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Characteristics of Centrifugal Pump Rotor System under Exciting Forces Generated by Shroud Leakage Flows

Quan-zhong Liu, Ru-zhi Gong, Hong-jie Wang, Yang Yao
School of Energy Science and Engineering, Harbin Institute of Technology, Harbin 150001, P.R China

Abstract

In order to investigate the flow-induced vibration in the impeller clearance of centrifugal pump, and make the rotordynamic prediction of the centrifugal pump rotor system, a rotordynamic force model is established which is indispensable to the impeller-rotor system dynamics analysis. The coefficient in the dynamic force model is solved by quasi-steady CFD solution. Multiple quasi-steady solutions of an eccentric three-dimensional model at a series of relative spin velocity yield the rotordynamic forces at different whirl frequency ratios, and a second-order least-square fitting of the rotordynamic forces generates the skew-symmetric stiffness, damping, and mass matrices. Based on the work, the finite element method is applied to establish the model of turbo pump impeller-rotor system. The phenomenon of the fluid oscillation and some typical features are explained from the perspective of oscillation fluid mechanics. The results show complicated frequency characteristics presents in the turbo pump rotor vibration due to the exciting forces generated by shroud leakage flows.

Key words

centrifugal pump, impeller-rotor system, rotordynamic force, numerical computation, Stability.

Introduction

As the fluid pressure and rotor flexibility increasing, the fluid-induced vibration in the clearance of centrifugal pump impeller becomes a very important factor to reduce the stability of centrifugal pump rotor system. The clearance structure and the flow feature lead to the complexity of the equations of fluid dynamics [1]. And it is difficult to describe the rotordynamic force by analytical expression, resulting in great difficulty for the dynamics analysis. The impeller clearance can be divided into shroud passage and wearing seal based on the structure and flow feature [2]. The bulk flow model is mainly used for the wearing seal, which is based on film assumption [3]. The Black-Childs model [4, 5] and Muszynska model [6] both are given on this, and are generally applied to the dynamic analysis of rotor-seal system. But for the shroud passage, its physical dimension of channel section is not constant; and the velocity magnitude and direction between the sections are also different. So the unit radius and axial velocity cannot be used to describe the flow in it, and the existing rotordynamic force model of the channel fluid is limited to the shroud passage flow. Some scholars have combined CFD method and hydrodynamic lubrication theory to solve the time-averaged Navier-Stokes equations of the clearance flow [7~9] and to get the fluid-induced vibration characteristics of the impeller clearance flow.

In this paper, a rotordynamic force model is put forward. The coefficient in the impeller clearance rotordynamic force model is solved by quasi-steady CFD solution of an eccentric 3-D model at different whirl frequency ratios and rotation speeds. After the confirmation of the rotordynamic force model, the finite element method is applied to establish the model of a turbo pump impeller-rotor system. And the Newmark method is used to analyze the dynamic response of impeller-rotor system. From the perspective of oscillation fluid mechanics, the phenomenon of the fluid oscillation and typical features of the impeller-rotor system are explained.

The formulation of the exciting force of impeller clearance

For wearing seal, the general models to describe the rotordynamic force are based on the small gap circulation theory. Muszynska model is one of the most typical cases, which can be expressed as:

$$-\begin{bmatrix} F_{cx} \\ F_{cy} \end{bmatrix} = \begin{bmatrix} K_c - m_{cf}\tau^2\omega^2 & \tau\omega D_c \\ -\tau\omega D_c & K_c - m_{cf}\tau^2\omega^2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} D_c & 2\tau m_{cf}\omega \\ -2\tau m_{cf}\omega & D_c \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} m_{cf} & 0 \\ 0 & m_{cf} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} \tag{1}$$

where K_c , D_c , m_{cf} reflect the perturbed motion stiffness, damping and inertial effects on the rotor respectively, which can be defined as:

$$K_c = K_{c0}\left(1-e^2\right)^{-n}, \quad D_c = D_{c0}\left(1-e^2\right)^{-n},$$
$$n = 0.5 \sim 3, \quad \tau = \tau_0\left(1-e\right)^b, \quad 0 < b < 1$$

where $e = \sqrt{x^2+y^2}\big/c$ is the relative center displacement; c is the clearance of the seal. Generally, $\tau_0 < 1/2$. K_{c0}, D_{c0} and m_{cf} can be determined by the following equation [5]:

$$K_{c0} = \mu_3\mu_0, \quad D_{c0} = \mu_1\mu_3 T, \quad m_{cf} = \mu_2\mu_3 T$$

