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SOURCE MODELS FOR NOISE GENERATED IN CORRUGATED PIPES

HGD Goyder
Cranfield University
Shrivenham United Kingdom

ABSTRACT

Corrugated pipes are used in the oil and gas industry because they are flexible. Such pipes may generate large levels of noise when carrying a gas flow. The noise source is due to the cavities in the corrugations in which vortices form and interact with acoustic waves. The resulting flow-acoustic interaction may result in noise levels sufficient to cause structural vibration which may lead to pipework failure due to fatigue. The interaction between the vortex shedding and the acoustic wave is that of a self-sustained oscillation. The objective of the paper is to attempt to produce an analytical model of these oscillations starting from first principles. Although the model does require some experimental input much information is obtained concerning the details of the mechanism and factors controlling how it scales with the geometry, flow velocity and other relevant parameters. The model requires three constants to complete its formulation. These three constants describe the source strength at low acoustic amplitudes, the nonlinearity as the amplitude is increased and a delay term that relates the vortex shedding to the local acoustic velocity. It emerges that the nonlinear parameter is the most important for determining the maximum acoustic amplitude.

1 INTRODUCTION

The oil and gas industry use corrugated pipes to carry gas between the sea bed and platforms. Such pipes are known as risers and can accommodate platform motions because they are flexible. On the sea bed corrugated pipes are used to connect pipework and are known as jumpers. An illustration of a corrugated pipe (taken from ISO 13628-11, 1997) is shown in Figure 1. The purpose of the corrugations is to maintain the circular form of the pipe even when the internal pressure is reduced (e.g. to atmospheric pressure) and the external pressure is large due to the sub sea hydrostatic depth.

Within the corrugation cavities the flow is stagnant and thus a shear layer forms at the edge of the cavity between the moving gas in the pipe and the still fluid in the cavity. This shear layer rolls up into vortices which are ejected from the cavity creating a noise source (see illustration in Figure 2). The noise is typically a pure tone and may have an amplitude sufficient to cause structural vibration in pipework at the terminations of the corrugated pipe. The structural vibration can be large enough to cause fatigue failure after only hours of operation. The release of gas due to a pipe fracture is unacceptable and consequently all the issues associated with noise from corrugated pipes need to be fully understood.

This paper examines the source of the noise and suggests an appropriate noise model. From the start is should be noted that the problem is complex and that there is a need for experimental investigations and data. The aim of this work is to provide a model that will be useful if there are some measured data and there is a need to interpolate or extrapolate to other circumstances. The model also provides insight that can assist in identifying where more experimental data is needed.

The approach taken here is to use the fundamental equations for fluid-acoustic interaction to provide a foundation and then to model the remaining parameters. The result is an analytic model that involves only three unknown parameters. It is further shown that only one of these parameters is needed to predict the amplitude of the noise.

The basic ideas concerning noise from flow over cavities are described in the textbook by Howe [2004].The production of noise from a corrugated pipe seems to have been first identified by Petrie and Huntley [1980] and then by Ziada [1991]. More recent publications are Belfroid et al [2008], Debut Antunes and Moreira [2008] and Tonon et al. [2010]. The effect on structural vibration induced by the corrugation noise has been described by Goyder et al [2006].

2 NOMENCLATURE

B	Total enthalpy
G	Green's function
L	Pipe length
R	Pipe Radius
S	Source
St	Strouhal number
T	Absolute temperature
U_c	Velocity of shear layer, convection velocity
U_0	Mean flow velocity
c	Speed of sound
e	Internal energy
f	Frequency in Hz
k	Wavenumber
ℓ	Corrugation gap width
m	Number of vortices in corrugation
p	Acoustic pressure
r	Radial coordinate
s	Entropy
t	time
u	Flow velocity
u	Acoustic velocity
x	Coordinate along pipe
φ	Acoustic velocity mode shape
ρ	Gas density
λ	Acoustic wavelength
ζ	Acoustic damping ratio
$\boldsymbol{\omega}$	Gas vorticity
ω	Frequency in radians per second

3 GENERAL SPECIFICATIONS AND CHARACTERISTICS

Corrugated pipes cover a wide range of lengths from short jumpers of 20 m to long risers of 500 m or more with all lengths producing noise. Typical pipe dimensions are an internal diameter of about 0.2 m with the corrugation width being 5 to 10 mm with a pitch of around 25 mm. The gas within the pipe is typically methane under high pressure with a density of about 100 kg / m^3. The gas flow velocity is in the range 2 to 15 m/s. The speed of sound can be high at about 400 m/s. The Mach number is thus small. The noise is typically a pure tone with a frequency that depends on flow velocity but with values of 100 to 500 Hz. Although it is important to note that there is no known lower or upper frequency bound and in principle any frequency can be produced. In typical cases the wavelength of the noise is large compared to the corrugation width and pitch with there being tens or hundreds of corrugation gaps in a wavelength.

