Proceedings of the ASME 2010 3rd Joint US-European Fluids Engineering Summer Meeting and
8th International Conference on Nanochannels, Microchannels, and Minichannels
FEDSM-ICNMM2010
August 1-5, 2010, Montreal, Canada

FEDSM-ICNMM2010-30151

THE DYNAMIC BEHAVIOR OF PROTEIN BUBBLE IN CASSON FLUID

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ABSTRACT

A Casson model is selected to describe the flow behavior of blood. Considering the viscoelasticity of protein film, and the finite deformation of protein bubble under the action of a Casson fluid, a nonlinear equation describing the dynamic behavior for a single protein bubble in blood is developed. The numerical method is used to study the effect of viscoelasticity of the protein film, the characteristic parameters of Casson fluid and the effect of surface tension on the dynamic behavior of protein bubble. The results show that, increasing the viscosity of Casson liquid will accelerate the amplitude decaying of protein bubble wall; the period becomes longer. In the condition that the viscosity of protein film is greater than that of Casson fluid, increasing the viscoelasticity of protein film will also accelerate the amplitude decaying of protein bubble wall. Further more, the greater the viscoelasticity of protein film is, the stronger its load bearing capacity is. Under the consideration of surface tension, the bubble wall will vibrate with higher decaying velocity of amplitude. Without considering the surface tension will lead to a greater relative deformation of protein bubble.

INTRODUCTION

Protein bubbles are widely used in the fields of life science, biomedical engineering. For example, an effective way to increase the quality of sonograms is to use the micron-size protein bubbles as ultrasound contrast agents [1, 2]. Micron-size protein bubbles (<10 μm) are also used to deliver genes or drugs to cure the tumor and diseases in the brain [1-3]. Protein bubbles can also be used to package liquid substances which are easy to gasify such as per-fluorocarbons (PFCS). They flow with the blood to the tumor. The bubbles will be destroyed at the designated locations, the PFCS will gasify and the gas in blood vessel can prevent the blood and oxygen supplying for tumor. By this way it can achieve the purpose of treatment [4]. Therefore, the study of the dynamic behavior of protein bubbles in blood will be important for better applications of protein bubbles.

In engineering applications, the independent protein bubbles are formed by physical methods and then flow with the blood to perform the duties of drug or gene delivering, and other materials. To study the dynamic behavior of independent protein bubbles in a non-Newtonian liquid, three key factors should be considered, those are (1) the constitutive relation for the bubble wall, that is, the protein film is treated as made of elastic material or viscoelastic material; (2) the deformation of the protein bubble is treated as a small deformation or a large deformation (i.e. a finite deformation) under the action of surrounding fluid; (3) characteristics of the fluid around the bubble, that is, it is Newtonian fluid or non-Newtonian fluid.

Church [5] defined the bubble wall as an elastic solid shell to study the vibration of a single protein bubble in an incompressible Newtonian fluid. In his study, the fluid is static, and the volume variation of the protein bubble is minimal and the process is defined as a small deformation under the action of fluid. Friking and de Jong [6] defined the bubble wall as a viscoelastic layer to study the vibration of a single protein bubble in viscous fluid, and the deformation is treated as a small deformation too. Based on the study of Friking and de Jong, Khismatullin [7] studied the effect of viscosity on the frequency response of the protein bubble under the action of ultrasound.

In the above studies, as the main applications of protein bubbles are their responses to ultrasound, the deformation of the bubble is treated as a small deformation. However, in many cases, large deformation (i.e. finite deformation) of the protein bubble would be considered when the pressure of liquid is much higher than that of gas inside the bubble. For example, when the protein bubble (the diameter is smaller than 10μm) is flowing with blood in vein vessel, as the pressure in different vessels is not identical, the pressure of blood may be higher than the gas pressure inside the bubble, so a large deformation caused by the high pressure difference will take place. In addition, the fluid is not always a Newtonian fluid. In many cases, the fluid may be a non-Newtonian fluid.

