# FEDSM-ICNMM2010-30~० <br> A study on pairs of stream-wise flexible circular cylinders in cross flow 

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#### Abstract

The importance of the in-flow oscillation of a single cylinder in cross-flow has been in the spotlight since the accident in a FBR-type reactor. In-flow oscillations can also be observed in heat exchanger tube arrays. Previous reports show some interesting phenomena on the oscillation of cylinder arrays. In this paper, detailed observations on the effect of the pitch ratio for pairs of cylinders, in parallel and in tandem, is highlighted in the range of low flow velocities, where each cylinder can move only in a given direction. The motion of the cylinders is measured by attached strain gages and by a high-speed digital video camera.


## NOMENCLATURE

D : Diameter of cylinder
$\mathrm{f}_{\mathrm{K}}$ : Karman-type alternate vortex shedding frequency
h : Damping ratio
P, T : Pitch between cylinders
St : Strouhal number
$\mathrm{U}, \mathrm{U}_{\mathrm{G}}, \mathrm{U}_{\infty}$ : Flow velocity, Gap \& Approach velocities
x : amplitude of cylinder
$\xi$ : non-dimensional amplitude of cylinder $(\xi=x / D)$

## 1. INTRODUCTION

The stream-wise oscillations of a circular cylinder in cross-flow caused by alternate vortex shedding behind the cylinder, is well known. Similar oscillations due to symmetric vortex shedding became famous after the event in a fast breeder reactor "Monju" in Japan, where a slender cylinder broke as a result of vibrations in the stream-wise direction due to symmetric vortex shedding. Based on some research works, a guideline to avoid tube rapture by any type of vortex shedding has been developed for a circular cylinder contained in a pipe [1].

The Karman-type alternate vortex shedding frequency is expressed as a linear function of flow velocity using the following equation,

$$
\begin{equation*}
f_{K}=S t \frac{U}{D} . \tag{1}
\end{equation*}
$$

Karman-type alternate vortex shedding does not act only in the cross-flow (lift) direction; it does also in the stream-wise direction at double the frequency in the lift direction [2]. There, however, has been reported another type of
symmetrical vortex shedding behind cylinders at a higher Strouhal number. Thus, based on reported results there are three resonant velocities when the flow velocity increases [3].

The JSME's guideline [1] presents methods to avoid severe accidents for single circular cylinders. This method may not work for cylinders arrays. One of this paper's authors found that a similar vibration phenomenon to that for a single cylinder is observed even for an array of cylinders [4]. Similar results had been obtained in Weaver's earlier work [5] and some published papers, e.g. [6], etc.

There is significant research work on the flow around a pair of cylinders [7], [8] etc., and flows through arrays of cylinders [9]. However, there is no detailed research work on the case where the cylinders are restricted to move only in the stream-wise direction, yet this problem is useful for the understanding of the stream-wise excitation force both by Karman-type alternate vortex shedding and by symmetric vortex shedding.

One of the authors has initiated a study to find the excitation mechanism for stream-wise direction fluid forcing in a cylinder array [10,11], and found some interesting results, where two possible excitation mechanisms in the stream-wise direction were observed. These mechanisms cannot occur for an isolated single cylinder, but they have been observed in cylinder arrays, combined with the vibration modes of cylinders immersed in liquid. The pitch between cylinders in tandem has a strong effect on the cylinder vibration response [12]. Furthermore, resonant vibrations are not only caused by cylinder interaction with alternate vortex shedding vortex, but can also be caused by the symmetric vortex shedding. This result suggests that the symmetric vortex shedding in the inflow direction has to be considered from the Strouhal number 0.2 to 1.0 , and possibly 2.0 as well.

