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A NUMERICAL STUDY OF THE AEROACOUSTIC INTERACTION OF A CAVITY WITH A CONFINED FLOW: EFFECT OF EDGE GEOMETRY IN CORRUGATED PIPES

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ABSTRACT

In the present work a numerical study is carried out to investigate the aeroacoustic interaction of a 2D cavity with a confined flow. Incompressible flow simulations are performed with imposed incoming velocity perturbations. Equivalent time averaged acoustic source power is calculated through enthalpy differences using the theory of Vortex Sound. Information required for the estimation of acoustic source strength in high Reynolds number flows are acquired from relatively low Reynolds number, Re = O (10^3) simulations. This is done by an extrapolation method in which the effect of friction is estimated by considering a reference flow in a straight channel.

The proposed methodology is employed to explain the whistling phenomenon observed in corrugated pipes, which can be considered as rows of acoustically coupled cavities. The method successfully predicts the Strouhal number ranges where acoustic energy production and absorption occur. The method can also explain the effect of edge geometry on sound production, which is known to be important in corrugated pipes. In accordance with the earlier experimental study, *round upstream - sharp downstream* edge case leads to higher sound production levels than *sharp upstream – round downstream* case.

INTRODUCTION

Cavities in confined flows appear in various industrial applications including corrugated pipes, which can be modeled as a system composed of a series of axisymmetric cavities connected to each other with straight pipe segments, as shown in Figure 1.

There has been considerable attention to the mechanism of sound generation in corrugated pipes [3, 5, 6, 7, 10, 17, 20]. Flow separation occurring at the upstream edge of each cavity generates a shear layer, which is a source of unsteadiness. This unsteady flow induces an unsteady force on the walls of the bounding flow and the reaction force of the walls to this hydrodynamic force is a source of sound. The coupling of shear layer with an acoustic standing wave along the pipe can result into self-sustained oscillations, which is called whistling. It is important to note that the flexibility of the tube is not a necessary facet for the sound generation in corrugated tubes as shown by Nakamura and Fukamachi [13], but it can also be important as shown by Ziada [22].

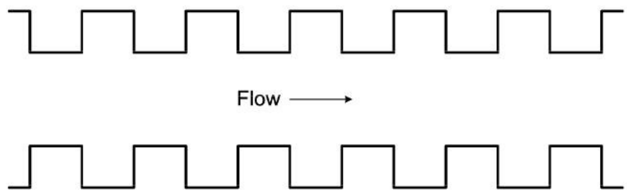

Figure 1: A typical cross-section of a corrugated pipe with sharp cavity edges.

Flow induced whistling phenomenon in corrugated pipes has a number of characteristics that are demonstrated extensively in the literature [3, 4, 6, 11, 13, 17, 20, 22]. Stepwise increase of the whistling frequency as a function of flow velocity is one of these particularities. It has been also noted that these whistling frequencies are close to the acoustic resonance frequencies of the system. A second characteristic is that all the consecutive whistling frequencies are close to a single non-dimensional frequency, called Strouhal number:

$$St = \frac{f\,L}{U} \quad (1)$$

where f is the whistling frequency, L is the characteristic length and U is the flow velocity.

The wavelength of the corrugations, pitch (p), is a commonly used characteristic length [6, 13, 19], see in Figure 2. However, recent studies [1, 11, 14] have demonstrated that the corrugation width (W) is the most suitable length scale. Keeping the cavity width fixed and varying the distance between the two corrugations, plateau (l), does not alter the Strouhal number based on the width. This suggests that the sound production is a local fluid dynamics phenomenon within a single cavity [1, 17, 22]. Thus, it is feasible to model a single cavity instead of modeling the whole corrugated segment as proposed by Popescu [18].

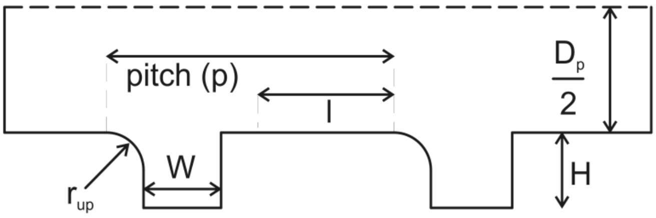

Figure 2: The geometric parameters of corrugated pipes.

