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EFFECT OF FLUID FLOW NONLINEARITIES ON THE DYNAMIC BEHAVIOUR OF CYLINDRICAL SHELLS SUBJECTED TO A SUPERSONIC FLOW

R. RAMZI

Department of Mechanical Engineering, Ecole Polytechnique de Montréal,C.P. 6079 Succursale Centre-ville, Montréal, H3C 3A7, Canada

A. A. LAKIS

Department of Mechanical Engineering, Ecole Polytechnique de Montréal,C.P. 6079 Succursale Centre-ville, Montréal, H3C 3A7, Canada

ABSTRACT

An analytical model is presented to predict the influence of nonlinearities associated with supersonic fluid flow on the dynamic and stability behavior of thin isotropic cylindrical shells.

The method developed is a combination between finite element method, sander’s shell theory and nonlinear aerodynamic theory (third-order piston theory). The shell is subdivided into cylindrical finite elements, the displacements functions are derived from exact solutions of Sanders equations for thin cylindrical shells and the influence of stress stiffening due to internal or external pressure and axial compression is also taken into account. Expressions for the masse and stiffness matrices are determined by exact analytical integration.

With the nonlinear dynamic pressure, we develop nonlinear matrices: stiffness, damping and coupling matrices for flow. The nonlinear equation of motion is then solved using a fourth-order Runge-kutta numerical method. Frequency variations are determined with respect to the amplitude of the motion for different cases. This is a powerful model to predict linear, nonlinear vibrations and stability characteristics of cylindrical shells subjected to external supersonic flow that can be applied for the aeroelastic design of aerospace vehicles.

INTRODUCTION

Analysis of thin shells subjected to supersonic flow has been the focus of many investigations. Most of these researches efforts have involved linear analysis of thin shells subjected to supersonic flow [1,2,3,4,5,6,7], but the obtained results are not satisfied sufficiently for accurate satisfactory design. Therefore a non-linear analysis becomes necessary [2,8]. This study presents the effect of the nonlinearities of fluid pressure in the dynamic behavior of cylindrical shells under supersonic airflow. The method is based on a combination of finite element method and linear sanders theory [9,10,11,12] and nonlinear aerodynamics theory. The finite element proposed was cylindrical (Figure 1) with four degree of freedom at each node: axial, radial, circumferential, and rotation. This geometry made it possible to use Sanders theory to determine displacements functions.

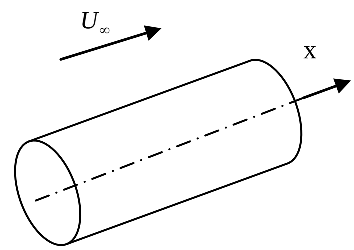


Figure1: Cylindrical shell and flow direction

This method satisfies the finite element method convergence criteria [9] and shows greater accuracy than the more usually chosen polynomial functions.

The treatment of this problem involves two steps:

1 - Using the linear strain-displacement and stress-strain relationships which were introduced into Sanders equation of equilibrium, we determined the displacement functions by solving the linear equation system. We then determined the mass and stiffness matrices for the finite element and assembled the matrices for the complete shell.

2 - From third order piston theory we derived an expression of nonlinear pressure as a function of:

- The nodal displacement of the shell element
- A combination of non linear effects.

Using this dynamic pressure, we obtained three nonlinear matrices from second order and four nonlinear matrices from third order piston theory.

