle motion. The right-hand term is named as sound source vector and it is composed of the nodal forces which are reaction of the fluid forces on the tubes. Hereafter it is referred to as 'sound source vector'. Its characteristics are mentioned in detail in section (3).

(2) Vortex Shedding

In the case of tube vibration problem, applying momentum conservation theorem to the supposed control volume, response of wake oscillator (referred to as w_r) by tube vibration (y_r) is derived as follows[16]:

$$\ddot{w}_r(t) + \left\{ a_2' \frac{1}{UD} \dot{w}_r(t)^2 - (a_1' - a_4') \frac{U}{D} \right\} \dot{w}_r(t) + \omega_s^{\,2} w_r(t) = a_3' \ddot{y}_r(t) + a_4' \frac{U}{D} \dot{y}_r(t) \qquad (2)$$

Hereafter we regard variable y_r as acoustic particle displacement instead of tube vibration displacement. There is a non-linear term in the left-hand side which includes squared term of acoustic particle velocity. This non-linear term is needed to explain two phenomena: first, without acoustic disturbance, vortices are shed. And second, there is a limit cycle in the amplitude of the wakes, in turn sound pressure.

(3) Interaction Force

When the tube which shed vortices is approximated as a dipole source of sound radiation, this source is considered as a reaction of the fluid force on the tube. In the process to derive equation (2), the relation between the fluid force on the tube and relative response of wake oscillator to the tube was derived as follows[16]:

$$f_v(t) = a_1 \rho D^2 \{ \ddot{w}_r(t) - \ddot{y}_r(t) \} + a_2 \rho D U \{ \dot{w}_r(t) - \dot{y}_r(t) \} \qquad (3)$$

Therefore in translating tube vibration phenomena to acoustic oscillation phenomena, this equation may be used with some modification. Here, as the tube is stationary, tube displacement y_r becomes zero or it is eliminated. And supposing the sinusoidal behavior, equation (3) becomes as follows in frequency domain:

$$f_v(\omega) \approx (a_1 \rho D^2 \omega + a_2 \rho D U) \cdot \dot{w}_r(\omega) \qquad (4)$$

This relation means that response of wake oscillator can be estimated from the fluid force on the tube, and this force may be estimated by measuring the pressure on the tube. This fact is important in the identification of parameters which represent vortex/sound interaction mechanism by statistical method. Its detail is mentioned in section 4.2.3. Here, $f_v(t)$ and $f_v(\omega)$ in the equations (3) and (4) are the fluid forces on the tube of unit length. This force is incorporated in the sound source vector in the equation (1) and sound source matrix in the equation (5), and feedback loop is formed.

4. INTERACTION MODEL : STATISTICAL APPROACH

4.1 Outline of Modeling Method

In this section, in order to consider the random characteristics of vortex shedding, statistical expression is introduced. Here, instead of equation (1), governing equation of sound field in frequency domain becomes as follows:

$$\mathbf{S_y}(\omega) = \mathbf{K}_r(\omega) \cdot \mathbf{S_{fv}}(\omega) \cdot \mathbf{K}_r^{\,H}(\omega) \qquad (5)$$

^{H}: *Hermitian operator*

$\mathbf{S}_y$ is cross spectrum matrix of acoustic particle velocity and $\mathbf{K}r$ is matrix which is composed of modal parameters of acoustic resonance modes. $\mathbf{S_{fv}}$ is sound source matrix and its elements are cross spectrum between the elements of sound source vector $\bar{f}_v$ of equation (1). Therefore only when both locations are those of tube positions, the corresponding elements of $\mathbf{S_{fv}}$ are likely to have non-zero values. Here, the main issue is to derive the formula for the cross spectrum which reflect the statistical characteristics of wake oscillator and its relation with resonant sound field. The process of this work is as follows:

a : Definition of statistical characteristics of resonant sound field

b : Expression of statistical relation between vortex shedding and acoustic resonance.

c : Derivation of sound source matrix

Here, $\mathbf{S_{fv}}$ is composed of the reaction of fluid force on the tubes. Based on the equation (4), this fluid force is described by the valuables of wake oscillator behavior. In the following sections, the process of derivation is mentioned and in deriving the equation of cross, power and coherence spectrum, detail description is shown in Appendix.