The corrugation gap is a helix around the pipe inner wall but will be modelled for simplicity in this paper as a sequence of circular ring gaps as shown in Figure 3.

As the flow rate in a corrugated pipe is increased from zero there is initially no noise with noise starting to occur when a threshold velocity is crossed. The prediction of this threshold velocity is important because for some operators it marks the maximum flow rate that will be allowed in the pipe. Such operators decide not to take the risk of noise in the pipeline causing vibration and possible pipework fatigue. Other operators decide to allow higher flow rates and to have noise in the pipe but then take care to assess their pipework for the potential damaging effects of vibration.

As the flow rate is increased beyond the initial threshold a pure tone is maintained over a range of flows. At some higher flow rate the tone will switch to a new higher frequency in a step like manner. If the flow is increased still further then new higher frequencies will be observed each of which maintains, to a good approximation, the same frequency over a range of flow rates. If the flow rate is decreased the frequencies also decreases stepping down to a new, lower, frequency as some flow rate is passed. (The step up and step down flow rates do not necessarily coincide for increasing and decreasing flows.)

4 VORTEX SHEDDING

The noise source is attributed to vortex shedding from the corrugation gaps. Figure 4 illustrates the behavior. The shear layer between the moving fluid and the still fluid rolls up into discrete vortices which are convected from the leading edge to the trailing edge. There may be one or more vortices in the gap. The relationship between flow velocity and frequency was first developed by Rossiter [1962] and is given in detail in Howe's textbook [2004]. In each cycle one vortex is formed at the leading edge and one is ejected at the trailing edge. Each vortex is convected by the local velocity U_c which is about 0.4 to 0.6 U_0 where U_0 is the mean velocity in the pipe. Thus the time taken to cross the corrugation gap of width ℓ is ℓ / U_c. If f is the frequency of vortex shedding then this time may be related to the frequency by

$$\frac{\ell}{U_c} = \frac{m}{f} \tag{1}$$

where m is the number of vortices in the gap. This equation may be generalized by allowing for additional delays and, at high Mach numbers, for the effects of the speed of sound. See Howe [2004] for details.

The usual nondimensional group used to characterize the vortex shedding frequency is the Strouhal number which is given by $St = f\,\ell / U_0$. Thus the Strouhal number is related to the Rossiter theory by

$$St = \frac{f\,\ell}{U_0} = m\frac{U_c}{U_0} \tag{2}$$

The above formula has been examined in detail by Belfroid et al. [2007] who largely find agreement but report on the difficult of defining ℓ exactly when the corrugation edges are rounded.

The above equation gives the natural vortex shedding frequency. Under the influence of an imposed external flow oscillation this natural frequency can change to the imposed frequency a process called lock-on. It is for this reason that the same frequency is observed over a wide range of flow rates with the frequency only jumping to a new frequencies following a significant change in flow rate.

The frequencies selected by the vortex shedding mechanism are the acoustic natural frequencies of the whole pipework system. In a long pipe there are very many acoustic natural frequencies. However, most of these frequencies are associated with large acoustic damping and are therefore difficult to excite. However, a few natural frequencies have small acoustic damping and it is these frequencies that are selected. A method for determining these frequencies, which depends on the reflection coefficients at the ends of the pipe, has been described by Goyder [2009].

The vortex shedding mechanism is not only influenced by the imposed flow oscillation frequency but is also influenced by the amplitude of the imposed oscillations. Thus there is feedback between the source and the induced acoustic field. The feedback is positive for small source strengths increasing the output. Thus the system is a self sustained oscillation. For large amplitudes the system saturates with the feedback becoming negative and decreasing the amplitude. The balance between the positive feedback and the negative feedback is an important issue to be investigated in this paper.

5 THEORETICAL BACKGROUND

The theory of aerodynamic noise has been well developed by Howe [2004]. The relationship between the noise (acoustic pressure) and the vorticity in the flow is given by

$$\left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) B = \nabla . (\boldsymbol{\omega} \times \mathbf{u}) \tag{3}$$

where c is the speed of sound, t is time, B is the total enthalpy (which is simply related to the acoustic pressure), $\boldsymbol{\omega}$ is the vorticity and $\mathbf{u}$ is the velocity. This equation is a simplification of a more complex version and ignores the effect of Mach number. This is permissible for low mean flow velocities and may be permissible for larger flow rates if acoustic standing waves form (this will be discussed subsequently).

From the start it is noted that this equation and the processes it represents is far too complex to attempt a direct solution. However, this equation does provide a good starting point even if it is impossible to solve directly.