NOMENCLATURE

a_∞	=	freestream speed of sound
$[B]$	=	defined by Equation 7
D	=	defined by Equation 3
E	=	Young's modulus
$\{F_p\}$	=	force vector due to the aerodynamic pressure field
h	=	shell thickness
K	=	bending stiffness of shell, defined by Equation 3
$[K]$	=	global stiffness matrix for a shell
$[K_f]$	=	global aerodynamic damping matrix
$[K_I]$	=	global initial stiffness matrix for a shell
$[k]$	=	stiffness matrix for a shell element
$[k_f]$	=	local aerodynamic stiffness matrix
$[k_I]$	=	initial stiffness matrix for a shell element
L	=	shell length
M	=	Mach number
$[M_s]$	=	global mass matrix for a shell
n	=	circumferential number
$[m]$	=	mass matrix for a shell element
N_x N_θ, $N_{x\theta}$	=	stress resultant for a circular cylindrical shell
$\overline{N}_x$, $\overline{N}_\theta$	=	stress resultant due to shell internal pressure and axial compression
$[N]$	=	defined by Equation 5
P_a	=	aerodynamic pressure
P_∞	=	freestream static pressure
P_m, P_x	=	shell internal pressure and axial compression
$[P]$	=	elasticity matrix
Q_x, Q_θ	=	transverse stress resultant for a circular cylindrical shell
R	=	shell radius
r	=	radial coordinate
U	=	axial displacement
U_i	=	potential energy due to initial strain
V	=	circumferential displacement
W	=	radial displacement
x	=	longitudinal coordinate
δ_i, δ_j	=	displacement at node *i* and *j*
λ_j	=	complex roots of the characteristic equation
υ	=	Poisson's ratio
γ	=	adiabatic exponent
θ	=	circumferential coordinate
ρ	=	shell density
ρ_f	=	fluid density
ω	=	oscillation frequency
φ_{xx}, $\varphi_{x\theta}$, φ_n	=	elastic rotations for a circular cylindrical shell
$\{\sigma\}$	=	stress vector

I - Structural model

1- Masse and Stiffness Matrices

The equations of motion of an isotropic cylindrical shell in terms of U, V, and W (axial, tangential and radial displacements) (Figure. 2) are written as follows:

$$L_J(U,W,V,P_{ij}) = 0 \tag{1}$$

Where $L_J\,(J=1,2,3)$ are three nonlinear partial differential equations presented in [13], and P_{ij} are elements of the elasticity matrix which, for an isotropic shell are given by:

$$[p] = \begin{bmatrix} D & \upsilon D & 0 & 0 & 0 & 0 \\ \upsilon D & D & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{D(1-\upsilon)}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & K & \upsilon K & 0 \\ 0 & 0 & 0 & \upsilon K & K & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{K(1-\upsilon)}{2} \end{bmatrix} \tag{2}$$

Where,

$$K = \frac{Eh^3}{12(1-\upsilon^2)}$$
$$D = \frac{Eh}{1-\upsilon^2} \tag{3}$$

The displacement functions associated with the circumferential wave number are assumed in the normal manner, as:

$$U(r,x,\theta) = \sum_n u_n(x)\cos(n\theta)$$
$$W(r,x,\theta) = \sum_n w_n(x)\cos(n\theta) \tag{4}$$
$$V(r,x,\theta) = \sum_n v_n(x)\sin(n\theta)$$

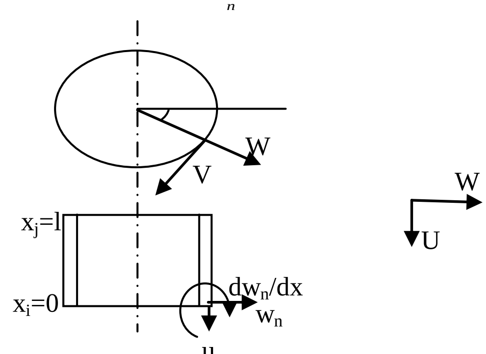


Figure 2: Finite element discretization and nodal displacement

The final form of the displacement functions was obtained from Lakis and Paidoussis study [9]:

$$\begin{Bmatrix} U(x,r,\theta) \\ W(x,r,\theta) \\ V(x,r,\theta) \end{Bmatrix} = [N] \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} \quad (5)$$

where

$$\{\delta_i\} = \begin{Bmatrix} u_{ni} \\ w_{ni} \\ (dw_n/dx)_i \\ v_{ni} \end{Bmatrix} \quad (6)$$

and [N] represents the displacement function matrix.
The constitutive relation between the stress and deformation vector of cylindrical shells is given as:

$$\{\sigma\} = [P][B] \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} \quad (7)$$

