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OUT-OF-PLANE VIBRATION OF A CURVED PIPE DUE TO PULSATING FLOW (NONLINEAR INTERACTIONS BETWEEN IN-PLANE AND OUT-OF-PLANE VIBRATIONS)

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ABSTRACT

A great deal of study has been done on the dynamics of straight pipes conveying fluid. In contrast, only a few studies have been devoted to the dynamics of curved pipe conveying fluid. In this paper, a theoretical and experimental investigation was conducted into out-of-plane vibration of a curved pipe for the case that the fluid flow contains a small time-dependent harmonic component. The nonlinear out-of plane vibrations of a curved pipe, which is hanging horizontally and is supported at both ends, are examined when the frequency of the pulsating fluid flow is near twice the fundamental natural frequency of out-of-plane vibration. The main purpose of this paper is to investigate the nonlinear interactions between the in-plane and the out-of-plane vibrations analytically and experimentally. The partial differential equations of out-of-plane motions are reduced into a set of ordinary differential equations, which govern the amplitude and phase of out-of-plane vibration, using the method of Lyapnov-Schmidt reduction. It is clarified that the excitation of the in-plane vibration produces significant responses in the out-of-plane vibrations. Finally, the experiments were conducted with a silicon rubber pipe conveying water. The typical features of out-of-plane vibration are confirmed qualitatively by experiment.

NOMENCLATURE

u Displacement in the radial direction
v Out-of-plane displacement
w Displacement in the axial direction
χ Angle of rotation around the pipe axis
Π Combined force
V_s Dimensionless mean flow velocity
α Ratio of the flexural rigidity to the torsional rigidity
β Ratio of the fluid mass to the total mass
γ Ratio of the gravity force to the elastic force
μ Dimensionless polar moment of inertia
ε Dimensionless amplitude of the pulsating fluid flow
ν Dimensionless frequency of the pulsating fluid flow

1 INTRODUCTION

The vibration and stability of a flexible pipe conveying fluid, which has been the subject to many investigations, is one of the attractive phenomena from the viewpoint of nonlinear dynamics. Presently, the vibration and stability of a flexible pipe conveying fluid are a new paradigm in dynamics, providing a field to search for new dynamical features and to develop mathematical techniques [1]. Many of these examined the dynamics of straight pipes, which was restricted in a plane. In contrast, only a few

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studies have been devoted to the dynamics of curved pipe conveying fluid although the heat exchanger tubes and system piping are vulnerable to flow-induced vibration, especially in the U-bend regions. Geometrical considerations make the analysis somewhat more difficult. There are many points which must be clarified. In the present situation, the literature on nonlinear vibrations of fluid-conveying curved pipes is limited. Nonlinear out-of-plane vibration is one of the problems to be solved.

Theoretical developments in the field of out-of-plane vibration of a curved pipe conveying fluid started with the work of Chen [2, 3]. Chen conducted a theoretical linear study into in-plane and out-of-plane vibration of the curved pipe. Subsequently, Misra re-examined the linear dynamics and stability of fluid-conveying curved pipes with the idea of the modified inextensible theory [4]. The equations of motion could be derived concerning a curved pipe by Dupuis and Rousselet [5]. These equations were not written in the form of pipe displacements. In recent years, some papers deal with nonlinear dynamics of the curved pipe conveying fluid. Wang take up the problem of the harmonic excitation at the free-end of the curved pipe [6]. Among a series of studies on the dynamics of the curved pipe conveying fluid, mentioned above, some theoretical studies have been carried out, especially on the linear stability analysis, but only few have been followed up by experiments.

In this paper, the theoretical and experimental investigation conducted herein deals with the nonlinear out-of-plane vibrations of a curved pipe conveying fluid, from the view point of nonlinear dynamics. The in-plane and out-of-plane vibrations for the curved pipe are discussed under the influence of gravity. This pipe, clamped both ends and situated vertically, conveys fluid, whose velocity has a small time-dependent harmonic component. For this case, the in-plane vibrations are forced excited due to the pulsating component of the fluid flow and this flow may also induce the parametric resonance of the out-of-plane pipe vibration. The out-of-plane vibration and a torsional vibration around a pipe axis are coupled through linear and nonlinear terms. On the other side, the out-of-plane vibration is coupled with in-plane vibration only through nonlinear terms. The effects of the nonlinear interactions between in-plane and out-of-plane vibrations are elucidated in this paper.

To begin, we discuss the effects of gravity on the static equilibrium state of the curved pipe conveying fluid. The equations govern the nonlinear dynamics around the static equilibrium state are derived. The complex amplitude equation of the out-of-plane pipe vibration in case of the principal parametric resonance is derived by using the orthogonal condition between the eigenfunction and its adjoint function of the governing equation of the pipe vibration. The out-of-plane vibration is excited by the pulsating fluid flow and the nonlinear interaction between in-plane and out-of-plane vibrations.

Finally, the spatial motions of a curved pipe due to the pulsating fluid flow were observed by using the image processing system, which was based on images from two CCD cameras. The flow velocities were also measured with the electro-magnetic flow-meter. The deflections of the pipe were measured under quasi-stationary sweep of the frequency of the pulsating fluid flow, which was nearly twice the natural frequency of the out-of-plane vibration for the first mode, i.e. principal parametric resonance. The pipe motions resulting from the nonlinear analysis and experiments were complex and had several bifurcations.

2 ANALYTICAL MODEL AND BASIC EQUATIONS
2.1 Analytical model

The system under consideration (Fig.1) consists of a flexible curved pipe, built-in at both ends. The pipe conveys an incompressible fluid which discharges to atmosphere from one end. The pipe is flexible in bending yet inextensible. It also has uniform circular cross section and a length far in excess of its diameter. The pipe, of length ℓ, flexural rigidity EI, torsional rigidity GJ, mass per unit length m_p and bore area S_f is hung vertically under the influence of gravity g in its static state. The gravity force acting on the pipe is not neglected compared with the restoring force due to the bending rigidity of the pipe. The axial flow velocity v_f relative to the pipe motion is assumed as follows:

$$v_f = v_s + \delta v = v_s(1+\varepsilon \sin Nt) \tag{1}$$

where v_s is a mean flow velocity, ε is an amplitude of the pulsating flow component and N is the frequency of the pulsating flow. The density of fluid is ρ_f.

The pipe motions investigated herein are three-dimensional. The coordinate system used has its origin O at the center of the pipe at one of the built-in ends. The OX direction is along the downward vertical axis. The OY is in a horizontal plane and another built-in end is taken on the extension of the line OY. The direction OZ completes the Cartesian set. The pipe is initially in X-Y plane, having an arbitrary centerline shape, i.e. the radius of curvature is R_0.