It should also be noted that axisymmetry of the cavities is not a necessary feature for the whistling phenomenon observed in periodic systems. Experiments performed with multiple side branch system, which is a non-axisymmetry system, have shown that corrugated pipes and multiple side branch systems have similar whistling behaviors [14, 21].

Depending on the application, structural requirements and manufacturing technique geometric parameters of corrugated pipes (e.g. cavity width, pipe diameter, edge radius, cavity depth etc.) may vary in a fairly large range. It is known that these geometric parameters have significant effects on the whistling phenomenon [1, 14, 17]. Thus, it is an essential asset for industry to understand the effects of these parameters.

The primary purpose of the present work is to introduce a methodology that is developed to study numerically the aeroacoustic response of high Reynolds number confined flows to acoustic excitations. In the next section, this methodology is explained in detail. Later this approach is used to investigate the effect of cavity edges on the whistling phenomenon and compared with the available experimental data. In the last section, the conclusions about the proposed methodology and its capability of predicting the effect of cavity edge geometry is stated.

METHOD

In the first part of this section, the numerical simulations that were performed are explained. In the following part, calculation of average acoustic source power is discussed.

Numerical simulations

In corrugated pipes the cavity width of the corrugations and respectively in multiple side branch system, the diameter of the side branches are usually small compared to the wave length of the standing waves [7, 21]. The flow in such a cavity/side branch is locally incompressible. Thus, incompressible, unsteady, 2D planar simulations were performed [12] for a single cavity in a confined flow. The domain and the relevant geometric parameters are shown in Figure 3.

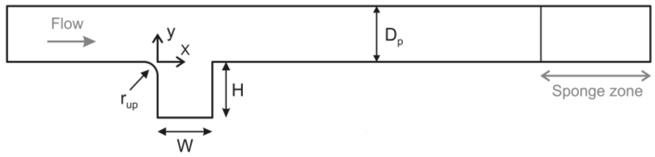

Figure 3: Computational domain and relevant parameters.

Duct height is denoted by D_p, the depth of the cavity by H, the width of the cavity by W and the radius of curvature of the upstream cavity edge by r_{up}.

Flow inlet is located at 3W upstream of the cavity and the outlet is placed at 9W downstream of the cavity. Cavity depth is taken equal to the cavity width. Previous experimental study [15] showed that H/W = 1 is in the range, where a saturation behavior is observed in the amplitude of pressure fluctuations with an almost constant peak whistling Strouhal number. In this range, variations in H/W ratio have no influence on the amplitude and peak whistling Strouhal number.

A finite volume commercial code, Fluent 6.3, was used. A pressure-based segregated solution algorithm, SIMPLE [16] was employed. The second-order implicit time discretization scheme together with the second-order upwind space discretization for convective terms was chosen. No turbulence modeling was applied. For each simulation, initially a steady flow solution is performed with an unexcited fully developed turbulent velocity profile $u(y,t) = u(y)$ as the inlet boundary condition, which has an average velocity of U. The iterations were continued until all the residuals dropped below 10^{-12}. Then a velocity perturbation $u'(t)$ with a certain frequency (f) and amplitude ($|u'_{max}|/U$):

$$u'(t) = \frac{|u'_{max}|}{U} \sin(2\pi\, f\, t) \qquad (2)$$

is superposed to the inlet velocity profile $u(y,t) = u(y) + u'(t)$. Different computation times were checked. A typical computation time of 5 periods of the excitation frequency appeared to be sufficient. Simulations with longer computation times provide the same results. The time step size was chosen as $\Delta t = 0.01\, W/U$.

The computational domain contains approximately 70000 quadrilateral cells which are clustered close to the opening of the cavity and to the walls, where there are high gradients of velocity due to shear layer and boundary layer, respectively, see Figure 4. In the domain between 6W and 9W downstream of the cavity, shown as sponge zone in Figure 3, cells with high aspect ratio ($\Delta x/\Delta y \gg 1$) are employed. By doing so vorticities travelling down the duct are artificially smoothed out. Thus, problems that could arise due to reverse flow at the outlet boundary condition, which is $\partial u/\partial x = 0$, are avoided. A study on mesh dependency was also carried out. The same computation was performed also with quadrilateral cell numbers of approximately 160000 and 320000. The difference in the calculated acoustic source power is less than 5%.