The mass and stiffness matrices for each element are derived using the classical finite element procedure and can then be expressed as:

$$\begin{aligned} [m] &= \rho h \iint [N]^T [N] dA \\ [k] &= \iint [B]^T [P][B] dA \end{aligned} \quad (8)$$

Where ρ is the density of the shell, h its thickness, $dA = r dx d\theta$. The matrices [m] and [k] are obtained analytically by carrying out the necessary matrix operations over x and θ in the equation, the global matrices $[M_s]$ and $[K_s]$ may be obtained, respectively, by superimposing the mass and stiffness matrices for each individual finite element.

2- Stress Matrix:

The potential energy formulation is given as [14]:

$$U_i = 1/2 \int_0^l \int_0^{2\pi} \left[\bar{N}_x \phi_{\theta\theta}^2 + \bar{N}_\theta \phi_{xx}^2 + (\bar{N}_x + \bar{N}_\theta) \phi_n^2 \right] R d\theta dx \quad (9)$$

Where $\bar{N}_x$, $\bar{N}_\theta$ are stress resultants due to shell internal pressure P_m and axial compression P_x respectively given by:

$$\begin{aligned} \bar{N}_x &= -\frac{P_x}{2\pi R} \\ \bar{N}_\theta &= P_m R \end{aligned} \quad (10)$$

l is the element length, ϕ_{xx} is the strain rotation about x, $\phi_{\theta\theta}$ is about normal to the $x\theta$ plane and ϕ_n is the rotation about the normal to a shell element. This rotation vector is expressed as [12]:

$$\begin{aligned} \phi_{\theta\theta} &= -\frac{\partial W}{\partial x} \\ \phi_{xx} &= \frac{1}{R}(V - \frac{\partial W}{\partial \theta}) \\ \phi_n &= \frac{1}{2}(-\frac{1}{R}\frac{\partial U}{\partial \theta} + \frac{\partial V}{\partial x}) \end{aligned} \quad (11)$$

After replacing the displacements expressions given in Eq. (5), we obtain the potential energy in terms of nodal degree of freedom, and can then obtain the initial stiffness matrix for each element in the following form:

$$[k_I] = \int_0^l \int_0^{2\pi} [N]^T [C_0]^T \begin{bmatrix} \bar{N}_x & 0 & 0 \\ 0 & \bar{N}_\theta & 0 \\ 0 & 0 & \bar{N}_x + \bar{N}_\theta \end{bmatrix} [C_0]^T [N]^T R d\theta dx \quad (12)$$

where $$[C_0] = \begin{bmatrix} 0 & -\frac{\partial}{\partial x} & 0 \\ 0 & -\frac{1}{R}\frac{\partial}{\partial \theta} & 0 \\ -\frac{1}{2R}\frac{\partial}{\partial \theta} & 0 & \frac{1}{2}\frac{\partial}{\partial x} \end{bmatrix}$$

The global initial stuffiness matrix may be obtained, by assembling the matrix for each individual finite element, and adding to get the global stiffness matrix.

II- Aerodynamic model:

Piston theory was first proposed by Lighthill for application to vibrating airfoils, and it was first introduced into the field of aeroelasticity by Ashley and Zartarian[15].

The pressure on the body surface may be approximated by the pressure experienced by a piston moving in the flow with a velocity equivalent to the superposition of the velocity due to the changing body shape and the velocity due to the rigid-body motion of the body.