The left hand side is a linear operator which is the standard acoustic wave equation. The right hand side is the source or forcing term. The linear formulation may be exploited to break the problem down into simple components that may be used as building blocks for the whole problem. The configuration of Figure 5 is thus taken as a simplified version of the problem. Here there is just one corrugation in a pipe at location x_n. In Section 8 the effect of many corrugations will be assembled by superimposing the solution to this initial problem.

The total enthalpy B and its differential are given by

$$\begin{aligned} B &= e + \frac{p}{\rho} + \frac{1}{2} u^2 \\ dB &= T\, ds + \frac{dp}{\rho} + d\left(\frac{1}{2} u^2 \right) \end{aligned} \tag{4}$$

where e is the internal energy per unit mass, p is the pressure, ρ the fluid density, T the temperature, s the entropy and u the flow velocity. Here the pressure and flow velocity include both the mean and the fluctuating components. Within the pipe, away from the source, the mean flow velocity is a constant and equal to U_0. Also the entropy is a constant. Thus in this region, which is the region where the acoustic pressure is required, the perturbation in enthalpy is equal to the perturbation in acoustic pressure divided by the density. Thus $\rho\ dB$ is equal to the acoustic pressure.

Equation 3 has a formal solution involving a Green's function. Although it cannot be solved directly it gives clues on how the solution can be formulated. Thus

$$\frac{p}{\rho} = \int_{vol} \int_{time} G(\mathbf{x}, \mathbf{x}', t, t') \nabla . (\boldsymbol{\omega} \times \mathbf{u}) d\mathbf{x}\, dt' \tag{5}$$

where G is the Green's function $\mathbf{x}'$ is the dummy variable space dimension (3 coordinates) and t' the dummy variable time dimension. The volume integration is taken over the region containing the source where the vorticity is non zero.

The integrand consists of two terms; the Green's function and the source term. The Green's function can be found exactly but the source term is more difficult and will require modeling.

6 GREEN'S FUNCTION

Formally the Green's function in Equation 5 is the solution to the partial differential equation, 3, with the source replaced by a point source in space and time. The solution to this problem gives rise to the mean flow and all the fluctuating acoustic modes. However, the solution to the complete problem is not required. Only the special case where vortex shedding gives rise to one pure tone need be investigated. This special case involves one acoustic mode which will be assumed to be a plain wave within the pipe that sets up a fluctuating perturbation of B.

The problem to be solved is thus that given in Figure 5 where the source is located in a thin ring around the outside of the pipe.

The nature of the source needs some consideration. Typically the flow over a cavity gives rise to monopoles and dipoles. If the cavity is deep it can have resonant acoustic modes trapped within it giving rise to a strong monopole source. However, in a shallow cavity, such as those considered here where the acoustic wavelength is much greater than any of the cavity dimensions a strong monopole is not possible. Howe [2004] analyses a shallow cavity where there are both monopole and dipole sources and looks at their interaction. He deduces that unless they can interact the dipole source will be dominant. This observation is backed up by experimental measurements of Tonon et al [2010]. Here a special experiment was conducted in which those cavities which were favorable to dipole sources were blocked. In this blocked condition no noise was produced. Alternatively, if those cavities were blocked which were favorable to a monopole source then there was no difference to the noise produced. Thus it is concluded that a dipole source is most likely. The possibility of a monopole source interacting with the dipole source should not be ignored. Belfroid et al. [2007] note that there is a dependence on cavity volume which would suggest that some monopole effect is present.

The differential equation with a unit dipole source is thus

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)\frac{p}{\rho}=\delta'(x-x_n)\delta(r-R)\delta(t-t_0) \qquad (6)$$

where $\delta'(x\text{-}x_n)$ is the derivative of a space delta function located at x_n, $\delta(r\text{-}R)$ is a space delta function located at the pipe radius R and $\delta(t\text{-}t_0)$ is time delta function. The derivative of a delta function can be considered as a distribution due to two close sources which are out of phase; this corresponds to a dipole source.

The equation is a liner partial differential equation which is separable into a time and space solutions. The space solution will be that of the acoustic mode of interest. The time solution will be the amplitude of that mode. It is the amplitude of the mode, and all the factors that control the amplitude, that needs to be determined.

Let the j^{th} mode be excited with an acoustic natural frequency, in radians per second, of ω_j a wavelength λ_j and consequently a wave number $k_j = \omega_j/c$. Here the speed of sound, c, includes a modification to account for the presence of the cavities; this is a minor modification easily taken into account by including the cavity volume in the bulk modulus of the gas (see Lighthill [1978]).

For this mode the acoustic pressure may be written

$$p(t,x)=p_j(t)\cos k_j x \qquad (7)$$

where p_j is the time dependant amplitude of the acoustic pressure standing wave. The standing wave shape is a cosine wave that has the origin of the x coordinates at a pressure antinode. (The location of the origin is arbitrary so this is just a convenient formulation.)