We define the angle of rotation around the pipe axis χ and combined force Π. The combined force $\Pi = pS_f - T$ involves both axial pipe tension T and pressure force pS_f. The pipe displacements, angle of rotation and Π are expressed as functions of coordinate s along the pipe axis and time t. The equations governing the spatial behavior of the pipe are derived under the assumption that the pipe axis is inextensible [7].

Shown in figure 2, $\boldsymbol{e}_{z0}$,$\boldsymbol{e}_{x0}$ and $\boldsymbol{e}_{y0}$ are tangential, normal and binormal unit vector. A position vector d of any point on the pipe axis is expressed as follows:

$$\boldsymbol{d} = u\boldsymbol{e}_{x0} + v\boldsymbol{e}_{y0} + w\boldsymbol{e}_{z0}. \tag{2}$$

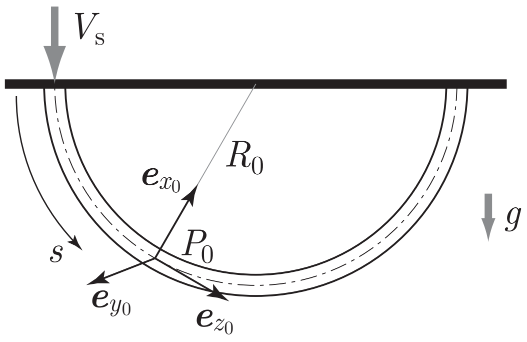


FIGURE 1. Analytical Model

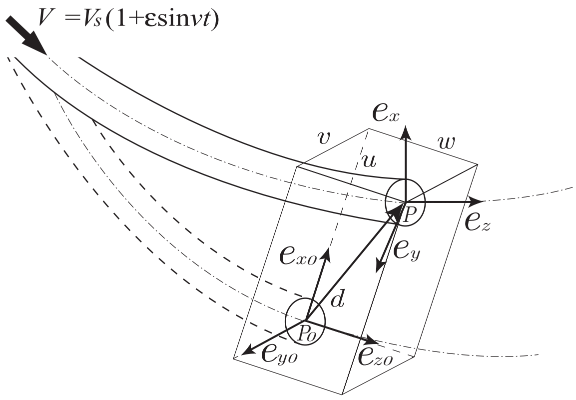


FIGURE 2. Deformation of a Curved Pipe

2.2 Basic equations

We take up the spatial behavior of the pipe under the assumptions that $u(s,t)$, $v(s,t)$ and $w(s,t)$ are small but finite. Some dimensionless variables are introduced (denoted with *): $s = R_0 s^*$, $u = R_0 u^*$, $v = R_0 v^*$, $w = R_0 w^*$, $\Pi = EI\Pi^*/R_0^2$, $t = \sqrt{(m_p+\rho_f S_f)/EI}R_0^2 t^*$. The equations governing the spatial pipe motion under the influence of gravity are expressed in the dimensionless form as follows [7]:

$$\ddot{u} - \mu(\ddot{u}'' + \ddot{w}') + 2\sqrt{\beta}V(\dot{u}' + \dot{w}) + (\Pi + V^2)(1 + u'' + w') \\ +\gamma\left\{\cos s(u' + w) + \sin s\right\} + u'''' + w''' = n_1(u,v,w,\chi,\Pi) \quad (3)$$

$$\ddot{v} - \mu\ddot{v}'' + v'''' - \chi'' - \alpha(\chi'' + v'') + 2\sqrt{\beta}V\dot{v}' + (\Pi + V^2)(v'' \\ -\chi) + \gamma\left\{v'\cos s - \chi\sin s\right\} = n_2(u,v,w,\chi,\Pi) \quad (4)$$

$$\Pi = \Pi(0) - \int_0^s \ddot{w}ds - \mu\int_0^s(\ddot{u}' + \ddot{w})ds + u'' + w' - u''(0) \\ -\sqrt{\beta}\dot{V}s + \gamma\sin s - \gamma\int_0^s \sin s(u' + w)ds + n_3(u,v,w,\chi,\Pi) \quad (5)$$

$$v'' - \chi + \alpha\chi'' + \alpha v'' - 2\mu\ddot{\chi} = n_4(u,v,w,\chi,\Pi) \quad (6)$$

$$w' - u = n_5(u,v,w,\chi,\Pi) \quad (7)$$

where $\dot{(\cdot)}$ and $(\cdot)'$ denote derivatives with respect to t and s, respectively. n_1, n_2, n_3, n_4 and n_5 represent the nonlinear terms and are omitted on account of limited space [7]. Except for section 4, the astrisks indicating the dimensionless variables in Eq.(3) are hence forward omitted. Each of the Eq. (3) $\sim$ Eq. (6) could be identified as being related to one of the co-ordinates. Eq. (3) corresponds to the radial transverse deformation, Eq. (4) to out-of-plane transverse deformation, Eq. (5) to longitudinal displacement, and the last to the twist of the pipe. Eq. (7) express the in-extensible condition. The boundary conditions for both ends of the pipe are expressed: $u = u' = v = v' = w = w' = \chi = 0$.

As a result, the spatial behavior of the curved pipe is described by five equations and fourteen boundary conditions with respect to the unknown variables u, v, w, χ and Π. In-plane vibrations can be expressed with u and w and Out-of-plane vibrations can be expressed with v and χ. From Eq. (3) through Eq. (7), it is clear that the in-plane and the out-of-plane vibrations are only coupled with nonlinear terms.

There are seven dimensionless parameters involved in Eqs. (3) through (7), i.e., the dimensionless mean flow velocity $V_s = R_0 v_s\sqrt{\rho_f S_f/EI}$, the ratio of the flexural rigidity to the torsional rigidity of the pipe $\alpha = GJ/EI$, the ratio of the fluid mass to the total mass $\beta = \rho_f S_f/(m_p + \rho_f S_f)$, the ratio of the gravity force to the elastic force to the pipe $\gamma = (m_p + \rho_f S_f)R_0^3 g/EI$, the dimensionless polar moment of inertia $\mu = \rho_p I/R_0^2(m_p + \rho_f S_f)$, the dimensionless amplitude of the pulsating fluid flow $\varepsilon = max|\delta v|/v_s$ and dimensionless frequency of the pulsating fluid flow $\nu = N\sqrt{(m_p + \rho_f S_f)/EI}R_0^2$.