For the shroud passage, considering the symmetry of the model, the rotordynamic force model based on gap annular flow dynamic model can be expressed as [2]:

$$-\begin{Bmatrix} F_{tx}/\rho\pi R^3 L \\ F_{ty}/\rho\pi R^3 L \end{Bmatrix} = \begin{bmatrix} K_t\omega^2 & k_t\omega^2 \\ -k_t\omega^2 & K_t\omega^2 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{bmatrix} C_t\omega & c_t\omega \\ -c_t\omega & C_t\omega \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} M_t & m_t \\ -m_t & M_t \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} \tag{2}$$

where, ρ is the fluid density, R is the impeller outlet radius, L is the impeller outlet width, and ω is the rotating speed.

Combining Eq.(1) and Eq.(2), the exciting forces generated by impeller clearance can be given by:

$$-\begin{bmatrix} F_x \\ F_y \end{bmatrix} = -\begin{bmatrix} F_{cx}+F_{tx} \\ F_{cy}+F_{ty} \end{bmatrix} = \left[K_{sd}\right]\begin{bmatrix} x \\ y \end{bmatrix} + \left[C_{sd}\right]\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \left[M_{sd}\right]\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} \tag{3}$$

Where:

$$\left[K_{sd}\right] = \begin{bmatrix} K_s\omega^2 + (K_c - m_{cf}\tau^2\omega^2) & k_s\omega^2 + \tau\omega D_c \\ -k_s\omega^2 + \tau\omega D_c & K_s\omega^2 + (K_c - m_{cf}\tau^2\omega^2) \end{bmatrix}$$

$$\left[C_{sd}\right] = \begin{bmatrix} C_s\omega + D_c & c_s\omega + 2\tau m_{cf}\omega \\ -c_s\omega - 2\tau m_{cf}\omega & C_s\omega + D_c \end{bmatrix}$$

$$\left[M_{sd}\right] = \begin{bmatrix} M_s + m_{cf} & m_s \\ -m_s & M_s + m_{cf} \end{bmatrix}$$

$$\left[K_s, k_s, C_s, c_s, M_s, m_s\right] = \left[K_t, k_t, C_t, c_t, M_t, m_t\right]\cdot \rho\pi R^3 L \text{。}$$

The quasi-steady solution method of dynamic parameters

The exciting forces will be presented here in non-dimensional form by dividing the forces by $\rho\pi\omega^2 R^3 L$. Thus

$$\begin{Bmatrix} F_x(t) \\ F_y(t) \end{Bmatrix} = \left[A\right]\begin{Bmatrix} x(t)/R \\ y(t)/R \end{Bmatrix} = -\begin{bmatrix} M & m \\ -m & M \end{bmatrix}\begin{Bmatrix} \ddot{x}/R\omega^2 \\ \ddot{y}/R\omega^2 \end{Bmatrix} - \begin{bmatrix} C & c \\ -c & C \end{bmatrix}\begin{Bmatrix} \dot{x}/R\omega \\ \dot{y}/R\omega \end{Bmatrix} - \begin{bmatrix} K & k \\ -k & K \end{bmatrix}\begin{Bmatrix} x/R \\ y/R \end{Bmatrix} \tag{4}$$

where the non-dimensional form of $M_{sd}, m_{sd}, C_{sd}, c_{sd}$,

K_{sd}, k_{sd} are:

$$M, m = \frac{M_{sd}, m_{sd}}{\rho \pi R^2 L},$$

$$C, c = \frac{C_{sd}, c_{sd}}{\rho \pi R^2 L \omega},$$

$$K, k = \frac{K_{sd}, k_{sd}}{\rho \pi R^2 L \omega^2}$$

Let $x = e \cdot \cos \Omega t$ and $y = e \cdot \sin \Omega t$, where Ω is the whirl velocity of impeller. An alternative notation is to define the exciting forces, F_n and F_t, that are normal and tangential to the circular whirl orbit at the instantaneous position of the centre of rotation. It follows that