Substituting Equation 7 back into the differential equation gives

$$\left(\frac{1}{c^2}\ddot{p}_j+k_j^2 p_j\right)\frac{\cos k_j x}{\rho}=\delta'(x-x_n)\delta(r-R)\delta(t-t_0) \qquad (8)$$

This equation may now be multiplied by cos $k_j x$ and integrated over the volume of the pipe to give an ordinary differential equation for the pressure.

$$\left(\frac{1}{c^2}\ddot{p}_j+k_j^2 p_j\right)\frac{\pi R^2 L}{2\rho}=2\pi R k_j \sin k_j x_n\,\delta(t-t_0) \qquad (9)$$

here L is the length of the pipe. Note that the source depends strongly on its location relative to the acoustic mode shape. If the source is on a node of the pressure mode shape it has a maximum effect while if it is on the antinode of the pressure mode shape it has a zero effect. This is a consequence of the dipole source which couples best with the mode shape at those locations where its gradient is a maximum.

The differential equation for the pressure is for a unit source strength. The actual source strength depends on the right hand side of Equation 3. A dimensional analysis shows that the source strength has the units of length cubed over time squared.

7 THE SOURCE MODEL

The source is the most difficult aspect to model. A certain amount of progress can be made by looking at the nature of the term $\nabla.(\boldsymbol{\omega}\times\mathbf{u})$ but, unfortunately, there is no hint of the positive or negative feedback effects that are actually observed.

Following the treatment in Howe [2004] an estimate may be made of the vorticity. Returning to the discrete vortex model illustrated in Figure 4 the circulation around each vortex may be estimated as follows. If the vortex shedding frequency is f then in a time $1/f$ a shear layer of length U_c/f leaves the leading edge of a cavity. Hence integrating around a contour embracing all of this shear layer gives a net circulation equal to $U_0 U_c/f$. The vorticity concentrated on each vortex is thus proportional to this value. Furthermore, as the vorticity vector points out of the paper and the velocity is in the perpendicular direction the cross product is proportional to $U_0 U_c^2/f$. The strength of the source is determined by the gradient of this value which is zero for simple convection across the gap but reaches a large value as the vortex is ejected from the cavity at the trailing edge. The introduction of the vorticity at the leading does not add to the source strength because it emerges smoothly from the leading edge without a significant gradient. This argument is given in Howe [2004] who notes that this is due to a Kutta condition.

The equation modelling the pressure may now be written as

$$\ddot{p}_j + \omega_j^2 p_j = \frac{4\rho c \omega_j \ell U_0^2}{RL} \frac{(U_c/U_0)^2}{f\,\ell/U_0} \sin(k_j x_n)\, S \tag{10}$$

where the various terms have been included in nondimensional groups and the wave number k_j has been replaced by ω_j/c. The dimensionless term S represents the remaining terms that need to be modelled. Rather awkwardly there is both the frequency ω in radians per second and the frequency f in Hz. However, this agrees with the term on the denominator being the Strouhal number.

The main feature that needs to be modelled is the response of the vortices to the external flow field. This has two manifestations. Firstly the lock-in effect whereby the natural vortex shedding frequency changes to the external frequency and secondly the feedback effects that control the growth and saturation. In order to make progress only the growth and saturation will be modelled. Thus it is assumed that the system is operating at the natural vortex shedding frequency i.e. $\omega_j = 2\pi f$. This is the condition for which the largest acoustic amplitudes occur.

The growth and saturation may be modelled by a linear term when the amplitude is small and by a cubic term when the amplitude is large. The complete model is then given by

$$\begin{aligned}&\ddot{p}_j + \omega_j^2 p_j = \\ &\frac{4\rho c \omega_j \ell U_0^2}{RL} \frac{(U_c/U_0)^2}{f\,\ell/U_0} \sin(k_j x_n)\, \alpha \\ &\times \frac{u(t-\gamma/f)}{U_0}\left(1-\beta^2\left(\frac{u(t-\gamma/f)}{U_0}\right)^2\right)\end{aligned} \tag{11}$$

The growth and saturation has been written in terms of the local acoustic velocity u divided by the mean flow velocity. The parameters α and β are assumed to be constants that must be established from experiment. A third constant γ controls a time delay for the acoustic velocity terms. The feedback terms with growth and saturation are similar to those of a van der Pol oscillator. Such a formulation for the corrugation noise has been examined by Debut et al. [2008]. However, their formulation was in terms of pressure rather than velocity and was a phenomenological approach rather than one based on the details of the flow.

The reason for modelling the growth in terms of velocity is because the source is compact in the sense that the size of the corrugation gap is small compared to the wavelength of the sound. Thus the flow over the gap appears locally to be incompressible and just a mean flow with a fluctuating component. It can thus only depend on flow velocity and local factors such as gap geometry. The reason for assuming that α, β and γ are constant is that it is hoped that the main effects of geometry are modelled by the use of the convection velocity and Strouhal number. However, details such as the radius and depth of the cavity may play a role.