In this report, to examine the resulting effects in detail, the number of cylinders is limited totwo, but their configuration is varied. The cylinders vibrate only in the stream-wise direction

## 2. TEST APPARATUS

### 2.1 Test loop

Fig. 1 shows the whole test apparatus. The main flow is through a rectangular cross sectional channel of 90 mm by 190mmsection, where water flow is generated by a pump below


Figure 1 Test apparatus
the main pipe. The tank on the left is a reservoir.
Although the intensity of the flow turbulence is not measured, flow straightening is achieved with a mesh unit composed of an array of plastic tubes at the up-stream region of the test section. The flow velocity is controlled with a valve and an inverter power unit, and is measured by an ultrasonic flow meter at the up-stream region of the test section. The maximum Reynolds number is approximately 780.

### 2.2 Test cylinders

As indicated in Fig.1, the test section is set in the rectangular channel, where a maximum number of fifteen cylinders can be installed. The cylinders are 20 mm in diameter, and 84 mm in length. These are very light weight tubes made of plastic. They are supported with a stainless steel plate of 1.5 mm thickness and 6 mm in width as shown in Fig.2, on which strain gages are mounted to measure the cylinder response.
Table 1 shows the configurations tested including the case of a single cylinder. The stream-wise pitch, P, between the cylinders is varied from $\mathrm{P} / \mathrm{D}=1.25$ to 7.0. Tests were, however, mainly conducted for smaller pitch up to 3.0 according to the test results presenteded later. The transverse pitch, T , is changed from $\mathrm{T} / \mathrm{D}=1.25$ to 3.0. Table 2 shows the measured


Figure 2 Concept of test cylinders
Table 1 Measured cylinder patterns

| Single <br> cylinder | Two cylinders <br> in tandem | Two cylinders <br> in parallel |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\square \mathrm{O}$ |  |  |  |  |  |
| $\mathrm{P} / \mathrm{D}$ <br> or <br> $\mathrm{T} / \mathrm{D}$ | $1.25,1.5,1.75,2.0$, <br> $2.25,2.5,2.75,3.0$, <br> $6.0,7.0$ | $1.25,1.5,1.75,2.0,2.25$, <br> $2.52 .75,3.0$ |  |  |  |

Table 2 Vibrational characteristics of cylinders

| Pattern |  |  | Single cylinder | Two in tandem | Two in parallel |
| :---: | :---: | :---: | :---: | :---: | :---: |
| In air |  | f | 15.2 Hz | $15.1-15.4 \mathrm{~Hz}$ | $15.1-15.4 \mathrm{~Hz}$ |
|  |  | h | 0.4\% | 0.4-0.5\% | 0.4-0.5\% |
| In water | Before test | f | 10.3 Hz | $10.1-10.8 \mathrm{~Hz}$ | $10.1-10.4 \mathrm{~Hz}$ |
|  |  | h | 2.2\% | 0.5-1.9\% | 0.5-2.2\% |
|  | After test | f | 10.3 Hz | $10.0-10.7 \mathrm{~Hz}$ | $10.1-10.7 \mathrm{~Hz}$ |
|  |  | h | 1.7\% | 0.5-2.2\% | 0.5-2.1\% |

natural frequencies and corresponding damping ratios of the test cylinders in air and in water. These were measured by tapping tests in still water and in air. All cylinders are set to have similar vibration frequency.

### 2.3 Test method

Cylinder responses are measured as flow velocity is increased via strain gages mounted on the support plates. In some cases the flow is observed with a high speed digital video camera. Black ink is injected into the upstream region for flow visualization.

For the parallel cylinder patterns, the flow velocity is expressed as gap flow velocity using the following relation.

$$
\begin{equation*}
U_{G}=\frac{T}{T-D} U_{\infty} \tag{2}
\end{equation*}
$$

## 3. TEST RESULTS (RESPONSE AMPLITUDE)

### 3.1 Single cylinder

The response of a single cylinder is shown in Fig.3, compared with the previous published data [11.12], where the data indicated as " 2009 " is the present data.