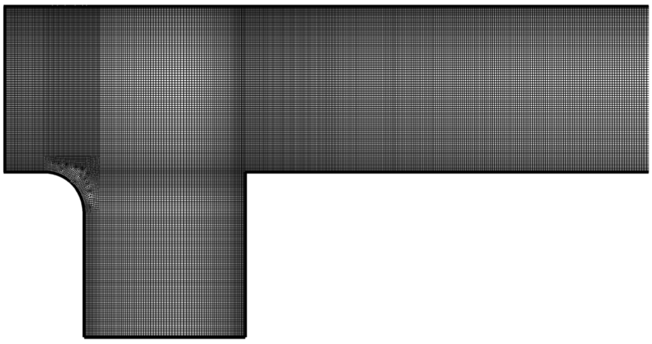

Figure 4: Part of the computational grid close to cavity.

Calculation of acoustic source power

A formal relationship between vortex shedding and sound generation can be established using a Helmholtz decomposition of the flow field $\vec{u}$ as:

$$\vec{u} = \nabla(\varphi_0 + \varphi') + \nabla \times \vec{\Psi} \qquad (3)$$

where, $\vec{\Psi}$ is the stream function, φ_0 and φ' are the steady and unsteady components of the scalar potential, respectively. Recognizing that the solenoidal vector field is incompressible, $\nabla.(\nabla \times \vec{\Psi}) = 0$. Acoustic field corresponds to the potential component of the flow, which is compressible. Then, acoustical flow velocity $\vec{u'}$ can be defined [9] as:

$$\vec{u'} = \nabla\varphi' \qquad (4)$$

For an incompressible flow with sufficiently high Reynolds number the effect of friction in the bulk of the flow can be neglected. Assuming a homentropic flow the momentum equation can be written as follows:

$$\nabla B = -\frac{\partial \vec{u}}{\partial t} - \vec{\omega} \times \vec{u} \qquad (5)$$

where $\vec{\omega} = \nabla \times \vec{u}$ is the vorticity and B is the total enthalpy:

$$B = \frac{1}{2}|\vec{u}|^2 + \int \frac{dp}{\rho} = \frac{1}{2}|\vec{u}|^2 + i \qquad (6)$$

where ρ is the fluid density and i is the specific enthalpy. Here it can be noted that the first term in the right hand side of the momentum equation $(-\partial\vec{u}/\partial t)$, is related to the potential flow solution and the second term corresponds to Coriolis force for a unit volume, $\vec{f_c} = -\rho(\vec{\omega} \times \vec{u})$, experienced by an observer moving with the flow.

Using the Howe's energy corollary [9], time average acoustic source power $\langle P_{source}\rangle$ due to the Coriolis force can be estimated at low Mach numbers as follows:

$$\langle P_{source}\rangle = -\rho \left\langle \int (\vec{\omega} \times \vec{u}).\vec{u'}\, dV \right\rangle \qquad (7)$$

where ρ is the fluid density, V is the volume in which $\vec{\omega}$ is non-vanishing and $\langle\, .\, \rangle$ is the time average. Using the Howe's energy corollary (eq. 7) together with the momentum equation (eq. 5), $\langle P_{source}\rangle$ can be calculated as follows:

$$\langle P_{source}\rangle = \underbrace{\rho \left\langle \int \nabla B'.\vec{u'} dV \right\rangle}_{\text{Source term}} + \underbrace{\rho \left\langle \int \frac{\partial \vec{u'}}{\partial t}.\vec{u'} dV \right\rangle}_{\text{Potential term}} \qquad (8)$$

Knowing that in a compact source region $\nabla.\vec{u'}$ is negligibly small; the source term $\nabla B'.\vec{u'}$ can be replaced by $\nabla.(\overrightarrow{u'}B')$. Also recognizing that $\vec{u'}$ is a harmonic function (eq. 2); due to time averaging over a period, the potential term is zero. Then using the divergence theorem $\langle P_{source}\rangle$ can be calculated as follows:

$$\langle P_{source}\rangle = \rho \left\langle \int (\vec{u'}\, B').\vec{n}\, dS \right\rangle \qquad (9)$$

where B' is the fluctuating total enthalpy. Considering a pipe segment including the cavity with a constant acoustical flow velocity, it is seen from eq. 9 that the acoustic source power generated in a volume can be calculated through the surface integral of fluctuating total enthalpy difference over the boundary of the volume.