This pressure is given by [15,16]:

$$\frac{p}{p_\infty} = \left\{ 1 + [(\gamma - 1)/2] \left(\omega / a_\infty \right) \right\}^{2\gamma/(\gamma-1)} \quad (13)$$

Where ω is the velocity of the piston. Selecting terms up to the third order in the binomial expansion of Eq. (13), we obtain

$$p - p_\infty = \rho_\infty a_\infty^2 \left\{ \left(\frac{\omega}{a_\infty} \right) + \frac{\gamma+1}{4} \left(\frac{\omega}{a_\infty} \right)^2 + \frac{\gamma+1}{12} \left(\frac{\omega}{a_\infty} \right)^3 \right\} \quad (14)$$

Where

$$\omega = \frac{\partial W}{\partial t} + U_\infty \frac{\partial W}{\partial x} \quad (15)$$

Substituting (15) in (14), the pressure relation is given by:

$$p_a = -\gamma p_\infty \left[\begin{array}{l} \left(M \frac{\partial W}{\partial x} + \frac{1}{a_\infty} \frac{\partial W}{\partial t} \right) + \frac{\gamma+1}{4} \left(M \frac{\partial W}{\partial x} + \frac{1}{a_\infty} \frac{\partial W}{\partial t} \right)^2 \\ + \frac{\gamma+1}{12} \left(M \frac{\partial W}{\partial x} + \frac{1}{a_\infty} \frac{\partial W}{\partial t} \right)^3 \end{array} \right] \quad (16)$$

the direction of the pressure loading is always into the body surface, opposite to outward pointing surface normal of the structure.

The form of the radial displacement [9] is:

$$W = \sum_{j=1}^{8} e^{i(\lambda_j \frac{x}{R} + \omega t + n\theta)} = \sum_{j=1}^{8} W_j \quad (17)$$

where λ_j is the j^th^ root of the characteristic equation and ω the natural angular frequency.

Substituting Eq. (17) into Eq. (16), we obtain the equation for the aerodynamic pressure on the cylinder wall:

$$\begin{aligned} p_a = -\gamma p_\infty \Bigg\{ & M \left(\frac{i\lambda_j}{R} \right) [T][R_f][A^{-1}] \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} + \frac{1}{a_\infty} [T][R_f][A^{-1}] \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix} + \\ & \frac{\gamma+1}{4} M^2 \left(-\frac{\lambda_j^2}{R^2} \right) \left([T][R_f][A^{-1}] \right)^2 \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix}^2 + \frac{\gamma+1}{4} \frac{1}{a_\infty^2} \left([T][R_f][A^{-1}] \right)^2 \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix}^2 + \\ & \frac{2M}{a_\infty} \left(\frac{i\lambda_j}{R} \right) \left([T][R_f][A^{-1}] \right)^2 \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix} + \frac{\gamma+1}{12} M^3 \left(\frac{i\lambda_j}{R} \right)^3 \left([T][R_f][A^{-1}] \right)^3 \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix}^3 + \\ & \frac{\gamma+1}{4} \frac{M^2}{a_\infty} \left(-\frac{\lambda_j^2}{R^2} \right) \left([T][R_f][A^{-1}] \right)^3 \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix}^2 \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix} + \frac{\gamma+1}{4} \frac{M^2}{a_\infty} \left(-\frac{\lambda_j^2}{R^2} \right) \left([T][R_f][A^{-1}] \right)^3 \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix}^2 \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix} + \\ & \frac{\gamma+1}{4} \frac{M}{a_\infty^2} \left(\frac{i\lambda_j}{R} \right) \left([T][R_f][A^{-1}] \right)^3 \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix}^2 + \frac{\gamma+1}{12 a_\infty^3} \left([T][R_f][A^{-1}] \right)^3 \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix}^3 \Bigg\} \end{aligned} \quad (18)$$

By introducing the displacement function into the aerodynamic pressure expression and performing the matrix operation required by the finite element method, the damping and stiffness of second order and third order for fluid are obtained by evaluating the integral:

$$\{F_p\} = \iint [N]^T \{p_a\} dA \quad (19)$$

III- Nonlinear Analysis

The motion of a cylindrical shell subjected to an external supersonic flow is governed by the equation of motion which we shall write as:

$$\begin{aligned} & \left[M_s^{(L)} \right] \begin{Bmatrix} \ddot\delta_i \\ \ddot\delta_j \end{Bmatrix} - \left[C_f^{(L)} \right] \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix} + \left[K^{(L)} \right] \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} - \left[K_f^{(NL2)} \right] \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix}^2 - \left[C_f^{(NL2)} \right] \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix}^2 - \\ & \left[KC_f^{(NL)} \right] \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix} - \left[K_f^{(NL3)} \right] \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix}^3 - \left[K_f^{(NL2)} C_f^{(NL)} \right] \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix}^2 \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix} - \\ & \left[K_f^{(NL)} C_f^{(NL2)} \right] \begin{Bmatrix} \delta_i \\ \delta_j \end{Bmatrix} \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix}^2 - \left[C_f^{(NL3)} \right] \begin{Bmatrix} \dot\delta_i \\ \dot\delta_j \end{Bmatrix}^3 = 0 \end{aligned} \quad (20)$$

where subscripts s and f refer to the shell in vacuo and fluid, respectively and $\{\delta\}$ is the degrees of freedom vector for the total nodes .

$$\left\lfloor K^L \right\rfloor = [K_s] + [K_I] - \left\lfloor K_f^L \right\rfloor \quad (21)$$

$$[K_I] = \int_0^l \int_0^{2\pi} [N]^T [C_0]^T \begin{bmatrix} \bar N_x & 0 & 0 \\ 0 & \bar N_\theta & 0 \\ 0 & 0 & \bar N_x + \bar N_\theta \end{bmatrix} [C_0]^T [N]^T R d\theta dx$$

$$[K_s] = \iint [B]^T [P][B] dA.$$

Let us set :

$$\{\delta\} = [\Phi]\{q\}$$

$$q_i(\tau) = A_i f_i(\tau) \quad (22)$$

$$f_i(0) = 1 \text{ and } \dot f_i(0) = 0$$

Where $[\Phi]$ represents the square matrix for eigenvectors of the linear system and $\{q\}$ is a time-related vector. Numerical solution of the coupled system (20) is difficult and costly. We limit to solving the uncoupled system, after simplifying the equation, becomes:

$$\begin{aligned} & \ddot f - \eta_i \dot f_i + \omega_i^2 f_i - \lambda_i \left(\frac{A_i}{t} \right) f_i^2 - \zeta_i \left(\frac{A_i}{t} \right) \dot f_i^2 - \xi_i \left(\frac{A_i}{t} \right) f_i \dot f_i - \\ & \sigma_i \left(\frac{A_i^2}{t^2} \right) f_i^3 - \chi_i \left(\frac{A_i^2}{t^2} \right) f_i^2 \dot f_i - \psi_i \left(\frac{A_i^2}{t^2} \right) f_i \dot f_i^2 - \varphi_i \left(\frac{A_i^2}{t^2} \right) \dot f_i^3 = 0 \end{aligned} \quad (23)$$

where

$$\eta_i = \frac{c_{ii}^{(L)}}{m_{ii}}\ ;\omega_i^2 = \frac{k_{ii}^{(L)}}{m_{ii}}\ ;\lambda_i = \frac{K_{ii}^{(NL2)}}{m_{ii}}t\ ;\eta_i = \frac{c_{ii}^{(L)}}{m_{ii}}\ ;$$

$$\omega_i^2 = \frac{k_{ii}^{(L)}}{m_{ii}}\ ;\lambda_i = \frac{K_{ii}^{(NL2)}}{m_{ii}}t\ ;\zeta_i = \frac{C_{ii}^{(NL2)}}{m_{ii}}t\ ;\xi_i = \frac{KC_{ii}^{(NL)}}{m_{ii}}t\ ;$$

$$\sigma_i = \frac{K_{ii}^{(NL3)}}{m_{ii}}t^2\ ;\chi_i = \frac{K_{ii}^{(NL2)}C_{ii}^{(NL)}}{m_{ii}}t^2\ ;\psi_i = \frac{K_{ii}^{(NL2)}C_{ii}^{(NL)}}{m_{ii}}t^2\ ; \qquad (24)$$

$$\varphi_i = \frac{C_{ii}^{(NL3)}}{m_{ii}}t^2\ .$$

where t represents the shell thickness. The square root of coefficient k_{ii}/m_{ii} represents the i[th] linear vibration frequency of system. The solution $f_i(\tau)$ of these ordinary nonlinear differential equations which satisfies the initial conditions (22) is approximated by a fourth-order Runge-Kutta numerical method. The linear and non-linear natural angular frequencies are evaluated using a systematic search for the $f_i(\tau)$ roots as a function of time. The ω_{NL}/ω ratio is expressed as a function of the non-dimensional ration A/t .