4.2 Description of Sound Source Matrix

4.2.1 Statistical Characteristics of Sound Field

As shown in Fig. 4, sound field is described by the acoustic particle velocity and it is composed of resonant component and noise component:

$$\dot{y}_t(x_i,t) = \dot{y}_r(x_i,t) + \dot{y}_n(x_i,t) \qquad (6)$$

$$\dot{y}_r(x_i,t) = \dot{y}_{r0}(x_i)\cos(\omega t + \phi_r(x_i) + \theta_r(x_i))$$

: *resonant component*

$\dot{y}_n(x_i,t)$: *noise component*

Omitting the valuable of location x_i, complex amplitude of acoustic particle velocity is expressed by,

$$\dot{y}_t(\omega) = \dot{y}_r(\omega) + \dot{y}_n(\omega) \qquad (7)$$

Hereafter, we suppose that statistical factors are expressed by the randomness of phase of acoustic particle velocity and wake oscillator and that the phase is divided by two parts; deterministic part (shown by 'ϕ') and statistical part (shown by 'θ'). In order to express these ideas, we introduce 'the standard expression' of complex amplitude of acoustic particle velocity (ref.: equation (a-2) of Appendix 1). Based on this expression, resonant component becomes;

$$\dot{y}_r(\omega) = \dot{y}_{r0}e^{i\phi_r}e^{i\theta_r} \equiv \dot{Y}_r e^{i\theta_r} \qquad \dot{Y}_r \equiv \dot{y}_{r0}e^{i\phi_r} \qquad (8)$$

$$\dot{y}_{r0} \geq 0\,, \quad \phi_r = 0 \;\; or \;\; \pi$$

Applying same expression to noise component,

$$\dot{y}_n(\omega) = \dot{y}_{n0}e^{i\phi_n}e^{i\theta_n} \equiv \dot{Y}_n e^{i\theta_n} \qquad \dot{Y}_n \equiv \dot{y}_{n0}e^{i\phi_n} \qquad (9)$$

$$\dot{y}_{n0} \geq 0\,, \quad \phi_n = 0$$

Here, $\theta_n \; (-\pi \leq -\Theta_n \leq \theta_n \leq \Theta_n \leq \pi)$ and $\theta_r \; (-\pi \leq -\Theta_r \leq \theta_r \leq \Theta_r \leq \pi)$ means statistical valuables of phase of noise and resonant component, respectively. And in order to describe the statistical characteristics of these components, we introduce probability density function of the form expressed by the equation as follows:

$$p(\theta_r) = \frac{1}{2\Theta_r}, \quad \Theta_r \approx 0 \quad \therefore \theta_r = 0 \Rightarrow \dot{y}_r(\omega) = \dot{Y}_r \qquad (10)$$

$$p(\theta_n) = \frac{1}{2\Theta_n}, \quad \Theta_n = \pi \qquad (11)$$

Equation (10) means that resonant component of acoustic particle velocity is regarded as deterministic valuable. Based on this expression, coherence spectrum between resonant component and noise component becomes zero which seems to be a reasonable result.

$$\gamma_{yr,yn} = \left(\mathrm{sinc}(\Theta_r) \cdot \mathrm{sinc}(\Theta_n)\right)^2 = (1 \times 0)^2 = 0 \qquad (12)$$

4.2.2 Statistical Characteristics Wake Oscillator

Synchronization of vortex shedding becomes remarkable as the acoustic resonance increases. This feature is incorporated by statistical expression as follows:

$$\dot{w}(t) = \dot{w}_0 \cos\{\omega t + (\phi_r - \phi_w) + \theta_w\} \qquad (13)$$

$$\dot{w}(\omega) = \dot{w}_0 \cdot e^{i(\phi_r - \phi_w)} \cdot e^{i\theta_w} = \dot{W}_r e^{i\theta_w} \qquad (14)$$

$$\dot{W}_r \equiv \dot{w}_0 \cdot e^{i(\phi_r - \phi_w)}, \qquad \phi_r = 0 \;\; or \;\; \pi \qquad (15)$$