In this derivation attention should be drawn to two points. First, it is assumed in the momentum equation (eq. 5) that the effect of friction in the bulk of the fluid is small enough to neglect. Second, though the potential flow related part of the flow causes a difference in total enthalpy (eq. 5); which after averaging over a period has no contribution to the sound generation (eq. 9).
The recent study of Martinez *et al.* [12] showed that after the time averaging the potential term (eq. 8) could still have a non-

zero contribution to the $\langle P_{source} \rangle$. They concluded that it is essential to remove the potential term $(-\partial \vec{u}/\partial t)$, from the enthalpy difference (∇B) before taking the time averaging. Their technique is based on successive linear least-square fits of the total pressure jumps considering many measuring planes both at the upstream and the downstream of the cavity. This procedure provides promising results for the response of low Mach number flows to acoustic excitations. However, this approach has some shortcomings. First of all, the necessity of a stable unperturbed solution as initial condition, which could only be achieved around $Re \approx 1000$, limits the approach to low Reynolds numbers. Related to this flaw, $\langle P_{source} \rangle$ appears to be Reynolds number dependent, while it was assumed that the Reynolds number is sufficiently high to neglect the friction. Another drawback is that the bounds of the source region cannot be determined. Due to uncertainty in the spatial linear fit, $\langle P_{source} \rangle$ appears to depend on the arbitrary choice of the position of the measuring planes.

Here an alternative method is proposed where not only the potential flow related part but also the effect of friction is estimated. This is done by means of a reference flow simulation in a straight pipe with identical boundary conditions with the respective cavity simulation and using the same measurement sections in the duct, e.g. x_1 and x_2, see Figure 5. Then $\langle P_{source} \rangle$ can be estimated as follows:

$$\langle P_{source} \rangle = \rho \left\langle \frac{1}{4} \left([B'_{x2} - B'_{x1}]_{cav} - [B'_{x2} - B'_{x1}]_{ref} \right) u' \, \pi \, {D_p}^2 \right\rangle \qquad (9)$$

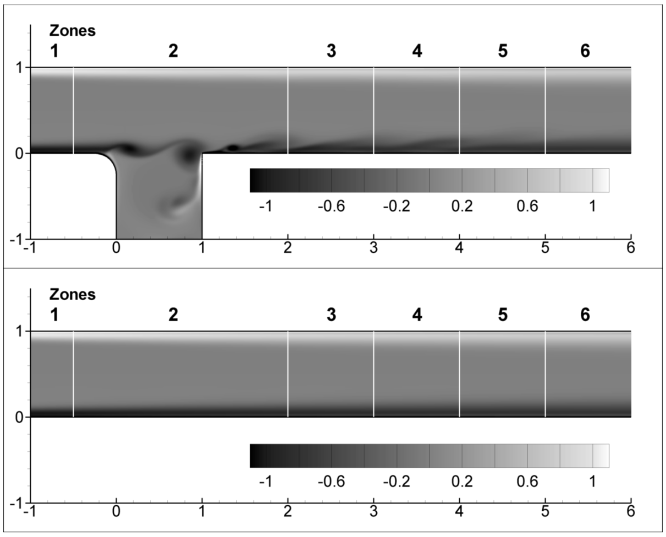

Figure 5: Normalized vorticity contours for a confined cavity flow and a reference flow in a straight pipe (Re = 4000, St = 0.6 and $|u'_{max}|/U = 0.2$). White lines represent the measurement sections upstream and downstream of the cavity.

In contrary to earlier approach, the proposed method can provide average acoustic source powers which are converging to a Reynolds number independent limit. This is demonstrated in Figure 6 for a simulation with perturbation amplitude of $|u'_{max}|/U = 0.2$ and at a $St = 0.5$. Other simulations that are considered here have also similar characteristics, in which above a certain Reynolds number, $\langle P_{source} \rangle$ can be assumed weakly depend on Reynolds number. This limit is determined as Re = 4000, which is used in all the simulations. By increasing the number of cells in the computational domain, simulations with higher Reynolds number can be achieved, which will increase the Reynolds number independency of the results.