It should be noted that the above mentioned range corresponds to the symmetric vortex shedding. Alternate vortex shedding appears after increasing the flow velocity, Vr, to more than 3.5.


Figure 3 Response of single cylinder

The slight mismatch between data measured in different years may come from some differences of the test cylinders or their boundary conditions.

### 3.2 Two cylinders in tandem

The response of the two cylinders for each pitch ratio is shown in Fig. 4 (a) - (j). "US" means the first (upstream) row and "DS" the second (downstream) one.

In these figures, the vibration mode of the cylinder pair is indicated as "In-phase" which means that the two cylinders oscillate in the same direction, and "Out-of-phase" which means two cylinders oscillate in opposite directions. It should be pointed out that these phases change between the pitch ratio, $\mathrm{P} / \mathrm{D}=1.5$ and 1.75 , and return from $\mathrm{P} / \mathrm{D}=2.25,2.5$ \& 3.0, although there is some complicated behavior for $\mathrm{P} / \mathrm{D}=2.75 \& 6.0$. $\mathrm{P} / \mathrm{D}=7.0$ shows a simple mode.

In nearly the whole range the vortex shedding is symmetric as indicated in these figures, and the mode-change does not correspond to a change of the pattern of vortex shedding. This is discussed further below.

As for the magnitude of the amplitudes, the responses of the two cylinders are similar for small pitch ratios from $\mathrm{P} / \mathrm{D}=1.25$ to 1.5 . However, from the pitch ratio of $\mathrm{P} / \mathrm{D}=1.75$ the response of the downstream cylinder shows a peak, although the response of the upstream cylinder continues to increase as the flow velocity increases. This trend changes in the case of $\mathrm{P} / \mathrm{D}=2.25$, where the response of the downstream cylinder is greater than that of the upstream one; the response of the downstream cylinder still has the peak in this case. When the pitch ratio increases to more than $\mathrm{P} / \mathrm{D}=2.5$, the response of the upsteam cylinder is greater than that of the downstream one up to $\mathrm{P} / \mathrm{D}=7.0$. There is no significant difference between the results for $\mathrm{P} / \mathrm{D}=3.0$ and 6.0 , where the trend of the vortex is reported to be changed [3].


Figure 4-1 (a)-(d) Response of two cylinders in tandem (To be continued)


Figure 4-2 (e)-(h) Response of two cylinders in tandem (To be continued)


Figure 4-3 (i)\&(j) Response of two cylinders in tandem

### 3.3 Two cylinders in parallel

The response of the two cylinders for each pitch ratio is shown in Fig. 5 (a) - (h). The two cylinders are identified as "Left" and "Right" in the figure, but there should be no much difference between their responses.

The first mode appears as the "In-phase" type, which means the two cylinders oscillate in the same direction, in almost all cases except $\mathrm{T} / \mathrm{D}=1.25$, followed by the second mode "Out-of-phase" response, which corresponds to the two cylinders oscillating in opposite directions.

The first mode appears over a very small reduced velocity range, and in many cases the vortex shedding looks symmetric. The alternate vortex shedding does not easily appear at the low flow velocities.

The transverse pitch does not greatly influence the results. Compared with the data shown in Fig.4, the response of these cylinders has a peak around the non-dimensional flow velocity $\mathrm{Vr}=2.5$; this peak is probably interrupted by the alternate vortex shedding which appears at higher flow velocities. This result is not unusual, but it is somewhat different from previously published results where there is no clear distinction between the peak by the symmetric vortex shedding and that by the alternate shedding, and instead one peak appears in the transition range between these two vortex mechanisms [13].

## 4. TEST RESULTS (FREQUENCY TREND)

From the measured cylinder responses some interesting trends are examined. These are closely related with the shift of


Figure 5 Response of two cylinders in parallel


Figure 6 Dominant frequency for single cylinder
the dominant frequency of the cylinders. In this section, some examples are presented to help clarify the above mentioned phenomenon, but most data has eliminated due to the limits of the space.