Here, as shown in Fig. 5, statistical part of phase θ_w $(-\pi \le -\Theta_w \le \theta_w \le \Theta_w \le \pi)$ has the equal probability in the limited range, and width of this range depends on the amplitude ratio of resonance component to total component of acoustic particle velocity at the location of vortex shedding. This ratio is referred to as 'resonance index' ,R, and defined as follows:

$$R \equiv \frac{\dot{y}_{r0}(\omega_r)}{\dot{y}_{t0}(\omega_r)} \qquad (16)$$

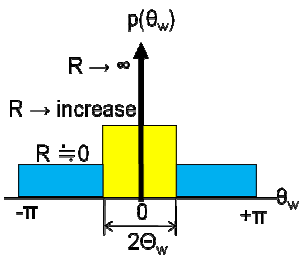

Fig.5 Probability density function of statistical component of phase of wake oscillator

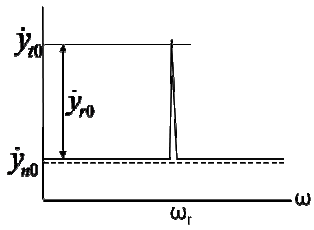

Fig. 6 Auto-spectrum of acoustic particle velocity

R may be evaluated approximately from auto-spectrum of acoustic particle velocity data shown in Fig. 6. Using this expression, feedback effect in synchronization of vortex shedding may be expressed as follows:

$$\Theta_w \propto (R^{-1})^q \quad \Theta_w = \alpha (R^{-1})^q = \alpha / R \qquad (17)$$

$$p(\theta_w) = \frac{1}{2\Theta_w} = \frac{R}{2\alpha} \quad in \ -\Theta_w \le \theta_w \le \Theta_w$$

$$= 0 \qquad in \ -\pi \le \theta_w \le -\Theta_w, \ \Theta_w \le \theta_w \le \pi$$

Here, Because of simplicity, value of q is supposed to be 1. Equation (17) means that increase of R leads to decrease of Θ_w and means that as acoustic resonance level increases, scattering of phase of vortex shedding in any location decreases. This feature seems to be rational because it coincides with the experimental facts that as acoustic resonance level increases, synchronization of vortex shedding becomes remarkable. Proportional constant α implies the sensitivity of vortex shedding to acoustic noise level and it may be interesting to evaluate this value for tube bundles of various tube pitch or tube arrangement.

4.2.3 Derivation and Identification of Sound Source Matrix

Sound source is a reaction of fluid force on the tube and according to equation (4), this force is proportional to wake oscillator velocity. Therefore instead of fluid force, we consider the behavior of wake oscillator. Applying the standard expression for wake oscillator, which is shown in equation (14), cross spectrum and coherence spectrum between wake oscillators in any two locations, becomes as follows:

$$\begin{aligned} S_{w1,w2}(\omega) &= E < \dot{w}_1^*(\omega) \cdot \dot{w}_2(\omega) > \\ &= \dot{W}_{r,1}^* \dot{W}_{r,2} \frac{\sin \Theta_{w,1}}{\Theta_{w,1}} \frac{\sin \Theta_{w,2}}{\Theta_{w,2}} \\ &= \dot{W}_{r,1}^* \dot{W}_{r,2} \cdot \mathrm{sink}(\Theta_{w,1}) \cdot \mathrm{sink}(\Theta_{w,2}) \\ &= \dot{W}_{r,1}^* \dot{W}_{r,2} \cdot \mathrm{sink}(\frac{\alpha}{R_1}) \cdot \mathrm{sink}(\frac{\alpha}{R_2}) \end{aligned} \qquad (18)$$

$$\begin{aligned} \gamma_{w1,w2}(\omega) &= \frac{S^*_{w1,w2}(\omega) \cdot S_{w1,w2}(\omega)}{S_{w1,w1}(\omega) \cdot S_{w2,w2}(\omega)} \\ &= \left(\frac{\sin \Theta_{w,1}}{\Theta_{w,1}} \right)^2 \left(\frac{\sin \Theta_{w,2}}{\Theta_{w,2}} \right)^2 \\ &= \{ \mathrm{sink}(\frac{\alpha}{R_1}) \cdot \mathrm{sink}(\frac{\alpha}{R_2}) \}^2 \end{aligned} \qquad (19)$$