2.3 Static equilibrium state

In order to understand the nonlinear spatial behavior of the curved pipe around the static equilibrium state, first static equilibrium states under the gravity are discussed. In the case of steady flow velocity $V = V_s$, we assume that the pipe rests slightly away from a semicircle, under the gravity g. Neglecting the time depending terms and the nonlinear terms in Eqs. (3) $\sim$ (7) and assuming $u = u_g$, $w = w_g$, $v = \chi = 0$, $\Pi = \Pi_s$, $\Pi_s(0) = -V_s^2 + \Pi_g(0)$, the following equations are obtained.

$$u_g'''' + w_g''' + (\Pi_s + V_s^2)(1 + u_g'' + w_g') + \gamma\left\{\cos s(u_g' + w_g) + \right.$$

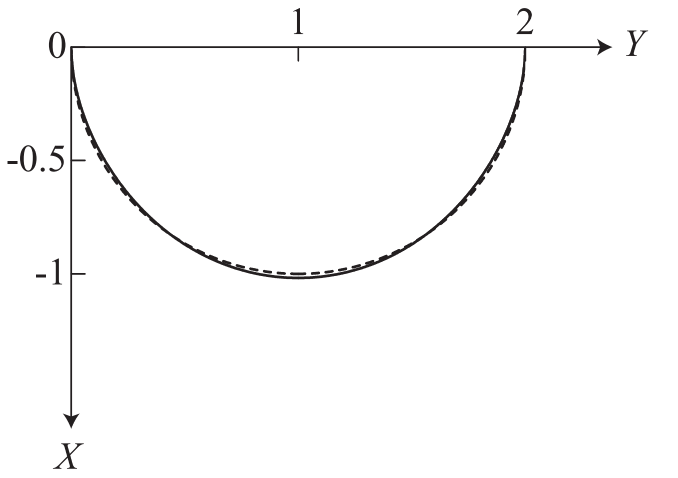


FIGURE 3. Static equilibrium state of the curved pipe (V_s=3.68,γ= 4.22,solid line:static equilibrium state, broken line:semicircle)

$$\sin s\} = 0, \tag{8}$$

$$w_g' - u_g = 0, \tag{9}$$

$$\Pi_s = -V_s^2 + \Pi_g(0) + u_g'' + w_g' - u_g''(0) + \gamma \sin s$$
$$-\gamma \int_0^s \sin s(u_g' + w_g)ds. \tag{10}$$

This boundary value problem describes small static deflections of a curved pipe being pulled downward by a gravitational force. This boundary value problem are solved using a power series of s satisfying the Eqs. (8) $\sim$ (10). $\Pi_g(0)$ is a reaction force acting on the origin O and is estimated to match the amount of sag $u_g(\pi/2)$ with experimental one.

Figure 3 shows the static equilibrium state of the curved pipe under the influence of gravity (V_s=3.68, γ= 4.22). The solid and broken lines show the static equilibrium state and a semicircle, respectively. At the middle of the span, the pipe deflects downward. Since an in-extensible condition is assumed, the pipe deflects inside the semicircle around $s = \pi/4$ and $s = 3\pi/4$. In this case, even though under the influence of gravity, the pipe holds almost a semicircular shape from sagging.

2.4 Equations of motion around the static equilibrium state

The dynamics of small deformation of the curved fluid-conveying pipe is investigated around its static equilibrium state. The unknown variables u, v, w, χ and Π of the Eqs. (3) through (7) are assumed as $u = u_s + u_d$, $v = v_s + v_d$, $w = w_s + w_d$,$\chi = \chi_s + \chi_d$ and $\Pi = \Pi_s + \Pi_d$. Keeping the nonlinear terms up to the third order with respect to u_d, v_d, w_d, Π_d and χ_d in Eqs. (3) $\sim$ (7), the differential equations of u_d, v_d, w_d, Π_d and χ_d are derived as follows:

$$\ddot{u}_d - \mu(\ddot{u}_d'' + \ddot{w}_d') + u_d'''' + w_d''' + 2\sqrt{\beta}V\dot{u}_d' + 2\sqrt{\beta}V\dot{w}_d$$
$$-\int_0^s \ddot{w}_d ds + u_d'' + w_d' - \mu\int_0^s (\ddot{u}_d' + \ddot{w}_d)ds - \sqrt{\beta}\dot{V}s$$
$$-\gamma\int_0^s (u_d' + w_d)\sin s ds + (\Pi_g(0) - \sqrt{\beta}\dot{V}s + \gamma \sin s)(u_d'' + w_d')$$
$$+\gamma(u_d' + w_d)\cos s - u_d''''(0) - w_d'''(0) - \Pi_g(0)u_d''(0) + \mu\ddot{u}_d'(0)$$
$$-u_d''(0) = N_1 \tag{11}$$

$$\ddot{v}_d + v_d'''' - \chi_d'' - \alpha(\chi_d'' + v_d'') - \mu\ddot{v}_d'' + 2\sqrt{\beta}V\dot{v}_d' + (v_d'' - \chi_d)$$
$$(\Pi_g(0) - s\sqrt{\beta}\dot{V} + \gamma \sin s) + \gamma(v_d' \cos s - \chi_d \sin s) = N_2 \tag{12}$$

$$\Pi_d = \Pi_d(0) - s\sqrt{\beta}\dot{V} - \gamma\int_0^s (u_d' + w_d)\sin s ds + w_d' + u_d''$$
$$-u_d''(0) - \int_0^s \ddot{w}_d ds - \mu\int_0^s (\ddot{u}_d' + \ddot{w}_d)ds + N_3 \tag{13}$$

$$v_d'' - \chi_d + \alpha\chi_d'' + \alpha v_d'' - 2\mu\ddot{\chi}_d = N_4 \tag{14}$$

$$w_d' - u_d = N_5 \tag{15}$$

where N_1, N_2, N_3, N_4 and N_5 are nonlinear terms and can be expressed with u_d, v_d, w_d and χ_d. In these equations, it was assumed that the influence of the polar moment of inertia was small (μ=0). Out-of-plane vibration and torsional vibration described by v_d and χ_d are only coupled through nonlinear terms with in-plane vibrations described u_d and w_d.

From the physical discussion of the Eqs. (11) $\sim$ (15), pulsating fluid flow excites the in-plane vibration and may also induce the parametric resonance of the out-of-plane pipe vibration. Forced excitation term of the in-plane vibration is $-\sqrt{\beta}\dot{V}s$ in Eq. (11). Terms, which may induce parametric resonance of out-of-plane vibration are $2\sqrt{\beta}V\dot{v}_d'$ and $-s\sqrt{\beta}\dot{V}(v_d'' - \chi_d)$.