IV- Results and discussion

An additional hypothesis was required to develop our analytical model. We simplified the parameters and limited our dynamic analysis strictly to consideration of those shell-fluid systems which lead to a symmetric matrix system of eigenvalues. This simplifying hypothesis is validated if the resultant eigenvalues come close to the original system. Tables 1-4 of Lakis and Laveau [13], show the variance between the eigenvalues in the original and simplified systems. These results seem to validate our approach since the two systems are seen to have comparable dynamic behavior.

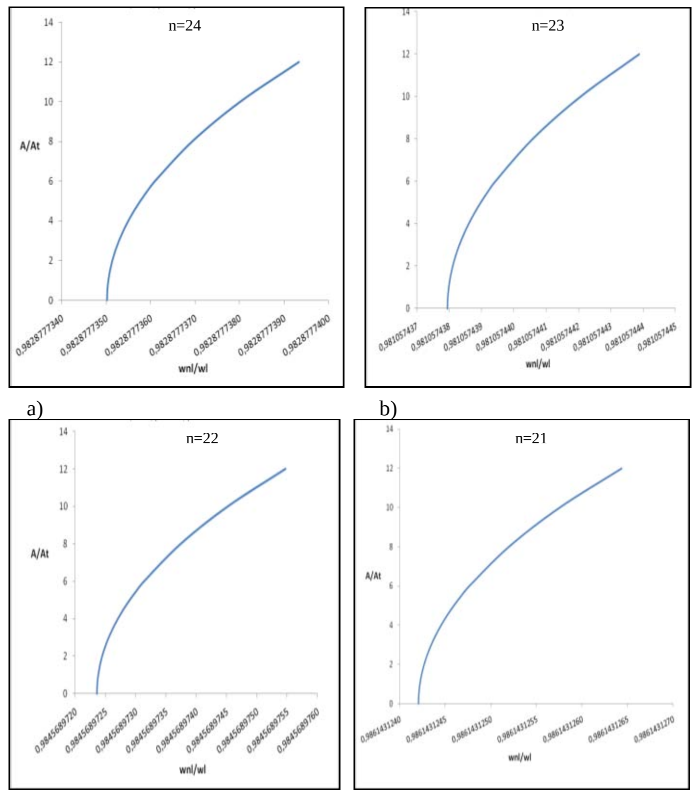
The shell and flow properties are given by:

$$E = 16\times10^6\ \ lb/in \quad (11\times10^{10}\ N/m^2)$$
$$\upsilon = 0.35$$
$$h = 0.0040\ \ in \quad (0.0001015\ \ m)$$
$$L = 15.4\ in \quad (0.381\ \ m)$$
$$R = 8.00\ \ in \quad (0.203\ \ m)$$
$$\rho_s = 0.000833\ \ lb - s^2/in^4 \quad (8900\ \ kg/m^3)$$
$$M = 3.00$$
$$a_\infty = 8400\ \ in/s \quad (213\ \ m/s)$$

The influence of non-linearities associated with nonlinear piston theory on the cylindrical shells subjecting to a supersonic flow is main focus of our analysis.

1- Effect of circumferential number

Figure 3 shows how the flutter frequency varies with flutter amplitude for different circumferential numbers. In all cases, the flutter frequency first decreases very slightly and then increases with amplitude at the higher amplitudes. The final hardening nonlinearity shown by the frequency curve in the figure may be attributed to the nonlinear terms in the piston equation. This same result is found in several publications [2,17,18].



c) d)

Figure3 : Non-dimensional flutter amplitude vs non-dimensional flutter frequency for pressurized shell (p_m = 0.5psi).