It should be noted that statistical characteristics of vortex shedding synchronization is intensively represented by the proportional constant α and it should be experimentally identified.

Hereafter we mention the experimental identification method of α based on the formula of coherence spectrum between acoustic particle velocity $\dot{y}_t$ and wake velocity $\dot{w}$. Both values are measurable in the experiment.

$$\gamma_{yt,w}(\omega) = \frac{\mathrm{S}^*_{\mathrm{yt,w}}(\omega) \cdot \mathrm{S}_{\mathrm{yt,w}}(\omega)}{\mathrm{S}_{\mathrm{yt,yt}}(\omega) \cdot \mathrm{S}_{\mathrm{w,w}}(\omega)} \qquad (20)$$

Here, suffixes t, r and n mean total, resonant and noise component. The formulas of the spectra included in equation (20) are derived as follows:

$$\begin{aligned} \mathrm{S}_{\mathrm{yt,w}}(\omega) &\equiv E < \dot{y}_t(\omega)^* \cdot \dot{w}(\omega) > = E < (\dot{Y}_r + \dot{Y}_n e^{i\theta_n})^* \cdot \dot{W}_r e^{i\theta_w} > \\ &= E < \dot{Y}_r^* \cdot \dot{W}_r e^{i\theta_w} > + E < \dot{Y}_n^* e^{-i\theta_n} \cdot \dot{W}_r e^{i\theta_w} > \\ &= \dot{Y}_r^* \cdot \dot{W}_r \cdot E < e^{i\theta_w} > + \dot{Y}_n^* \cdot \dot{W}_r \cdot E < e^{-i\theta_n} \cdot e^{i\theta_w} > \\ &\approx \dot{Y}_r^* \cdot \dot{W}_r \cdot \mathrm{sinc}(\Theta_\mathrm{w}) = \dot{Y}_r^* \cdot \dot{W}_r \cdot \mathrm{sinc}(\frac{\alpha}{\mathrm{R}}) \end{aligned} \qquad (21)$$

$$\begin{aligned} \mathrm{S}_{\mathrm{yt,yt}}(\omega) &= E < (\dot{Y}_r + \dot{Y}_n e^{i\theta_n})^* \cdot (\dot{Y}_r + \dot{Y}_n e^{i\theta_n}) > \\ &= \dot{Y}_r^* \dot{Y}_r + \dot{Y}_n^* \dot{Y}_n + \dot{Y}_r^* \dot{Y}_n \cdot E < e^{i\theta_n} > + \dot{Y}_n^* \dot{Y}_r \cdot E < e^{-i\theta_n} > \\ &= \dot{Y}_r^* \dot{Y}_r + \dot{Y}_n^* \dot{Y}_n \end{aligned} \qquad (22)$$

$$\mathrm{S}_{\mathrm{w,w}}(\omega) = E < \dot{W}_r^* e^{-i\theta_n} \dot{W}_r e^{i\theta_w} > = \dot{W}_r^* \dot{W}_r \qquad (23)$$

Here, approximation in equation (21) is based on the fact that $E < e^{i\theta_w} >$ and $E < e^{-i\theta_n} \cdot e^{i\theta_w} >$ have almost same magnitude (≈1), and noise component of acoustic particle velocity is negligible comparing to resonant component. As a result the coherence function becomes as follows:

$$\gamma_{yt,w}(\omega) = \frac{\dot{Y}_r^* \dot{Y}_r \cdot \dot{W}_r^* \dot{W}_r}{(\dot{Y}_r^* \dot{Y}_r + \dot{Y}_n^* \dot{Y}_n) \cdot \dot{W}_r^* \dot{W}_r} \{\mathrm{sinc}(\frac{\alpha}{\mathrm{R}})\}^2$$
$$= \frac{\dot{Y}_r^* \dot{Y}_r}{\dot{Y}_r^* \dot{Y}_r + \dot{Y}_n^* \dot{Y}_n} \{\mathrm{sinc}(\frac{\alpha}{\mathrm{R}})\}^2 \qquad (24)$$