The out-of-plane vibration is coupled with in-plane vibrations through nonlinear terms. Therefore, the forced excitation of the in-plane vibration most likely affects the behavior of the out-of-plane vibration through nonlinear terms. In the case that ν is nearly twice the natural frequency of out-of-plane vibration since the in-plane and out-of-plane vibrations could be excited with frequency of ν and $\nu/2$ respectively, the second order nonlinear terms in N_2 and N_4 affects the occurrence of out-of-plane vibration and torsional vibration. Some characteristic terms in N_2 and N_4 are indicated as follows: $N_2 = -2w_d v_d' - 2\chi_d' w_d'' - 10\alpha\chi_d' u_d''' + \cdots - \chi_d v_d'^2/2 - 3v_d' v_d'' v_d''' + \cdots$, $N_4 = -w_d v_d'/2 + \chi_d w_d' + \cdots$.

3 SOLUTION METHOD

3.1 Eigenvalue problem

Letting $\varepsilon = 0$ and Neglecting the nonlinear terms in Eqs. (11) $\sim$ (15), v_d and χ_d are independent of u_d and w_d. Eliminating variable u_d from two equations for u_d and w_d, we obtained following linearlized equation about w_d.

$$\ddot{w}_d'' - \ddot{w}_d + w_d'''''' + (2 + \Pi_g(0) + \gamma \sin s) w_d'''' + (1 - \gamma \sin s + \Pi_g(0)) w_d'' + 2\sqrt{\beta} V_s (\dot{w}_d''' + \dot{w}_d') - 2\gamma w_d \sin s + 2\gamma w_d''' \cos s + 2\gamma w_d' \cos s = 0 \tag{16}$$

Letting $w_d = \Phi_w e^{\Lambda t}$, $v_d = \Phi_v e^{\lambda t}$ and $\chi_d = \Phi_\chi e^{\lambda t}$, we can form the eigenvalue problems. In order to understand the nonlinear spatial behavior of the curved pipe, in the next section, a bifurcation analysis is to be carried out. In this analysis, the linear system and its solution play an important role. We give an outline of an eigenvalue problem for out-of-plane vibration. The eigenvalue problem associated with the out-of-plane vibration is described as follows:

$$\lambda^2 \Phi_v + 2\sqrt{\beta} V_s \lambda \Phi_v' + \Phi_v'''' - \Phi_\chi'' - \alpha(\Phi_\chi'' + \Phi_v'') + (\Pi_g(0) + \gamma \sin s)(\Phi_v'' - \Phi_\chi) + \gamma(\Phi_v' \cos s - \Phi_\chi \sin s) = 0, \tag{17}$$

$$(1+\alpha)\Phi_v'' - \Phi_\chi + \alpha \Phi_\chi'' = 0. \tag{18}$$

Boundary conditions for both ends of the pipe are: $\Phi_v = \Phi_v' = \Phi_\chi = 0$.

The eigenvalues Λ and λ are the roots of the complex characteristic equations, symbolically represented $f(\Lambda : V_s, \beta, \gamma, \Pi_g(0))$=0 and $g(\lambda : V_s, \alpha, \beta, \gamma, \Pi_g(0))$=0, and can be found numerically. The eigenvalues Λ and λ are equal to $i(\Omega_r + i\Omega_i)$ and $i(\omega_r + i\omega_i)$ respectively; Ω_r and ω_r is the linear natural frequencies and Ω_i and ω_i are the damping ratio. The complex eigenfunction Φ_w, Φ_v and Φ_χ are $\Phi_{wr} + i\Phi_{wi}$, $\Phi_{vr} + i\Phi_{vi}$ and $\Phi_{\chi r} + i\Phi_{\chi i}$ respectively.

Figure 4 shows the natural frequency Ω_r and ω_r as a function of V_s for the first mode of the system in the case of $\alpha = 0.667$, $\beta = 0.401$ and $\gamma = 0, 4.22$. The solid line denotes the theoretical results in the case of γ=4.22. The points • denote the experimental results. The theoretical results are good agreement with the experimental ones. The parameter values used in this calculation corresponds to the experimental ones. In the case of V_s=3.7, Ω_r and ω_r are 4.7 and 2.5, respectively. These values are used in the following section, are corresponding to the experimental data.

These theoretical results and results obtained from modified inextensible theory [4] show similar tendencies. Flow tends to reduce the first-mode natural frequencies of in-plane and out-of-plane vibrations, but does not cause divergence in the flow range investigated.

The following section uses the eigenfunctions Φ_v and Φ_χ for the first mode. These are represented using a power series of s. Figure 5 and Figure 6 shows the first mode shape Φ_v and Φ_χ. The eigenfunctions ϕ_v and ϕ_χ are the complex functions. ϕ_v and ϕ_χ are unsymmetric. Since Eq. (17) and Eq. (18) are non-self-adjoint equations, to obtain the periodic solution of the nonlinear problem we need the adjoint to the linear problem.

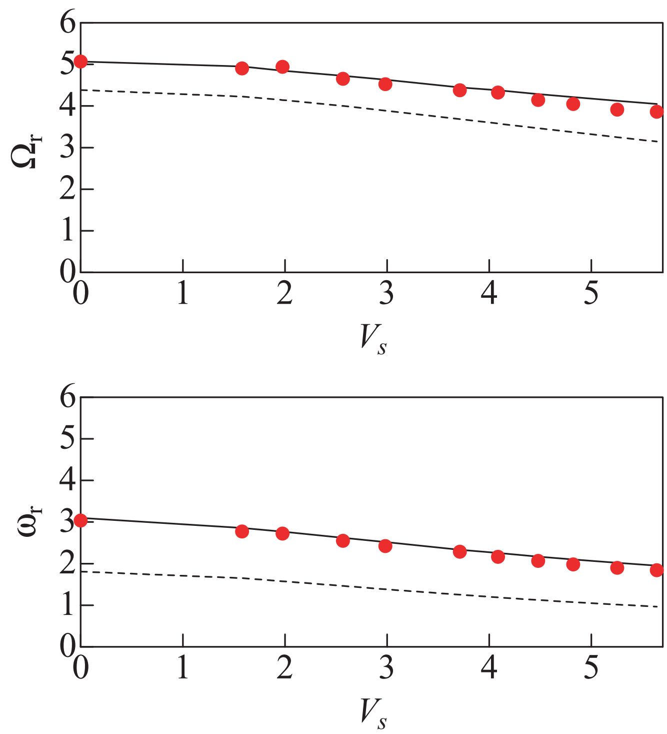


FIGURE 4. The natural frequency Ω_r and ω_r as functions of flow velocity for the first mode($\alpha = 0.667$, $\beta = 0.401$, solid line: theoretical result $\gamma = 4.22$, broken line: theoretical result γ=0, •:experimental result)