### 4.1 Single Cylinder

The dominant frequency of the cylinder response is shown in Fig.6. The frequency varies with flow velocity although it is in the resonant region with the symmetric vortex shedding.

### 4.2 Two Cylinders in tandem

Some examples of the dominant frequency variation from several cylinder responses are shown in Fig.7(a)-(d). The other data are similar to these results, thus $\mathrm{P} / \mathrm{D}=1.25$ is similar to $\mathrm{P} / \mathrm{D}=1.5$, or $\mathrm{P} / \mathrm{D}=1.75$ to $2.0, \mathrm{P} / \mathrm{D}=2.5$ to 2.25 , and $\mathrm{P} / \mathrm{D}=2.75$ to 3.0 etc .

As for the small pitch ratio, $\mathrm{P} / \mathrm{D}=1.25 \& 1.5$, the sudden change of the frequency in the out-of-phase mode comes from the impact between two cylinders, thus it should be neglected.

In some cases such as in Fig.7(c), the change of the vibration mode is closely related to the change of the dominant frequency, but in most cases the shift of the dominant frequency is almost linear in trend relative to the flow velocity, and the appearance of each mode does not have a clear rule depending on the excitation mechanism which has almost a fixed trend with the increase of flow velocity.

### 4.3 Two Cylinders in parallel

Two examples of the dominant frequency variation in different pitches are shown in Fig.8(a)\&(b). Other data are similar to these results, thus $\mathrm{T} / \mathrm{D}=1.25$ is similar to $\mathrm{T} / \mathrm{D}=1.5$, or and results for $\mathrm{T} / \mathrm{D}=1.75$ are similar to those for $\mathrm{T} / \mathrm{D}=2.0$, $2.25,2.5,2.75$, and 3.0.
The change of the vibration mode corresponds directly to the change of the dominant frequency. And in some cases, the dominant frequency changes in steps as seen in Fig.8(a). This has also been observed for $\mathrm{T} / \mathrm{D}=2.0$ and 3.0. The trend can easily be understood when combined with the natural frequency of each mode. However, there still remains some questions on why it does not appear for every pitch ratio.

This strange behavior may come from the structure of the flow itself, but there is no exact relation with the shift of the vortex mechanism, which is nearly a constant function ofthe flow velocity.


Figure 7 Dominant frequency of two cylinders in tandem

## 5. DISCUSSION

There are several unique results in the data presented above. These include:
(1) For tandem cylinders, the phase shift for each pitch ratio shows a complicated trend.


Figure 8 Dominant frequency of two cylinders in parallel
(2) In the cases of tandem cylinders with spacing $\mathrm{P} / \mathrm{D}=1.75$ and 2.0 , why does the response of the second cylinder decrease when the flow velocity increase around $\mathrm{Vr}=2-$ 2.5, and why is the response of the second cylinder in the case of $\mathrm{P} / \mathrm{D}=2.25$ greater than that of the first cylinder around $\mathrm{Vr}=1.5-3.0$ only in this case?
(3) Although the peak response of the cylinders seems to indicate resonance with the excitation vortex shedding frequency, the cylinder dominant frequency still shifts according to the flow velocity. This is unlike the result for the resonance by alternate vortex shedding, where the frequency remains constant during resonance.
The following discussion is an attempt to understand the above unique phenomena.

### 5.1 Mode shift of tandem cylinders

In the case of small pitch ratios such as $\mathrm{P} / \mathrm{D}=1.25 \& 1.5$, vortex shedding cannot occur in the narrow space available. The two cylinders therefore easily oscillate in phase. The symmetric vortices from the first cylinder seems to cover both cylinders and separate only from the second cylinder as shown in Fig.9.