This formula implies that when resonance occurs, resonance index 'R' increases and coherence spectrum between acoustic particle velocity and wake oscillator velocity comes close to 1. This means the progress of vortex shedding synchronization in the tube bundles in the duct. These results seem to be a reasonable conclusion. Applying this formula to the measured data of coherence spectrum between acoustic particle velocity and wake oscillator velocity, the value of α can be identified. Here, measurement of wake oscillator velocity is difficult and it should be alternated to the force or pressure on the tube because measurement of pressure data may be more feasible with the use of pressure tap on the tube. This replacement is validated by equation (4).

It should be noted that feedback mechanism as to synchronization of vortex shedding , which is represented by resonant level dependence of the phase of sound source cross spectrum, is incorporated in the proposed model by the measured data of proportional constant α in a single location. Another feedback mechanism, which is represented by resonant level dependence of the magnitude of sound source cross spectrum, may be identified based on equation (2) and it is one of the issues in future.

5. SUMMARY

Vortex shedding synchronization effect plays important role in acoustic resonance attack in multi-tube bundle heat exchanger. For this effect, simple model is proposed which is consistent with the interaction phenomena found in past experiments. Experimental method to identify the undefined parameters in the model is also proposed. We expect that application of this method to the experiments of various tube configurations will contribute to resonance attack prediction method.

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Appendix 1 : Cross Spectrum of Independent Time Series Data

Cross spectrum of sinusoidal time series data is expressed using probability density functions as follows;

$$S_{w1,w2}(\omega) = E < w_1^*(\omega) w_2(\omega) >$$
$$= \frac{1}{N} \sum_{k=1}^{N} w^*_{1,k}(\omega) w_{2,k}(\omega)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w_1^*(\omega) w_2(\omega) p(w_1(\omega), w_2(\omega)) dw_1 dw_2 \quad (a-1)$$

$w_i(t)$: *time series data*
$w_i(\omega)$: *complex amplitude of* $w_i(t)$
$p(\quad)$: *probability density function*
N : *total sample number*

Here, two hypothesizes are introduced:

・Each time series data are statistically independent.

・Statistical factor of time series data is expressed by phase of complex amplitude.

And 'standard expression' of complex amplitude is introduced as follows:

$$w_i(\omega) = w_{i0}(\omega) e^{i\{\phi_i(\omega)+\theta_i(\omega)\}} = W_i(\omega) \cdot e^{i\theta_i(\omega)} \quad (a-2)$$

$W_i(\omega) \equiv w_{i0}(\omega) e^{i\phi_i(\omega)}$: deterministic part of complex amplitude
$w_{i0}(\omega) > 0$: amplitude
$\phi_i(\omega)$: deterministic part of phase
$\theta_i(\omega)$: statistical part of phase

As each time series data are statistically independent,

$$p(\theta_1(\omega), \theta_2(\omega)) = p(\theta_1(\omega)) \cdot p(\theta_2(\omega)) \quad (a-3)$$