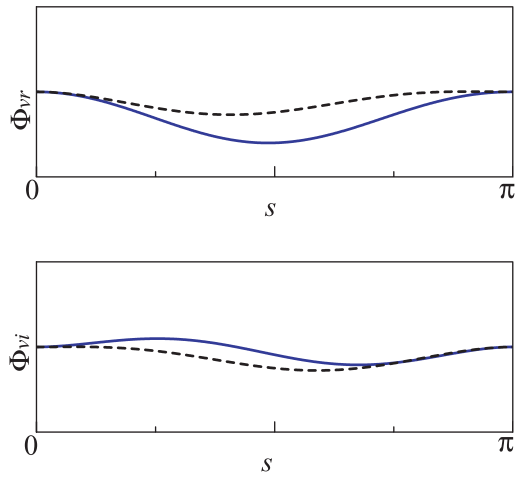


FIGURE 5. First mode shape Φ_v (V_s=3.68, solid line:$\gamma = 4.22$, broken line: γ=0)

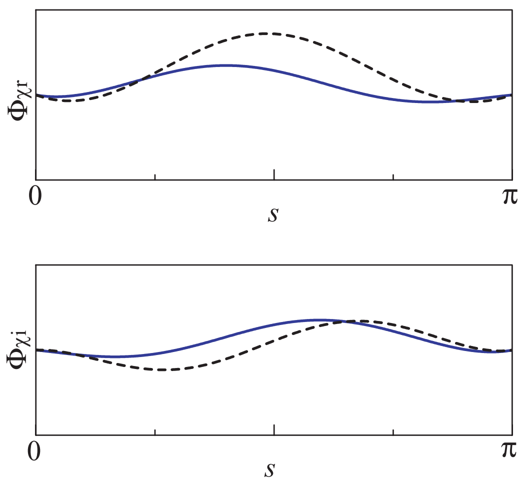


FIGURE 6. First mode shape Φ_χ (V_s=3.68, solid line:$\gamma = 4.22$, broken line: γ=0)

3.2 Forced excitation of in-plane vibration

We focused on the case where the natural frequency of the in-plane vibration slightly off from twice the natural frequency of the out-of-plane vibration. Therefore, it is not necessary to consider the influence of the out-of-plane vibration on the in-plane vibration at the first approximate solution, from the physical estimation and ordering of the governing equation.

Therefore, the influences of a pulsating fluid flow on the in-plane vibrations will briefly be considered first. In the case of $\varepsilon \neq 0$, forced in-plane vibration is governed the following equation:

$$\begin{aligned}&\ddot{w}_d'' - \ddot{w}_d + w_d'''''' + (2+\Pi_g(0)+\gamma\sin s)w_d'''' + (1-\gamma\sin s\\&+\Pi_g(0))w_d'' + 2\sqrt{\beta}V_s(\dot{w}_d''' + \dot{w}_d') - 2\gamma w_d\sin s + 2\gamma w_d''\cos s\\&+2\gamma w_d'\cos s - c_w\dot{w}_d = \sqrt{\beta}\dot{V}\end{aligned} \quad (19)$$

where the term of right hand side Eq. (19) produce a excitation term. Boundary conditions for both ends of the pipe are: $w_d = w_d' = w_d'' = 0$. In the presence of pulsating fluid flow $V = V_s(1+\varepsilon\sin\nu t)$, we assume that w_d is expressed as follows:

$$w_d = A(s)\sin\nu t + B(s)\cos\nu t. \quad (20)$$

Substituting Eq. (20) into Eq. (19), a set of equations associated A and B are obtained. A and B are determined using a power series of s. Figure 7 shows the frequency response of the in-plane vibration. The in-plane vibrations are excited at its own characteristic frequency ($\Omega_r = 4.7$) due to pulsating fluid flow.

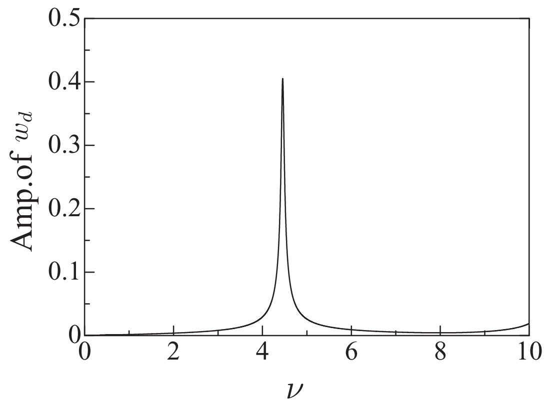


FIGURE 7. Frequency response curve of amp. of w_d (V_s=3.68,ε =0.05,c_w=0.091)

3.3 Equations in vector form and adjoint vector

By defining $\boldsymbol{v} = {}^t[v_d, \chi_d]$, the governing equations of the out-of-plane vibration are expressed in the vector form as follows:

$$\begin{bmatrix}\ddot{v}_d\\0\end{bmatrix} = \boldsymbol{L}\boldsymbol{v} + \boldsymbol{R}\boldsymbol{v} + \boldsymbol{N} \quad (21)$$

where

$$L = \begin{bmatrix} L_{11} & L_{12} \\ (1+\alpha)(\cdot)'' & -1+\alpha(\cdot)'' \end{bmatrix}, \tag{22}$$

$$R = \begin{bmatrix} -2\sqrt{\beta}V_d(\dot{\cdot})' + s\sqrt{\beta}\dot{V}_d(\cdot)'' & -s\sqrt{\beta}\dot{V}_d \\ 0 & 0 \end{bmatrix}, \tag{23}$$

$$\boldsymbol{N} = \begin{bmatrix} N_2(u_d, v_d, w_d, \chi_d) \\ N_4(u_d, v_d, w_d, \chi_d) \end{bmatrix}, \tag{24}$$

and $L_{11} = -(\cdot)'''' + \{\alpha - \Pi_g(0) - \gamma \sin s\}(\cdot)'' - 2\sqrt{\beta}V_s(\dot{\cdot})' - \gamma \cos s(\cdot)'$, $L_{12} = (1+\alpha)(\cdot)'' + \{\Pi_g(0) + 2\gamma \sin s\}$. L and R are the linear operators and vector $\boldsymbol{N}$ represents nonlinear terms. R contains the parametric resonant terms due to pulsating fluid flow. The boundary conditions for both ends are $v_d = v_d' = \chi_d = 0$.