The phasing of the feedback is particularly unclear. How does the vortex shedding synchronise with the flow? At what stage in a cycle does the vortex get ejected into the flow? It is for this reason that the parameter γ is included. This parameter controls the delay between the dipole acting on the acoustic wave and the instant at which the acoustic velocity reaches a maximum.

The parameters α, β and γ will be considered again when the modelling is completed.

8 SUPERPOSITION OF ALL THE SOURCES

So far only one source has been considered while in practice the sources are regularly distributed along the pipe. The model of Equation 11 has one source located at x_n where the local acoustic velocity is u. The effect of all the sources must now be taken into account.

The pipe may be subdivided into sections each containing one wavelength. The contribution of each source within the wavelength to the modal pressure p_j is achieved by appropriately summing all the contributions. Typically there are tens or hundreds of sources within one wavelength.

Each source has a factor that depends on its location through the term $\sin k_j x_n$. The source strength also depends on location through the acoustic velocity. The first step is to relate the velocity to location. It was noted in Section 6 that the pressure was separable into time and space factors. The acoustic velocity is also separable into time and space factors. The relationship between velocity and pressure is given by the linearised momentum equation:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{12}$$

Thus if the velocity is separated as

$$u(t,x) = u_j(t)\varphi_j(x) + U_0 \tag{13}$$

where u_j is the time dependant velocity amplitude φ_j is the velocity mode shape and U_0 the mean flow velocity it may be deduced, by substituting Equations 13 and 7 into Equation 12, that

$$\phi_j = \sin k_j x \tag{14}$$

and

$$\dot{u}_j = \frac{k_j}{\rho} p_j \tag{15}$$

It is now possible to express Equation 11 in terms of pressure and velocity amplitudes p_j and u_j respectivly. This gives

$$\ddot{p}_j + \omega_j^2 p_j = \frac{4\rho c \omega_j \ell U_0^2}{RL} \frac{(U_c/U_0)^2}{f\,\ell/U_0} \sin^2(k_j x_n)\,\alpha \times \frac{u_j(t-\gamma/f)}{U_0}\left(1-\beta^2\left(\frac{u_j(t-\gamma/f)}{U_0}\right)^2 \sin^2(k_j x_n)\right) \tag{16}$$

for a source located at x_n.

The task is now to combine all the sources at all the locations within one wavelength. If there are N sources in one wavelength the required summation involving all the terms which include x_n may be expressed as

$$\sum_{n=1}^{N} \frac{u_j}{U_0} \sin^2 2\pi \frac{n}{N} \left(1-\beta^2\left(\frac{u_j}{U_0}\right)^2 \sin^2 2\pi \frac{n}{N}\right) \tag{17}$$

which, due to the properties of trigonometric functions, can be calculated to be

$$\frac{N}{2}\frac{u_j}{U_0}\left(1-\frac{3}{4}\beta^2\left(\frac{u_j}{U_0}\right)^2\right) \tag{18}$$

This is the contribution to one wavelength. To obtain all the contributions from all the wavelengths it is necessary to multiply by L/λ. If the corrugations have a pitch of w then N is equal to λ/w. These relationships may be used in simplifications of formula.

With all the sources modelled the final form of the differential equations may be written as

$$\ddot{p}_j + 2\zeta_j \omega_j \dot{p}_j + \omega_j^2 p = \frac{2\rho c \omega_j U_0^2}{R} \frac{\ell}{w} \frac{(U_c/U_0)^2}{f\ell/U_0}\alpha \times \frac{u_j(t-\gamma/f)}{U_0}\left(1-\frac{3}{4}\beta^2\left(\frac{u_j(t-\gamma/f)}{U_0}\right)^2\right) \tag{19}$$

$$\dot{u}_j = \frac{\omega_j}{\rho c} p_j$$

where an acoustic damping term $2\zeta_j\,\omega_j$ has been introduced.

9 MAXIMUM PRESSURE

Numerical solutions of the above equation are shown in Figure 6. They show that the acoustic pressure increases until it saturates and reaches a limit cycle.

An analytical solution for the limit cycle may be obtained by using the method of multiple scales or by the method of averaging (Nayfeh and Mook [1979]). The amplitude of the limit cycle for small acoustic damping is found to depend only on the value of β. It is given by

$$\frac{u_j}{U_0} = \frac{4}{3\beta} \tag{20}$$

or

$$p_j = \frac{4}{3\beta}\rho c U_0 \tag{21}$$

The initial rate of growth depends on the value of α as might be expected. Numerical investigations suggest that the behaviour is relatively insensitive to the value of γ as long as it is close to 0.5.