Here, next formula for cross spectrum (and auto power spectrum) is delivered:

$$\begin{aligned} S_{w1,w2}(\omega) &= E < w_1^*(\omega) > E < w_2(\omega) > \\ &= W_1^*(\omega) \int_{-\pi}^{\pi} e^{-i\theta_1(\omega)\}} p(\theta_1(\omega)) d\theta_1 \cdot W_2(\omega) \int_{-\pi}^{\pi} e^{i\theta_2(\omega)\}} p(\theta_2(\omega)) d\theta_2 \\ &= W_1^*(\omega) \cdot E < e^{-i\theta_1(\omega)} > \cdot W_2(\omega) \cdot E < e^{i\theta_2(\omega)} > \end{aligned} \quad (a-4)$$

$$\begin{aligned} S_{wi,wi}(\omega) &= E < w_i^*(\omega) w_i(\omega) > \\ &= W_i^*(\omega) W_i(\omega) \int_{-\pi}^{\pi} e^{-i\theta_i(\omega)\}} e^{i\theta_i(\omega)} p(\theta_i(\omega)) d\theta \\ &= W_1^*(\omega) W_1(\omega) \end{aligned} \quad (a-5)$$

Here, we suppose that probability density function is expressed as follows:

$$\begin{aligned} p(\theta_i) &= \frac{1}{2\Theta_i} \quad for \quad -\Theta_i \le \theta_i \le \Theta_i \quad (0 \le \Theta_i \le \pi) \\ p(\theta_i) &= 0 \qquad for \quad other\, value\, of\, \theta_i \end{aligned} \quad (a-6)$$

Based on this condition, and using the formula in appendix 2, cross spectrum, power spectrum and coherence spectrum becomes as follows (ω is omitted):

$$\begin{aligned} S_{w1,w2}(\omega) &= W_1^* W_2 \frac{\sin \Theta_1}{\Theta_1} \frac{\sin \Theta_2}{\Theta_2} \\ &= W_1^* W_2 \operatorname{sinc}(\Theta_1) \cdot \operatorname{sinc}(\Theta_2) \end{aligned} \quad (a-7)$$

$$S_{wi,wi}(\omega) = W_i^* W_i = |W_i|^2 \quad (a-8)$$

$$\begin{aligned} \gamma_{w1,w2}(\omega) &= \frac{S^*_{w1,w2} \cdot S_{w1,w2}}{S_{w1,w1} \cdot S_{w2,w2}} \\ &= \frac{W_1 W_2^* \frac{\sin \Theta_1}{\Theta_1} \frac{\sin \Theta_2}{\Theta_2} \cdot W_1^* W_2 \frac{\sin \Theta_1}{\Theta_1} \frac{\sin \Theta_2}{\Theta_2}}{W_1^* W_1 \cdot W_2^* W_2} \\ &= \left(\frac{\sin \Theta_1}{\Theta_1}\right)^2 \left(\frac{\sin \Theta_2}{\Theta_2}\right)^2 = \{\operatorname{sinc}(\Theta_1) \cdot \operatorname{sinc}(\Theta_2)\}^2 \end{aligned} \quad (a-9)$$

$$(\therefore \quad 0 \le \gamma_{w1,w2}(\omega) \le 1)$$

Appendix 2 : Formula for E<exp(i θ)> and E<exp(-i θ)>

In the case that probability density function is expressed by equation (a-6), equations below are derived;

$$\begin{aligned} E < e^{i\theta} > &= \int_{-\pi}^{\pi} e^{i\theta} p(\theta) d\theta \\ &= \frac{1}{2\Theta} \int_{-\Theta}^{\Theta} e^{i\theta} d\theta = \frac{1}{2\Theta} \cdot \left. \frac{e^{i\theta}}{i} \right|_{-\Theta}^{\Theta} \\ &= \frac{\sin \Theta}{\Theta} = \operatorname{sinc}(\Theta) \end{aligned} \quad (a-10)$$

$$\begin{aligned} E < e^{-i\theta} > &= \int_{-\pi}^{\pi} e^{-i\theta} p(\theta) d\theta \\ &= \frac{1}{2\Theta} \int_{-\Theta}^{\Theta} e^{-i\theta} d\theta = \frac{1}{2\Theta} \cdot \left. \frac{e^{-i\theta}}{-i} \right|_{-\Theta}^{\Theta} = \operatorname{sinc}(\Theta) \end{aligned} \quad (a-11)$$

$$\operatorname{sinc}(0) = 1, \qquad \operatorname{sinc}(\pi) = 0 \qquad 0 \le \operatorname{sinc}(\Theta) \le 1$$