The following section uses the foregoing eigenfunctions $\boldsymbol{q} = {}^t(\Phi_v, \Phi_\chi)$ for the first mode. These are represented using a power series of s satisfying the condition $< \boldsymbol{q}, \boldsymbol{q} >$=1 . Here, the brackets denote the inner product $< \boldsymbol{x}, \boldsymbol{y} >= \int_0^\pi {}^t\boldsymbol{x}\bar{\boldsymbol{y}} ds$.

Moreover, from the condition:$< L\boldsymbol{q}, \boldsymbol{q}^* >=< \boldsymbol{q}, L^*\boldsymbol{q}^* >$, we can determine the adjoint vector $\boldsymbol{q}^* = {}^t(\psi_v, \psi_\chi)$ of $\boldsymbol{q}$. This, expressed as a power series of s, also satisfies the condition $< \boldsymbol{q}, \boldsymbol{q}^* >$=1.

3.4 Nonlinear stability

In this section, the nonlinear first-order ordinary differential equations, which govern the amplitudes and phases of the parametric resonance for the out-of-plane vibrations, are derived. The analysis is performed by Lyapunov-Schmidt reduction. In order to analyze the parametric resonance of out-of-plane vibrations, it is assumed that the frequency ν is near twice the natural frequency of the out-of-plane vibration. Therefore, ν is expressed as follows:

$$\nu = 2\omega_r(1 + \varepsilon\sigma). \tag{25}$$

The Banach space, which includes unstable vibration mode component $\boldsymbol{v}$, is expressed as $\boldsymbol{Z} = \boldsymbol{X} \oplus \boldsymbol{M}$ [8]. $\boldsymbol{X}$ is the eigenspace spanned by the eigenvectors $\boldsymbol{q}$, which correspond to the linear vibration mode parametrically excited. $\boldsymbol{M}$ is the subspace of $\boldsymbol{X}$. Therefore $\boldsymbol{v}$ can be expressed as follows:

$$\boldsymbol{v} = A(t)\boldsymbol{q}(s) + \boldsymbol{y} + c.c. \tag{26}$$

where $\boldsymbol{y}$ is the elements of $\boldsymbol{M}$. So $\boldsymbol{y}$, which is spanned by the linear stable vibration modes, become zero with time. We assume that all other mode vibrations are not parametrically excited, since $\varepsilon << 1$. So, although the system may be unstable, the growth-rate of the unstable vibrations is so small that the system could be treated by the weakly nonlinear theory. The schematic image of the adjoint operator is shown in Figure 8. Using the projection P onto $\boldsymbol{X}$, Eq. (21) with boundary conditions, decomposes as follows:

$$P \begin{bmatrix} \ddot{v_d} \\ 0 \end{bmatrix} = PL\boldsymbol{v} + PR\boldsymbol{v} + P\boldsymbol{N} \tag{27}$$

where $P\boldsymbol{x} =< \boldsymbol{x}, \boldsymbol{q}^* > \boldsymbol{q}$.

From Eq.(27), the complex amplitude equation of the out-of-plane pipe vibration in the case of the principal parametric resonance are derived. Letting the amplitude $A(t)$ be $C(t)e^{i\nu t/2}$, ω_r is the natural frequency of the first mode of a out-of-plane vibration, to transform in autonomous system, the complex amplitude equation is derived as follows:

$$\dot{C} = -i\varepsilon\sigma\omega_r C + (\varepsilon\zeta + \xi_1)\bar{C} + \xi_2|C|^2 C. \tag{28}$$

The resonant terms $\varepsilon\zeta\bar{C}$ in Eq. (28) is arisen from the preceding resonant terms due to pulsating fluid flow. Another resonant term $\xi_1\bar{C}$ is arisen from the second order nonlinear interaction terms between in-plane and out-of-plane vibration. The coefficient ξ_2 is determined by the third order nonlinear terms.

Letting $C = ae^{i\phi}/2$, separating the real and imaginary parts of Eq. (28), and averaging them by the period $4\pi/\nu$, we obtain the nonlinear first-order ordinary differential equations which govern the amplitude a and the phase ϕ as follows:

$$\dot{a} = (\varepsilon\zeta_r + \xi_{1r})a\cos 2\phi + (\varepsilon\zeta_i + \xi_{1i})a\sin 2\phi + \xi_{2r}\frac{a^3}{4} \tag{29}$$

$$\dot{\phi} = -\varepsilon\sigma\omega_r - (\varepsilon\zeta_r + \xi_{1r})\sin 2\phi + (\varepsilon\zeta_i + \xi_{1i})\cos 2\phi + \xi_{2i}\frac{a^2}{4} \tag{30}$$

The pipe deflection v is expressed as follows:

$$v(s,t) = a|\Phi_v|\cos\left(\frac{1}{2}\nu t + \phi + \psi\right) \tag{31}$$

where $\psi = \tan^{-1}(\Phi_{vr}/\Phi_{vi})$. The out-of-plane vibrations are parametrically excited with half of the frequency of pulsating flow.

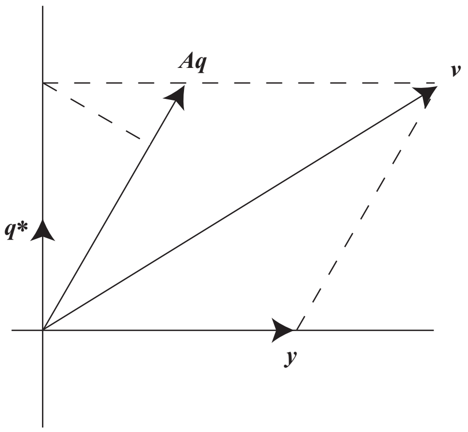


FIGURE 8. Schematic image of the adjoint operator

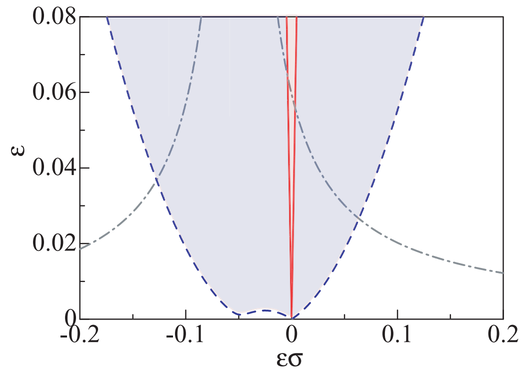


FIGURE 9. Neutral curves of the principal parametric resonance due to pulsating flow