10 ONSET THRESHOLD

The condition corresponding to the smallest flow velocity that will result in growth of an acoustic wave may be determined by looking at the balance between acoustic damping and the growth term in Equation 19. If the saturation term is dropped and the derivative of the first equation taken the second equation may be substituted to give a linear equation in pressure. The condition for onset then reads

$$\zeta_j < \frac{U_0}{\omega_j R}\frac{\ell}{w}\frac{(U_c/U_0)^2}{f\ell/U_0}\alpha \tag{22}$$

Interestingly the equation is independent of $\rho\ c$ and is proportional to the mean flow velocity squared. A difficulty here is that the acoustic damping does depend on the flow velocity and ρc so these parameters may return to the equation on the left hand side.

11 DISCUSSION

The importance of feedback effects emerges strongly from this investigation. Although it is useful to obtain the rate of growth, as determined by all the factors before the brackets on the right hand side of Equation 19, the most important factor is the maximum amplitude of the acoustic pressure. This does not depend on the rate of growth but only on the parameter β. Thus finding a good value for this parameter is essential. Values for β of about 10 fit reported experimental work.

Although it can be hoped that β is a universal value it can in principle depend on dimensionless groups such as Reynolds number, Strouhal number and geometric ratios such as corrugation rounding. To some extent the effect of Reynolds number is dealt with by choosing a value for the convection velocity. This velocity depends on the boundary layer and hence the Reynolds number. However, there may be other influences of Reynolds number which are not adequately included in the treatment of the convection velocity. Similarly the effect of Strouhal number may be adequately dealt with by its inclusion in the formula but the possibility of influences on β should not be disregarded.

It would be nice if the nonlinear term that suppresses growth of the boundary layer could be deduced from first principles. However, this seems difficult because the cause of the nonlinearity is not immediately obvious.

Another unobvious parameter is the delay term γ. This term must lie between 0.25 and 0.75 to enable the phase of the source to create positive feedback. When this parameter is 0.5 and the motion is nearly harmonic the forcing term on the right hand side is in phase with the derivative of the pressure. This represents a forcing term that acts directly like negative damping. As the value of γ moves away from 0.5 it can lead to effects that are out of phase with the pressure. Such terms would change the natural frequency of the system. This term is most interesting because it is not clear how the source comes into step with the velocity. If it is assumed that each source adjusts itself to have a maximum effect on the system then the value of $\gamma = 0.5$ would be most appropriate.

The equation for the threshold conditions (Equation 22) needs two values before it can be used. The first value is for the acoustic damping ratio and the second a value for α. Damping is always difficult to estimate and consequently the onset flow velocity may not be easy to deduce precisely.

The formulation of the above equations has taken a local viewpoint. The sources within one wavelength both contribute to and subtract from the production of noise. As a consequence the influence of Mach number may be small because phase delays due to the sound travelling with or against the mean flow may be minimal.

Damping has not been considered in detail here. The damping due to friction is local to the sources and the treatment in which one representative wave length is considered is appropriate. In contrast the damping due to energy lost from the ends of the pipe is not local. For very long pipes the energy lost from the ends may be negligible compared to that lost due to local friction. In particular, as investigated by Goyder [2009] an acoustic standing wave may not be generated if the losses from the ends of the pipe are significant.

12 CONCLUSIONS

The following conclusions may be drawn.

1. A formal calculation of the Green's function for flow-acoustic excitation reveals many of the terms that influence corrugation noise. A full solution is not however possible because feedback effects are not explicit in the defining equations.
2. The maximum acoustic pressure is governed not by the source strength but by the balance between the feedback effects. In the model this is controlled by a parameter which is the ratio of acoustic velocity to mean flow velocity. This leads to a very simple formula for the acoustic pressure for saturation conditions.
3. An equation for the smallest flow rate which will give rise to corrugation noise has been developed. This requires knowledge of acoustic damping and involves another parameter which must be deduced from experiment associated with the growth of vortex shedding.
4. An unknown phase enters the model which relates the instant the vortex is ejected into the flow to the position of the acoustic velocity within its cycle. The most unfavourable case can be assumed but this is an area that needs further investigation.