$$\begin{bmatrix} F_n \\ F_t \end{bmatrix} = \begin{bmatrix} \cos \Omega t & \sin \Omega t \\ -\sin \Omega t & \cos \Omega t \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix}$$

$$= \begin{bmatrix} \cos \Omega t & \sin \Omega t \\ -\sin \Omega t & \cos \Omega t \end{bmatrix} \begin{bmatrix} A_{xx} & A_{xy} \\ A_{yx} & A_{yy} \end{bmatrix} \begin{bmatrix} e / R \cdot \cos \Omega t \\ e / R \cdot \sin \Omega t \end{bmatrix} \quad (5)$$

where:

$$A_{xx} = M \left(\Omega / \omega \right)^2 - c \left(\Omega / \omega \right) - K,$$

$$A_{xy} = m \left(\Omega / \omega \right)^2 + C \left(\Omega / \omega \right) - k,$$

$$A_{yx} = -m \left(\Omega / \omega \right)^2 - C \left(\Omega / \omega \right) + k,$$

$$A_{yy} = M \left(\Omega / \omega \right)^2 - c \left(\Omega / \omega \right) - K 。$$

From the above analysis $F_n = A_{xx} = A_{yy}$ and $F_t = A_{yx} = -A_{xy}$. So,

$$\begin{cases} F_n = M (\Omega / \omega)^2 - c (\Omega / \omega) - K \\ F_t = -m (\Omega / \omega)^2 - C (\Omega / \omega) + k \end{cases} \quad (6)$$

Therefore, M, m, C, c, K, k can be obtained by fitting the second order curves of F_n, F_t to Ω / ω.

For a shroud passage flow field under a whirl ratio, the impeller housing is stationary, and the impeller shroud revolve about it center o at a speed of ω. At the meanwhile, it also rotates about the housing center o' at a speed of Ω. This unsteady flow can be solved using coordinate transformation to get quasi-steady 3-D flow field. As shown in Fig.1, the relative motion between housing and shroud, can be obtained by the rotating motion between the inside wall and outside wall of the shroud passage. The housing (outside wall) rotates on the fixed point o' with a speed of ω, and the shroud (inside wall) revolves on its center o with a speed of ($\Omega - \omega$), then flow in the impeller clearance keeps quasi-steady in this coordinate.

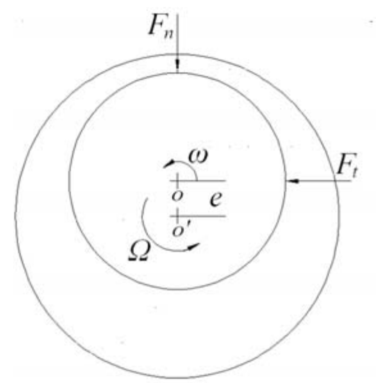


Fig.1 The schematic of impeller whirling

The control formulation of flow field and discrete method

The Reynolds-Averaged momentum equation is used to describe the incompressible flow, and the tensor form of the equations is:

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \frac{1}{\rho} \frac{\partial}{\partial x_j} \left[-p \delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) + \left(-\rho \overline{u_i' u_j'} \right) \right] \quad (7)$$

where, $u_i (i = 1,2,3)$ is the velocity of flow, p is the pressure of flow, μ is the kinematic viscosity, and $-\rho \overline{u_i' u_j'} (i, j = 1,2,3)$ is the Reynolds stress.

When solving the momentum equation and continuity equation in the rotating coordinate, the relationship between the absolutely velocity $\vec{v}$ and relative velocity $\vec{v}_r$ is as follows:

$$\vec{v}_r = \vec{v} - (\vec{\omega} \times \vec{r}) \quad (8)$$

where, $\vec{\omega}$ is the rotating angle velocity, and $\vec{r}$ is the position vector in the rotating coordinate.

Therefore, in the rotating coordinate, the left-hand of the momentum equation can be written in the form of the relative velocity:

$$\frac{\partial}{\partial t}(\vec{v}_r)+\nabla\cdot(\vec{v}_r\vec{v}_r)+(2\vec{\omega}\times\vec{v}_r+\vec{\omega}\times\vec{\omega}\times\vec{r})+\frac{\partial\vec{\omega}}{\partial t}\times\vec{r} \quad (9)$$

Since the whirl velocity is given when the quasi-steady method is used to compute the flow field in the impeller clearance, the $\frac{\partial\vec{\omega}}{\partial t}\times\vec{r}$ term, which is the angular acceleration of impeller rotating, can be omitted. The parts which denote the centrifugal force and Coriolis force can be moved to the source term in the right side of the momentum equation.