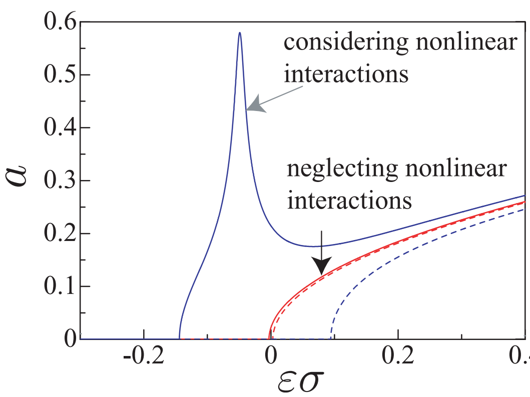


FIGURE 10. Frequency response curves of the out-of-plane vibration

Figure 9 shows the stability boundaries of the trivial solution of the out-of-plane vibration and torsional vibration. The solid and broken line each represents the stability boundaries of the trivial solution when nonlinear interactions are neglected and considered, respectively. Alternate long and short dash line represent the frequency response of the in-plane vibration. As shown in this figure, the amplitude of in-plane vibration greatly depends on $\varepsilon\sigma$. In the vicinity of $\varepsilon\sigma$ = -0.5, the amplitudes of the in-plane vibration are so large that unstable regions of out-of-plane vibrations are widely spread. It must be emphasized that the in-plane vibration are enough that non-linear interactions do come into play and change the stability limit predicted by a linear analysis.

Figure 10 shows the steady-state amplitude of the out-of-plane vibration. The solid and broken lines represent stable and unstable steady state. Nonlinear interactions increase the amplitude of out-of-plane vibration.

As shown in the figure above, the nonlinear interaction between in-plane and out-of-plane vibrations greatly affects the unstable region and the steady-state amplitude of the out-of-plane vibration. In the vicinity of $\varepsilon\sigma$ = -0.5, primary excited in-plane vibrations parametrically excited out-of-plane vibrations in spite of ν being apart from $2\omega_r$. As shown in figure 9, the amplitudes of in-plane-vibrations become small with $\varepsilon\sigma$. As the value of $\varepsilon\sigma$ increased, the influences of in-plane vibrations on out-of-plane vibrations become smaller and *a* approaches the values for the case of neglecting nonlinear interactions.

4 EXPERIMENT

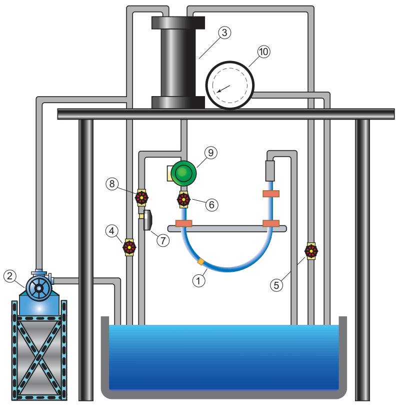
4.1 Experimental apparatus

The results of the theoretical analysis and the numerical calculations were qualitatively verified experimentally. The experimental setup is shown in figure 11. The experiments were conducted with the silicon rubber pipe of 6 mm external diameter, 4mm internal diameter and 550mm length. The distance between the both ends is 350mm. The equivalent bending rigidity EI was 3.66×10^{-4}Nm^2.The mass density ρ_f of the water was 1.0g/cm^3. The mean flow velocity v_s was 3.89 m/s. The values of α, β, γ and $\Pi_g(0)$ were determined experimentally as 0.067, 0.401, 4.22, and -7.12, respectively.

The spatial displacements of the flexible pipe were measured by the image processing system, is shown in figure 12, which could be performed measurements of the marker in a three dimensional space, based on the images from two CCD cameras. The pulsating flow velocity was measured by the electromagnetic flow meter.

4.2 Experimental results

The natural frequencies of the in-plane vibration and the out-of-plane vibration were 2.48Hz and 1.32Hz, respectively, at the flow velocity v_s=3.89m/s. The deflections u_d, v_d and w_d of



(1)Flexible pipe	(6)Flow velocity control valve
(2)Pump	(7)Electromagnetic valve
(3)Tank	(8)Pulsating flow control valve
(4)Pressure control valve	(9)Magnetic flow detector
(5)Air vent valve	(10)Pressure gauge

FIGURE 11. Diagram of experimental setup

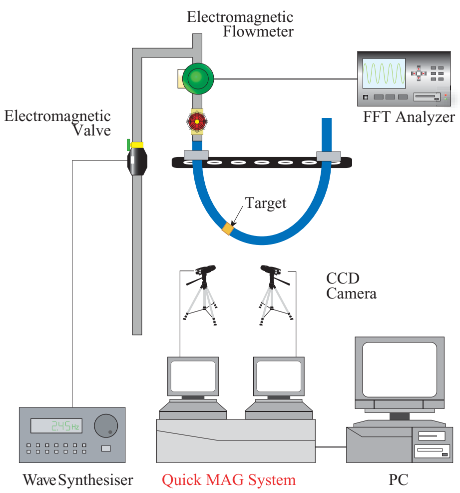


FIGURE 12. Measurement system of spatial pipe displacements

the pipe were measured under quasi-stationary sweep of the frequency of the pulsating fluid flow. In order to observe the principal parametric resonance, the frequency of the pulsating flow N was determined as near twice the natural frequency of the out-of-plane vibration for the first mode. Concerning the planned experimental verification of the nonlinear interactions between in-plane and out-of-plane vibrations, care was taken to make sure that the pipe initially is in one plane only. We conducted the experiment under quasi-stationary sweep of ν. The amplitudes of w_d and v_d settle at a constant value after a sufficiently long time and are measured.

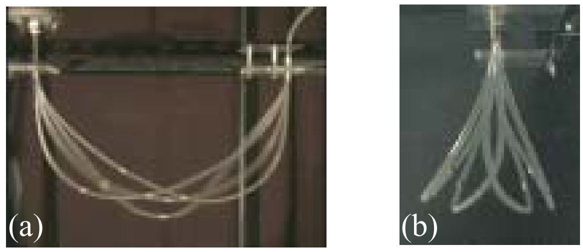


FIGURE 13. Photographs of the spatial behavior of the curved pipe, (a): in-plane vibration, (b): out-of-plane vibration

Figure 13 shows the photographs of in-plane and out-of-plane pipe vibrations at $N/2\pi$=2.68Hz. The pipe deflects in a binormal direction with a twist. Figure 14 shows the time histories of u_d, v_d, w_d and δv and their spectra analyses at $s=\pi/3$, $N/2\pi = 2.68$Hz. The dominant frequency of the out-of-plane vibration of the curved pipe was coincident with half of the frequency of the pulsating component of the internal fluid flow.