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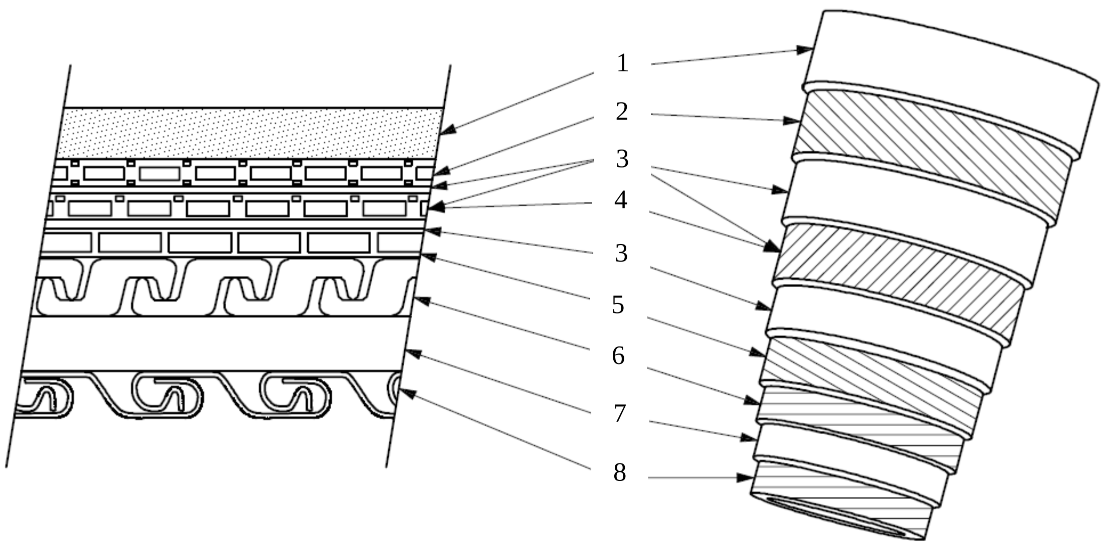

Figure 1. Construction of corrugated pipe. 1. Anti-friction layer, 2. Outer layer of tensile armour, 3. Anti-wear layer, 4. Inner layer of tensile armour, 5. Back-up pressure armour, 6. Interlocked pressure armour, 7. Internal pressure sheath, 8. Carcass.

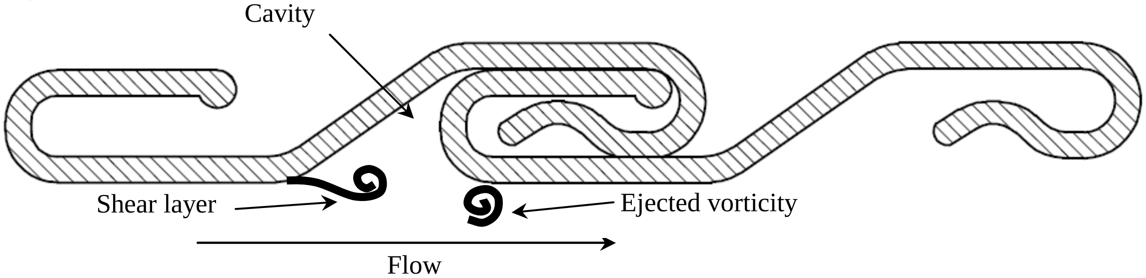

Figure 2. Corrugated pipe carcass showing cavity and shear layer.

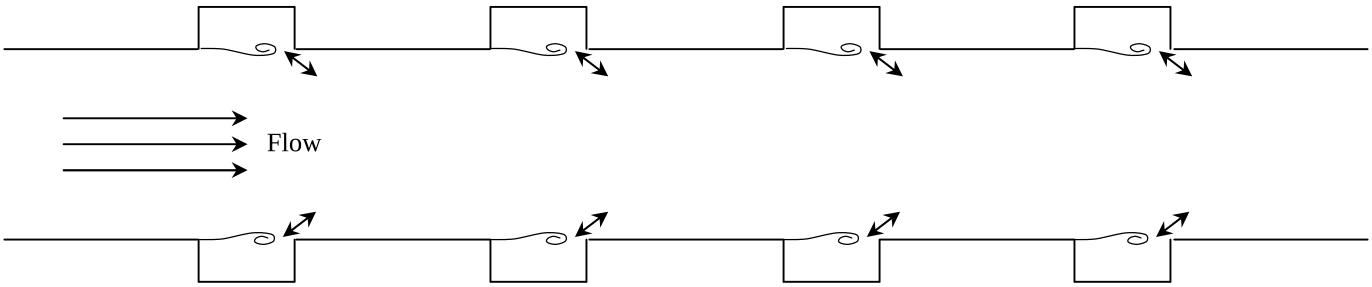

Figure 3. Diagrammatic representation of pipe, flow, cavities and shear layers. The sources are dipoles shown as arrows at the trailing edge.