This approach allows also the determination of the source region. In Figure 7 dimensionless average acoustic source power $\langle P_{source} \rangle / \rho \, U \, S \, |u'|^2$ that is produced in each zone in the duct is presented at a Strouhal number of 0.5. It is seen that the source region is mostly limited to zone 2. All the numerical simulations independent of the Strouhal number and perturbation amplitude have a similar bound for the source region, which is between 0.5W upstream of the cavity and W downstream of the cavity.

Simulations have been performed with various Strouhal numbers between 0.1 and 1.2, for a cavity with $D_p/W = 1$, $r_{up}/W = 0$, to determine the range of Strouhal numbers where there is sound production. For a perturbation amplitude of $|u'_{max}|/U = 0.2$, a dimensionless average acoustic source power of $\langle P_{source} \rangle / \rho \, U \, S \, |u'|^2 = 0.10$ is obseved at St = 0.6. This outcome is in good agreement with the simulations of Hofmans [8]. With an inviscid two-dimensional vortex blob method for the same amplitude $|u'_{max}|/U = 0.2$, he observed a dimensionless average acoustic source power of $\langle P_{source} \rangle / \rho \, U \, S \, |u'|^2 = 0.095$ at St = 0.6.

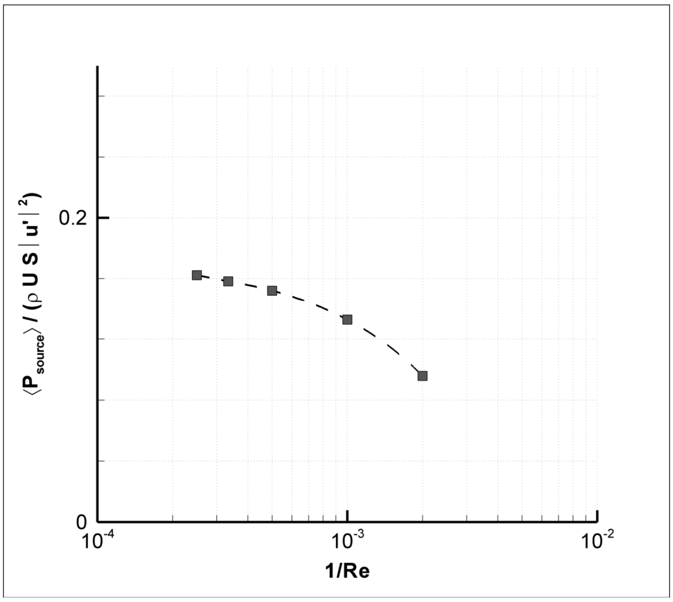

Figure 6: Reynolds number independency: Reynolds number is plotted against dimensionless average acoustic source power $\langle P_{source} \rangle / \rho\, U\, S\, |u'|^2$ at $St = 0.5$, $|u'_{max}|/U = 0.2$ and $r_{up}/W = 0.25$.

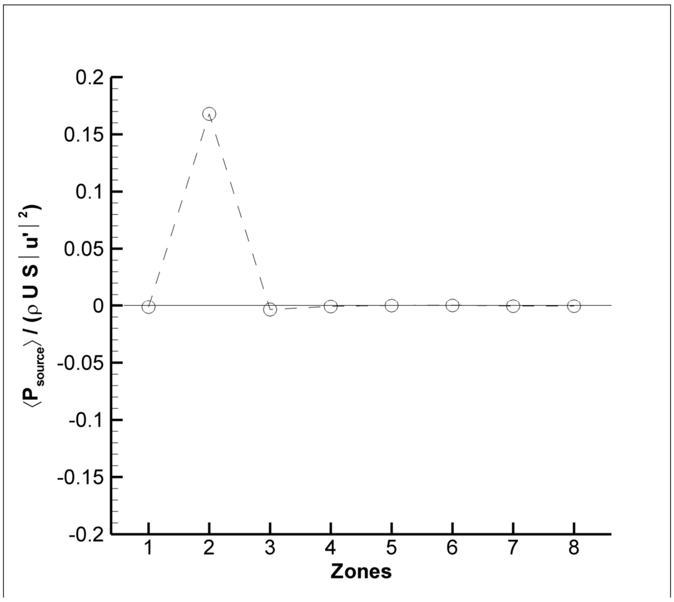

Figure 7: Dimensionless average acoustic source power $\langle P_{source} \rangle / \rho\, U\, S\, |u'|^2$ for each zone (the zone regions are shown in Figure 5) at $St = 0.5$, $|u'_{max}|/U = 0.2$ and $r_{up}/W = 0.25$.