The momentum equation will be closed when adopting the RNG k-ε model and combining the continuity equation. The RNG k-ε model can be expressed as:

$$\begin{cases}\frac{\partial k}{\partial t}+\frac{\partial v_i k}{\partial x_i}=\frac{\partial}{\partial x_i}\left[\left(v+\frac{v_t}{\sigma_k}\right)\left(\frac{\partial k}{\partial x_i}\right)\right]+p_r-\varepsilon \\ \frac{\partial \varepsilon}{\partial t}+\frac{\partial v_i \varepsilon}{\partial x_i}=\frac{\partial}{\partial x_i}\left[\left(v+\frac{v_t}{\sigma_\varepsilon}\right)\left(\frac{\partial \varepsilon}{\partial x_i}\right)\right]+\frac{C_{\varepsilon1}\varepsilon p_r-C_{\varepsilon1}\varepsilon^2}{k}\end{cases} \quad (10)$$

where p_r denotes the generation rate of turbulent kinetic energy.

$$p_r=v_t\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j} \qquad v_t=C_u\frac{k^2}{\varepsilon} \quad (11)$$

The five parameters in the model can be abstained through theoretical analysis: C_u =0.0845， $C_{\varepsilon1}$ =1.42， $C_{\varepsilon2}$ =1.68， σ_k=1， σ_ε=1.3.

The SEGREGATED implicit solver is used to solve the impeller clearance flow, and the SIMPLE pressure correction algorithm is used for solving the coupling of velocity and pressure. The turbulent kinetic energy, the turbulence dissipation and the momentum equation are discrete through second-order upwind scheme. The convergence is determined when the maximum residual reaches 10^{-4}.

Results and analysis

Based the analysis above, the 3-D model of centrifugal pump including the main flow in the impeller and the clearance is established as shown in Fig.2. The inlet and outlet pressure of the clearance can be obtained by solving the global flow in impeller and the clearance at the given rotating speed, then the clearance flow can be solved by the quasi-steady method independently at the given rotating speed, and the characteristics of the exciting forces generated by clearance flow can be analyzed. In other words, the influence of impeller whirl to the inlet and outlet pressure of the clearance flow is ignored in the quasi-steady computation in this paper.

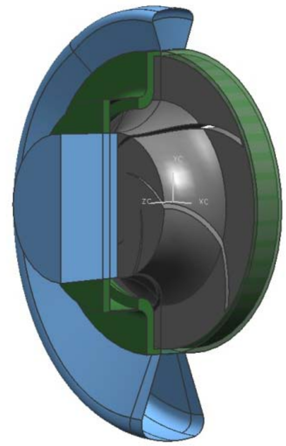


Fig.2 The geometric model of centrifugal pump

In order to get the coefficients in the rotordynamic force model of impeller clearance flow field, the impeller clearance flow field is calculated through quasi-steady method under a number of rotating speeds. Due to massive data and similar characteristics, the results of rotating speed at 3000r/min will be shown here only.

Under constant speed at 3000r/min, for the purpose of fitting the second order curve of the relationship between F_n, F_t and Ω/ω, three groups of Ω/ω and their corresponding normal force F_n and tangential force F_t are needed. In order to ensure the accurate calculation, eleven cases are taken in the range (0~1.5) of Ω/ω, and the impeller clearance flow field is calculated through quasi-steady method at every whirl ratio Ω/ω. The normal force F_n and tangential force F_t at each whirl ratio Ω/ω can be obtained through integration of the pressure on the impeller outer face.

Under the rotating speed of 3000r/min, with the calculation of the impeller clearance flow field through quasi-steady method and integration of the pressure on the impeller outer face, the normal force F_n and tangential force F_t , as well as the relationship between the two and the whirl ratio, are listed below. As shown in Fig.3, F_t^* and F_t^* are the fitting results of the computation results F_t and F_n in the range (0~1.5) of Ω/ω.