2- Effect of initial strain

In figure 4, the behavior is the same as that seen for the case of effect of circumferential number except, when the shell is not subjected to internal pressure, in which case the non-dimensional frequency is greater than those of the pressurized shell.

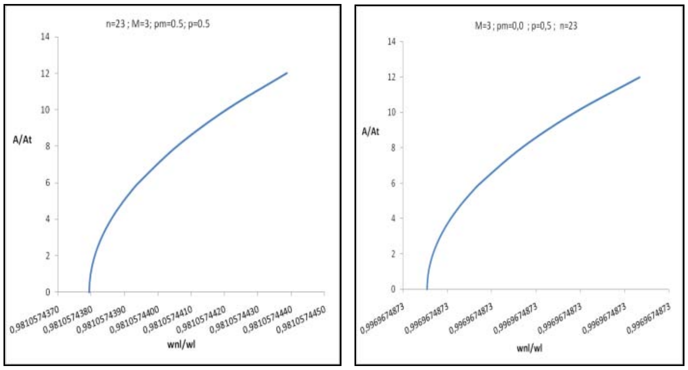


Figure4: Non-dimensional flutter amplitude vs non-dimensional flutter frequency for p_m = 0.0 psi and p_m = 0.5 psi.

V- Conclusion

This paper deals with some of the problems that arise when flow nonlinearities are considered in combination with a linear structural model in the study of dynamic and stability behavior of elastic isotropic cylindrical shells. An efficient linear hybrid finite element method and nonlinear aerodynamic piston theory were used to develop the nonlinear dynamic equations of the coupled fluid-structure system. By investigation several parameters that are dependent on structure and flow we can predict the limit cycle amplitudes, non-dimensional frequency and freestream static pressure, and the stabilizing effect of the internal pressure. In all cases, the uncoupled equation of motion gives wrong results, only the coupled problem gives correct nonlinear results for the presence of coupling between asymmetric and axisymmetric modes. This results in new knowledge concerning the contribution of flow nonlinearities on fluid-structure interaction in a supersonic regime.

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APPENDIX

Sander's linear equations for thin cylindrical shells in terms of axial, tangential and circumferential displacements are:

$$L_1(U,V,W,P_{ij}) = P_{11}\frac{\partial^2 U}{\partial x^2} + \frac{P_{12}}{R}\frac{\partial^2 U}{\partial x^2}(\frac{\partial^2 V}{\partial x\partial\theta} + \frac{\partial W}{\partial x}) - P_{14}\frac{\partial^3 W}{\partial x^3} + \frac{P_{15}}{R^2}(\frac{\partial^3 W}{\partial x\partial\theta^2} + \frac{\partial^2 V}{\partial x\partial\theta}) + (\frac{P_{33}}{R} - \frac{P_{63}}{2R^2})(\frac{\partial^2 V}{\partial x\partial\theta} + \frac{1}{R}\frac{\partial^2 U}{\partial\theta^2})$$
$$+(\frac{P_{36}}{R^2} - \frac{P_{66}}{2R^3})(-2\frac{\partial^3 W}{\partial x\partial\theta^2} + \frac{3}{2}\frac{\partial^2 V}{\partial x\partial\theta} - \frac{1}{2R}\frac{\partial^2 U}{\partial\theta^2})$$
$$L_2(U,V,W,P_{ij}) = (\frac{P_{21}}{R} - \frac{P_{51}}{R^2})\frac{\partial^2 U}{\partial x\partial\theta} + \frac{1}{R}(\frac{P_{22}}{R} + \frac{P_{52}}{R^2})(\frac{\partial^2 V}{\partial\theta^2} + \frac{\partial W}{\partial\theta}) - (\frac{P_{24}}{R} + \frac{P_{54}}{R^2})\frac{\partial^3 W}{\partial x^2\partial\theta} + \frac{1}{R^2}(\frac{P_{25}}{R} + \frac{P_{55}}{R^2})(-\frac{\partial^3 W}{\partial\theta^3} + \frac{\partial^2 V}{\partial\theta^2})$$
$$+(P_{33} + \frac{3P_{63}}{2R})(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 U}{R\partial x\partial\theta}) + \frac{1}{R}(P_{36} + \frac{3P_{66}}{2R})(-2\frac{\partial^3 W}{\partial x^2\partial\theta} + \frac{3}{2}\frac{\partial^2 V}{\partial x^2} - \frac{1}{2R}\frac{\partial^2 U}{\partial x\partial\theta})$$
$$L_3(U,V,W,P_{ij}) = P_{41}\frac{\partial^3 U}{\partial x^3} + \frac{P_{42}}{R}(\frac{\partial^3 V}{\partial x^2\partial\theta} + \frac{\partial^2 W}{\partial x^2}) - P_{44}\frac{\partial^4 W}{\partial x^4} + \frac{P_{45}}{R^2}(-\frac{\partial^4 W}{\partial x^2\partial\theta^2} + \frac{\partial^3 V}{\partial x^2\partial\theta}) + \frac{2P_{63}}{R}(\frac{1}{R}\frac{\partial^3 U}{\partial x\partial\theta^2} + \frac{\partial^3 V}{\partial x^2\partial\theta})$$
$$+\frac{2P_{66}}{R^2}(-2\frac{\partial^4 W}{\partial x^2\partial\theta^2} + \frac{3}{2}\frac{\partial^3 V}{\partial x^2\partial\theta} - \frac{1}{2R}\frac{\partial^3 U}{\partial x\partial\theta^2}) + \frac{P_{51}}{R^2}\frac{\partial^3 U}{\partial x\partial\theta^2} + \frac{P_{52}}{R^3}(\frac{\partial^3 V}{\partial\theta^3} + \frac{\partial^2 W}{\partial\theta^2}) + \frac{P_{55}}{R^4}(-\frac{\partial^4 W}{\partial\theta^4} + \frac{\partial^3 V}{\partial\theta^3})$$
$$-\frac{P_{21}}{R}\frac{\partial U}{\partial x} - \frac{P_{54}}{R^2}\frac{\partial^4 W}{\partial x^2\partial\theta^2} + \frac{P_{22}}{R^2}(\frac{\partial V}{\partial\theta} + W) + \frac{P_{24}}{R^2}\frac{\partial^2 W}{\partial\theta^2} - \frac{P_{25}}{R^3}(-\frac{\partial^2 W}{\partial\theta^2} + \frac{\partial V}{\partial\theta})$$

Matrices $[T]_{3\times3}$, $[R]_{3\times8}$, and $[A]_{8\times8}$ are defined as:

$$[T] = \begin{bmatrix} \cos(n\theta) & 0 & 0 \\ 0 & \cos(n\theta) & 0 \\ 0 & 0 & \sin(n\theta) \end{bmatrix}$$
$$R(1,i) = \alpha_i e^{\lambda_i x/R} \quad , \quad R(2,i) = e^{\lambda_i x/R} \quad , \quad R(3,i) = \beta_i e^{\lambda_i x/R} \qquad i = 1,2,...,8$$
$$A(1,i) = \alpha_i, \quad A(2,i) = 1, \quad A(3,i) = \frac{\lambda_i}{R}, \quad A(4,i) = \beta_i, \quad A(5,i) = \alpha_i e^{\lambda_i l/R}$$
$$A(6,i) = e^{\lambda_i l/R}, \quad A(7,i) = \frac{\lambda_i}{R}e^{\lambda_i l/R}, \quad A(8,i) = \beta_i e^{\lambda_i l/R} \qquad i = 1,2,...8$$

Matrix $\lfloor R_f \rfloor_{3\times8}$ is defined by:

$$[R_f] = \begin{bmatrix} 0 & ... & 0 \\ e^{i(\lambda_1 x/R)} & ... & e^{i(\lambda_8 x/R)} \\ 0 & ... & 0 \end{bmatrix}$$