EFFECT OF EDGE GEOMETRY

The developed numerical method is used to investigate the effect of cavity edges on the Strouhal number and on the amplitude of pressure fluctuations, which is known to be essential [1, 2, 14].

In the previous study [15], the effect of edge rounding on the whistling was investigated with a multiple side branch system. In Figure 8, measured dimensionless pressure fluctuation amplitude $p'_{max}/(\rho\, c\, U)$ for the 3rd acoustic mode[1] as a function of Strouhal number is presented for *round upstream - sharp downstream* case and *sharp upstream - round downstream* case. From this experiment, it was seen that *round upstream – sharp downstream* case has 3 times higher pressure fluctuation amplitudes than *sharp upstream – round downstream* case. This increase in pressure fluctuation due to the rounding of upstream cavity edge can be qualitatively predicted by vortex sound theory [9], as discussed in [2].

It was also concluded that when the cavity width $(W_{eff})^2$ is used as the characteristic length (L) in Strouhal number, *round upstream - sharp downstream* case has a lower peak whistling Strouhal number[3] than the *sharp upstream - round downstream* case. This is explained through an increase in the travel distance of the vorticity perturbation by rounding the upstream edge off. The downstream edge is less critical [2, 14] because the vorticity or the shear layer thickness is not localized as it approaches the downstream edge. The vortex core is typically large compared to the edge radius. Thus, it was clarified that the characteristic length used in the Strouhal number definition should be the sum of the cavity width and the upstream edge radius, $L = W + r_{up}$ [1].

In parallel to the experiments, two set of numerical simulations are performed for *round upstream – sharp downstream* case and *sharp upstream – round downstream* case, respectively. The radius of curvature is $r/W = 0.25$. A Strouhal number range between 0.2 and 1.2 is investigated with a perturbation amplitude of $|u'_{max}|/U = 5\%$, which is similar to the observed dimensionless pressure fluctuation amplitudes in the experiments. In Figure 9, for these two configurations dimensionless average acoustic source power $\langle P_{source} \rangle / \rho\, U\, S\, |u'|^2$ as a function of Strouhal number are presented.

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[1] Multiple side branch system is an acoustic analogous of corrugated pipes only for the lowest acoustic modes e.g. 2nd, 3rd [15, 21]. Here the results of the 3rd acoustic mode is provided, however it is an arbitrary choice. The effect of cavity edges on the whistling is independent of the acoustic mode [14].

[2] It should be noted that a main geometric difference between corrugated pipes and multiple side branch system is that the cross section of corrugation cavity is a slit whereas the side branches have circular cross sections. Thus, the side branch diameters are converted to effective cavity width ($W_{eff} = \pi D_{sb}/4$) as proposed by Bruggeman *et al.* [2] which is the average width of the side branch cross section.

[3] Peak whistling Strouhal numbers corresponds to the Strouhal number where the maximum amplitude in pressure fluctuations are registered [15].