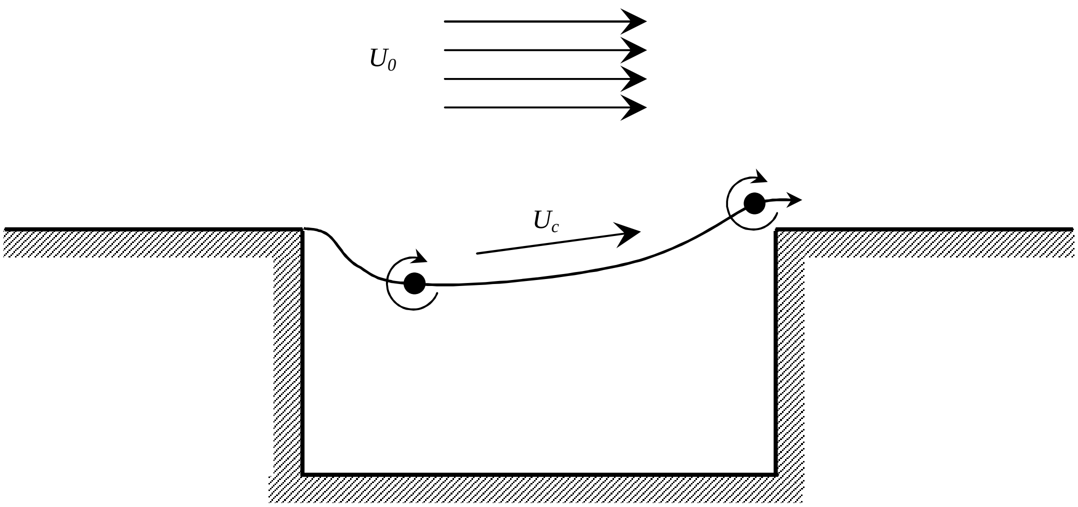

Figure 4. Illustration of vortices within a corrugation.

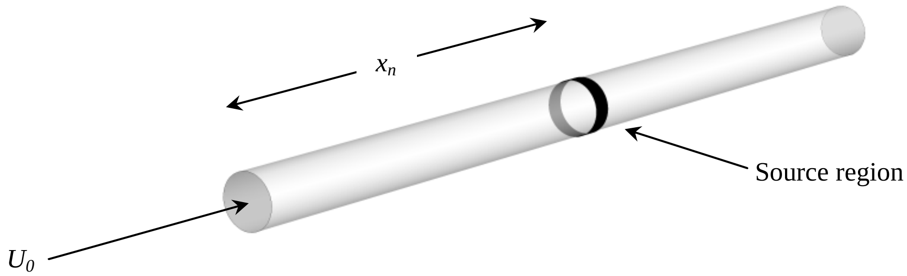

Figure 5. A pipe with a single source region as a ring around the pipe circumference

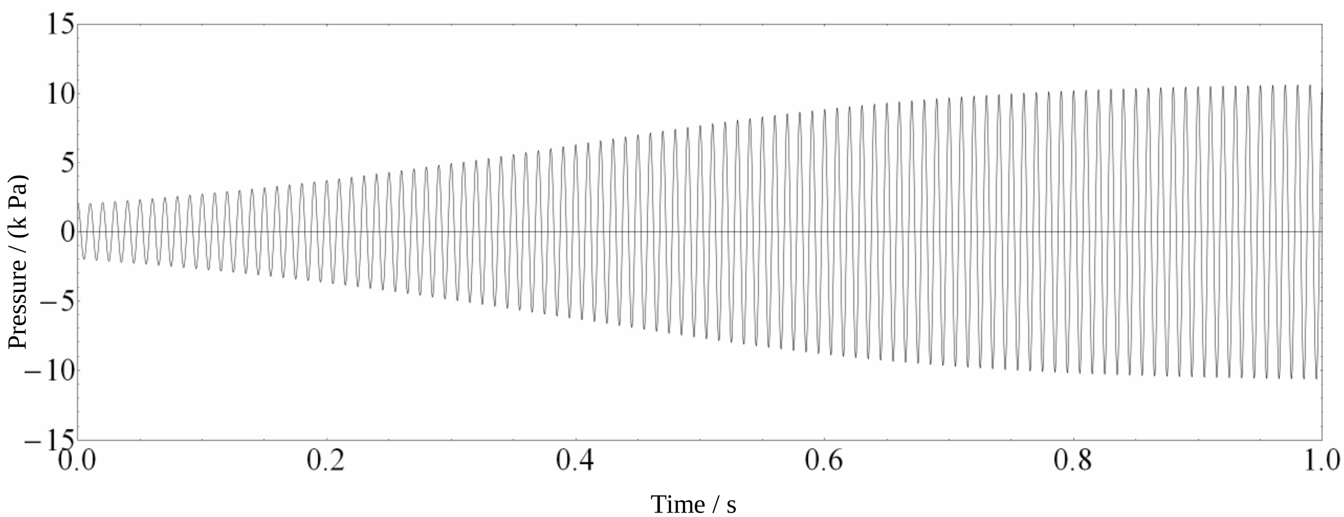

Figure 6. Numerical solution of differential equation. Density 90 kg/m^3, sound speed 400 m/s, damping ratio 0, natural frequency 100 Hz, mean flow velocity 3 m/s, corrugation width to pitch 0.4, $\alpha = 0.2$, $\beta = 13.33$, $\gamma = 0.5$.