Figure 15 and Figure 16 show the frequency response curves of the steady-state amplitudes of the in-plane and the out-of-plane vibration. $\circ$ represents the result obtained for forward sweep of ν. $\bullet$ represents the result obtained for downward sweep of ν. As shown in Figure 16, the out-of-plane vibration shows characteristics of the principal parametric resonance, as predicted in the theory. The in-plane vibrations were constantly excited, while the out-of-plane vibrations were only excited when the excited frequency was near twice the natural frequency. Unstable region of the trivial solution was much greater than the results of linear stability analysis for neglecting nonlinear interactions. Furthermore, when out-of-plane vibrations were excited, the amplitude of in-plane vibration decreases. From considering these results, the nonlinear interaction between in-plane and out-of-plane vibrations was confirmed, qualitatively.

5 CONCLUSION

Out-of-plane vibration of a curved pipe has been investigated theoretically and experimentally in this paper. First, the nonlinear equations of motion around the static equilibrium state for the curved pipe are described under the influence of gravity. Second, the complex amplitude equation of the out-of-plane pipe vibration in the case of the principal parametric resonance, are derived by using the orthogonal condition between the eigenfunction and its adjoint function of the governing equation of the pipe vibration. From the analytical investigation, the following conclusions may be drawn.

(1)The parametric excitation of the out-of-plane vibration and forced excitation of the in-plane vibration most likely occur by the presence of pulsating fluid flow. The out-of-plane vibrations are only coupled with in-plane vibrations through nonlinear

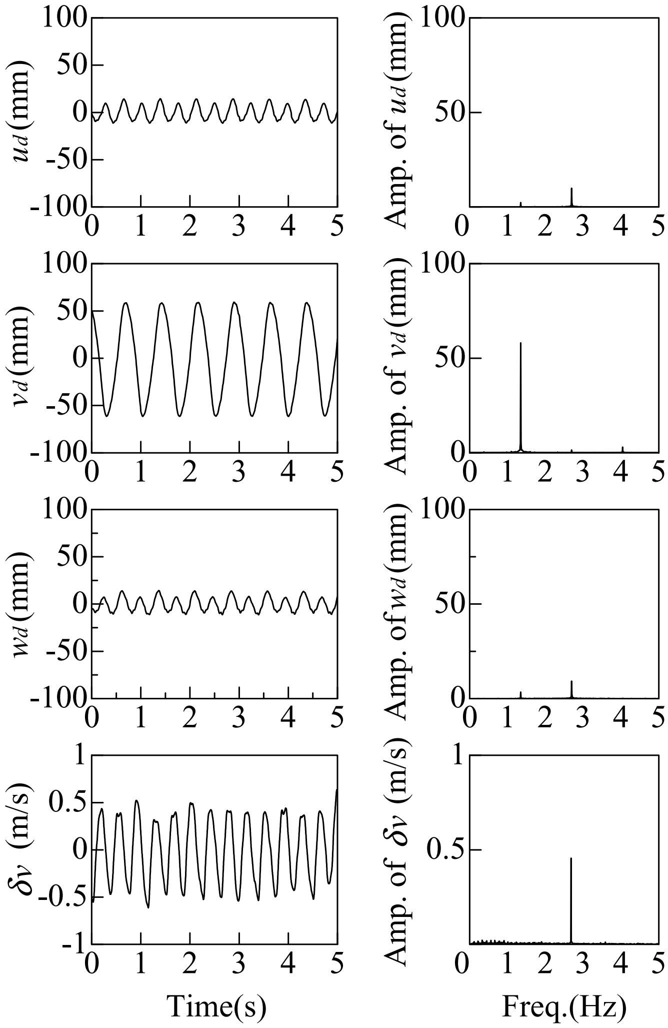


FIGURE 14. Time histories and their spectra at $N/2\pi$=2.68Hz

terms.

(2)The nonlinear interaction between in-plane and out-of-plane vibrations greatly affects the out-of-plane vibration for the principal resonance case. The excitation of the in-plane vibration produces significant responses in the out-of-plane vibration.

Finally, we conducted the experimental based on the analytical model. The deflections of the pipe were measured under quasi-stationary sweep of the frequency of the pulsating fluid flow. The frequency of the pulsating flow was near twice the natural frequency of the out-of-plane vibration for the first mode. We confirm that the in-plane vibration affects the out-of-plane vibration from the frequency responses.

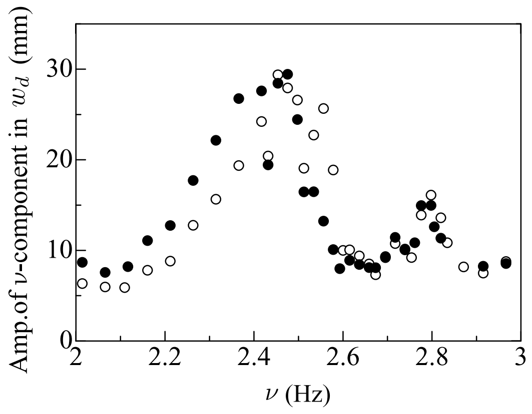


FIGURE 15. Frequency response of the in-plane vibration (○:forward sweep of ν,●:downward sweep of ν)

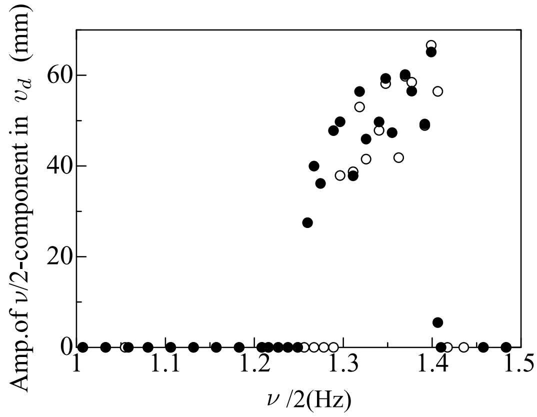


FIGURE 16. Frequency response of the out-of-plane vibration (○:forward sweep of ν,●:downward sweep of ν)

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