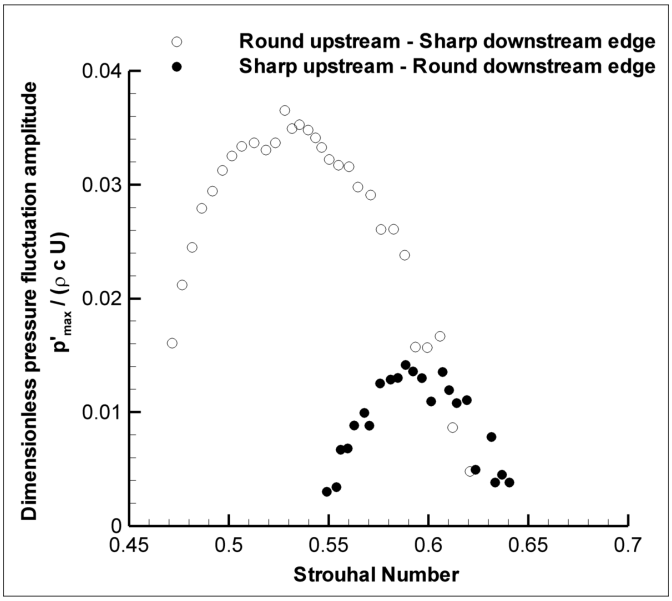

Figure 8: Measured dimensionless pressure fluctuation amplitude for the 3rd acoustic mode as a function of Strouhal number for *round upstream - sharp downstream* ($r_{up}/W = 0.1$) case and *sharp upstream - round downstream* ($r_{down}/W = 0.1$) case. (Nakiboglu *et al.* [15])

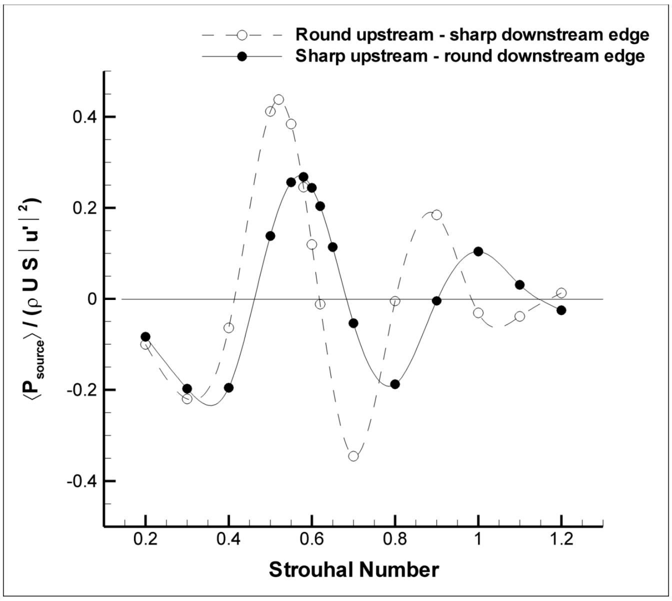

Figure 9: Calculated dimensionless average acoustic source power $\langle P_{source} \rangle / \rho\, U\, S\, |u'|^2$ as a function of Strouhal number for *round upstream– sharp downstream* ($r_{up}/W = 0.25$) edge and *sharp upstream – round downstream* ($r_{down}/W = 0.25$) edge configurations with a perturbation amplitude of $|u'_{max}|/U = 0.05$.

Similar to the experimental observations *round upstream – sharp downstream* case has a higher acoustic source power, almost 2 times, than *sharp upstream – round downstream* case. However a direct comparison of experimental and numerical results is not possible at this moment. In the experiments pressure fluctuation amplitudes are measured $p'_{max}/(\rho\, c\, U)$, in the numerical simulations, whereas normalized acoustic source power is calculated $\langle P_{source} \rangle / \rho\, U\, S\, |u'|^2$. To achieve a direct comparison, an energy balance model between the acoustic sources and losses is necessary. Using such a model [21], together with the calculated acoustic source power a finite fluctuation amplitude for the whistling can be predicted. Furthermore, in accordance with the experiments, peak whistling Strouhal number is lower for *round upstream – sharp downstream* case than *sharp upstream – round downstream* case.

CONCLUSION

A methodology, which combines relatively low Reynolds number Re = O (10^3), incompressible flow simulations and vortex sound theory, is proposed to investigate the aeroacoustic response of high Reynolds number Re = O (10^5) confined flows. The proposed method is computationally more demanding than an existing approach [12], due to the necessity of a reference flow. However it offers certain advantages, e.g. Reynolds independency, robustness and bounded source region Predicted Strouhal number ranges of acoustic energy production and absorption are in agreement with the earlier studies in the literature. The method is used to investigate the effect of edge geometry on sound production, which is known to be important. In line with the earlier experimental study on multiple side branch system, *round upstream - sharp downstream* edge case has a lower peak whistling Strouhal number and a higher acoustic source power than *sharp upstream – round downstream* case.

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