It also clearly seen that in the non-dimensional force acting on impeller from impeller clearance, the relationship between the tangential force and the whirl ratio is approximately linear, while the normal force varies much with the whirl ratio increases, and it reaches the minimum value when the whirl ratio is about 1. In addition, the normal force will increase no matter the whirl ratio increases or decreases.

Based on formulation (6), the six dynamic parameters can be obtained through the data above at the rotating speed of 3000r/min: M =2.90，C =2.05，K =4.33，m =0.06，c =8.11，k =2.03.

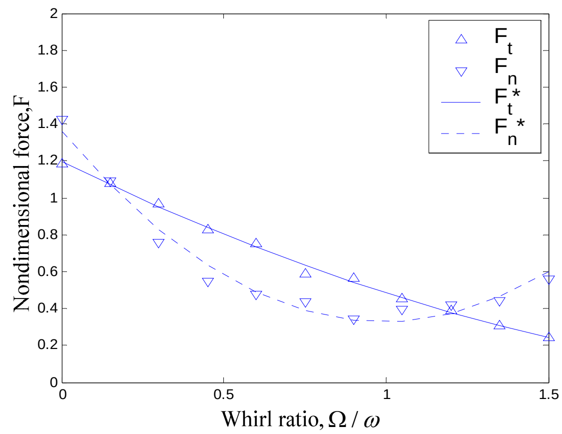


Fig.3 The relationship between the force and the whirl ratio at speed of 3000r/min

Using the same method, K、k、C、c、M、m at other speeds can be obtained. The results are listed in Table 1.

Tab.1 The dynamic parameters under different speeds

Rotating speed(r/min)	K	k	C	c	M	m
500	125.94	8.01	6.75	9.13	3.8	0.20
1000	32.12	4.42	5.42	9.13	3.85	0.19
1500	14.75	3.22	4.63	9.05	3.27	0.17
2000	8.67	2.63	3.55	8.85	3.13	0.15
2500	5.86	2.27	2.94	8.6	3.08	0.09
3000	4.33	2.07	2.05	8.11	2.90	0.06
3500	3.41	1.86	2.04	7.53	3.08	0.06
4000	2.81	1.73	1.98	7.31	3.08	0.06
4500	2.40	1.63	1.95	7.27	3.08	0.07
5000	2.11	1.55	1.95	6.75	2.93	0.07
5500	1.89	1.48	1.80	6.88	2.74	0.05
6000	1.72	1.43	1.77	6.36	3.37	0.05
6500	1.59	1.38	1.69	5.97	3.28	0.05
7000	1.49	1.34	1.51	6.02	3.21	0.06
7500	1.41	1.31	1.47	5.84	3.37	0.06
8000	1.34	1.28	1.42	5.45	3.17	0.07
8500	1.28	1.25	1.41	5.24	3.17	0.07
9000	1.24	1.23	1.37	4.82	3.27	0.07
9500	1.20	1.22	1.29	4.64	3.27	0.07

10000	1.16	1.19	1.25	4.33	3.27	0.06
10500	1.11	1.18	1.08	4.02	3.27	0.05
11000	1.11	1.15	1.25	3.85	3.11	0.05
11500	1.09	1.14	1.32	3.72	2.84	0.05
12000	1.07	1.13	1.04	3.89	2.62	0.05

With the twenty-four groups results under quasi-steady calculation, the parameters τ_0、n、b、 ξ、m_0、n_0、K_s、k_s、C_s、c_s、M_s、m_s in formulation (3) can be regressed using the least squares fitting method. The results are as follows:

The parameters about wearing seal: τ_0 =0.5，n =2，b =0.5，m_0 =-0.25，n_0 =0.079，ξ =0.97;

The parameters about shroud passage: M_s =1.926，C_s =0.376，K_s =0.35，m_s =0.037，c_s =0.112，k_s =0.15.

In the results above, since m_s is very small, and usually ignored. This is can be proved by experiment results [11].

Dynamic analysis of a centrifugal pump rotor system

The finite element method of a turbo pump rotor system is established as shown in Fig. 4. The model of spin shaft includes 10 units and 11 nodes. Node 5 and Node 8 are the position of bearing A and bearing B and the shaft is rigid support at both bearings. The stiffness and damping of both bearings are 9.8×10^7 N/m and 100 N*s/m respectively. Node 4 and Node 10 are the position of pump and turbo, and the exciting forces generated by shroud leakage flows as shown in equation 3 acts on Node 4.