However, since the natural frequency of the coupled mode for two tandem cylinders is the first in the out-of-phase mode, from the pitch ratio $\mathrm{P} / \mathrm{D}=1.75$ the out-of-phase mode appears at lower flow velocities and shifts to the in-phase mode at higher flow velocities.


Figure 9 Example of vortex shedding for $\mathrm{P} / \mathrm{D}=1.25, \mathrm{Vr}=1.78$


Figure 10 Example of vortex shedding for $\mathrm{P} / \mathrm{D}=1.75, \mathrm{Vr}=2.2$
This explanation can be adapted to the larger pitches $\mathrm{P} / \mathrm{D}=3.0,6.0$ and 7.0, but there are exceptional cases such as $\mathrm{P} / \mathrm{D}=2.25$ and 2.5 .

There is no clear explanation on this unique trend in the cases of $\mathrm{P} / \mathrm{D}=2.25$ and 2.5 , but a future study may help shed more light.

### 5.2 Change of response for tandem cylinders

For the pitch ratios, $\mathrm{P} / \mathrm{D}=1.75$ and 2.0 , the response of the second cylinder decreases near $\mathrm{Vr}=1.7$ and 2.0. Up to this flow velocity, both cylinders oscillate with the same magnitude in the out-of-phase mode.

This phenomenon can be understood by making the following assumption:
(1) The shed vortex separates from the first cylinder, and reaches the surface of the second cylinder with a certain time lag.
(2) If this vortex acts to prevent (damp) the motion of the second cylinder, the response of the second cylinder decreases.
Fig. 10 shows an example to verify the above assumption. Although it is not easy to demonstrate without the video film, the phenomenonhas been confirmed from video records. In the video, the symmetric vortex shed from the downstream surface of the first cylinder when the motion of the cylinder changes from the downstream direction to the upstream direction, and it reaches the second cylinder when the second cylinder moves toward the first cylinder. The drift speed of the shed vortex is not the same as the flow velocity itself and it could not be estimated from the video due lack of clarity in the movie.

As a simple computation of the time lag, when Vr is 1.8 , the practical flow velocity is around $400 \mathrm{~mm} / \mathrm{s}$, and the travel time from the first cylinder to the second cylinder is approximately 0.09 s at this speed if the space is 35 mm . Noting that half period of the cylinder oscillation lasts 0.05 s , both times are very close.

An additional test was conducted with the the upstream cylinder fixed. The result are compared in Fig. 11 with the data in Fig.4(c). When the upstream cylinder is fixed, there is no clear peak in the downstream cylinder response. This means that the vortex from the upstream cylinder has an important effect on the downstream cylinder. The result leads to the conclusion that the cylinder arrays with all fixed condition do not show the symmetric vortex shedding, where the alternative vortex shedding can be exist.

### 5.3 Shift of dominant frequency during resonance

Although the dominant frequency varies with flow velocity, the relationship is not linear. In Fig. 6 a trend is shown as an example for the single cylinder compared with the frequency calculated by Eq.(1) using $\mathrm{St}=0.5$. It is the same that the trigger of the resonance occurs at the first crossing flow velocity, but the dominant frequency is not constant and does notdepend on thevortex shedding excitation frequency.
In the case of alternate vortex shedding, it is reported that the dominant frequency remains relatively constant in the resonant region. This is a special feature in this test including the case of two cylinders. It may come from the fact that cylinder motion is here constrained to the stream-wise direction only.

## 6. CONCLUSION

In the case of tandem cylinders, there is a special pitch where the response of the cylinder in the wake region decreases. This special pitch depends on the phase between the wake and the cylinder motion. However, the symmetric vortex is suppressed when the upstream-cylinder is fixed. The vortex from the downstream-cylinder is also suppressed. Furthermore, unlike the case of alternate shedding resonance, in the lock-in range here the dominant frequency by the symmetric vortex shedding varies slightly with flow velocity.


Figure 11 Example of the downstream cylinder $(\mathrm{P} / \mathrm{D}=1.75)$

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