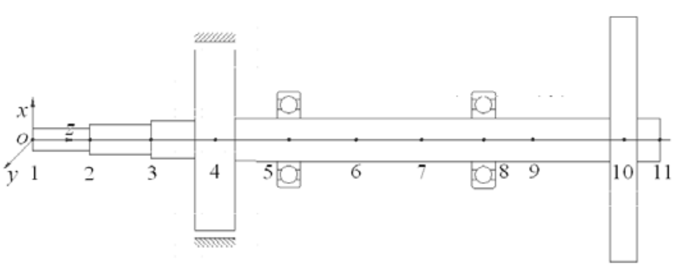
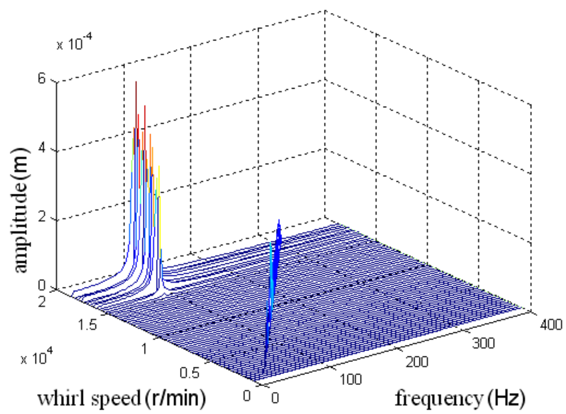


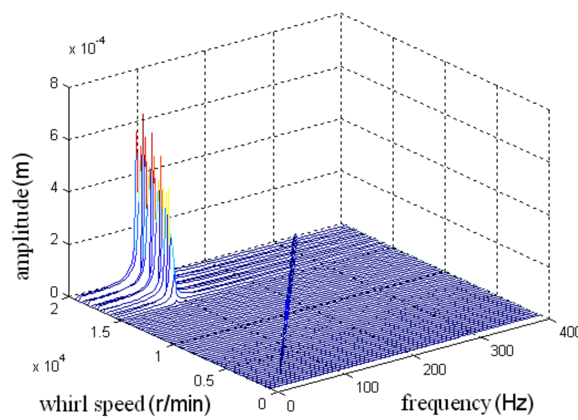
Fig.4 The finite element method of a turbo pump rotor system

The rotordynamic equation is solved by Newmark method to investigate the dynamic system behavior at different rotating speeds, and the control parameter γ and β of the solver are 0.5 and 0.25 respectively.

As is known, the vibration induced by gap annular flow is forward precession, and the frequencies of the vibration are in a low range which is independent of the spin velocity. But concerning the nonlinear character of the fluid induced forces, the frequencies of the vibration also have the complicated character occasionally.



(a) e =0.05mm



(b) e =0.02mm

Fig.5 Waterfall plot

The waterfall plot of the Node 4 with the fluid induced forces and eccentricity (e =0.05mm) was shown in Fig. 5(a). It is shown that there was a low frequency component (91.2Hz) which was sufficiently less than whirl frequency from the spin velocity (14760r/min), and this frequency component sufficiently closes to the natural frequency of rotor (5160r/min). As the increases of spin velocity, this frequency component does not changed, but the amplitude of vibration increases rapidly.

The waterfall plot of the Node 4 with the fluid induced forces and eccentricity (e =0.02mm) was shown in Fig. 5(b). It is shown the same character as Fig. 5(a), but the frequency component comes forth about 14660r/min, which is lower than the spin velocity when the eccentricity equals to 0.05mm. So we can get a conclusion that it is helpful for the stability of rotor system with the action of fluid induced forces through increasing the eccentricity reasonably.

Conclusion

Based on the analysis of the structure and flow characteristics of the centrifugal pump impeller clearance, a fluid-induced force model was given, and the parameters in this model were solved by quasi-steady CFD solution. Under the fluid-induced force, the dynamic response of a turbo pump impeller-rotor system was discussed. The results clearly showed that there was complicated vibration character of the rotor system with the action of fluid induced vibration, and it is helpful for the stability of rotor system with the action of fluid induced forces through increasing the eccentricity reasonably.

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