In this paper, the Maxwell viscoelastic model is selected to describe the constitutive relation of protein film, and the finite deformation is considered under the action of the pressure difference. As a non-Newtonian fluid, we select a Casson model to describe the flow property of the blood. According to the strain energy density function for finite deformation of the viscoelastic material, the relaxation function of the Maxwell model, and the deformation gradient tensor of the bubble, stress expressions for finite deformation of protein bubbles are derived. By using the stress expressions and the dynamic equation of bubble under the action of fluid, a nonlinear vibration equation describing the relation between the relative deformation rate of the inner radius and time is obtained for finite deformation. Based on this equation, the numerical simulation is carried out for the effect of factors such as the parameters of Casson liquid, the viscoelasticity of protein film and the effect of surface tension on the dynamic behavior of protein bubbles.

FINITE DEFORMATION STRESS EXPRESSIONS OF PROTEIN BUBBLE

(1) We consider a Cartesian system and a set of spherical coordinates (R,Θ,Φ) in R^3. $D_0=\{(r_0,\Theta,\Phi)\mid r_{01} \le r_0 \le r_{02},\ 0\le\Theta\le 2\pi,\ 0<\Phi\le\pi\}$ represents the static state of one point in the protein bubble wall at the instant t=0, and $d_0=\{(r,\theta,\varphi)\mid r_1 \le r \le r_2,\ 0\le\theta\le 2\pi,\ 0<\varphi\le\pi\}$ represents the deformed state of one point in the bubble wall at some instant t >0, where subscripts 1 and 2 denote the inner and outer surfaces of the bubble wall, respectively;

(2) The protein film of the bubble is isotropic and incompressible;

(3) The bubble deformation is homogeneous in the radial direction and no displacement occurs in the directions of θ and φ [8];

(4) As to the finite deformation of protein bubble, the energy density function and the right Cauch-Green strain tensor $C(t)$ satisfy Mooney-Rivilin relationship , that is:

$$\hat{\boldsymbol{W}}\left[\boldsymbol{C}(t)\right]=C_1(I_1-3)+C_2(I_2-3) \qquad (1)$$

where C_1 and C_2 are material constants, I_1 and I_2 denote the principal invariants of the right Cauchy–Green strain tensor.

According to the basic theory of viscoelasticity, for the viscoelastic material, the energy density function depends not merely on the strain $\boldsymbol{C}(t)$ at the current instant t, but also on the entire history s before that instant. The time-dependent strain energy density function can be defined as [9]:

$$\boldsymbol{W}(t)=\hat{\boldsymbol{W}}\left[\boldsymbol{C}(t)\right]+\int_0^t \dot{\mu}^*(t-s)\hat{\boldsymbol{W}}[\boldsymbol{C}(s)]ds \qquad (2)$$

where $\dot{\mu}^*(t-s)$ is the derivative of dimensionless relaxation function $\mu^*(t-s)$ to time. We have used Maxwell's two-parameter model to describe the viscoelasticity of protein bubble wall, and obtained the stresses in the directions of r,θ,φ during the course of finite deformation of protein bubble [10].

$$\tau_{M,rr}(r,t)=2\left(C_1\frac{r_0^4}{r^4}+2C_2\frac{r_0^2}{r^2}\right)-\frac{2}{\tau_0}\int_0^t e^{-(t-s)/\tau_0}\left(C_1\frac{r_0^4}{r^4}+2C_2\frac{r_0^2}{r^2}\right)ds \qquad (3\text{-a})$$

$$\tau_{M,\varphi\varphi}(r,t)=\tau_{M,\theta\theta}(r,t)=2\left(C_1\frac{r^2}{r_0^2}+C_2\left(\frac{r^4}{r_0^4}+\frac{r_0^2}{r^2}\right)\right)-\frac{2}{\tau_0}\int_0^t e^{-(t-s)/\tau_0}\left(C_1\frac{r^2}{r_0^2}+C_2\left(\frac{r^4}{r_0^4}+\frac{r_0^2}{r^2}\right)\right)ds \qquad (3\text{-b})$$

Where τ_0 is the relaxation time. These two equations denote the relation between stress and finite deformation for a spherical protein bubble.

FORCES ACTING ON THE BUBBLE WALL

The stress acting on the protein bubble wall is composed of two parts, one is the liquid pressure, and the other is the stress caused by the motion of viscous fluid. The viscous stress is closely related to the deformation of protein bubble. First, we establish the relationship between the stress generated by the motion of viscous fluid and the deformation of bubble.

Stress generated by the motion of Casson fluid follows the equation [11]

$$\begin{cases}\boldsymbol{\tau}=-\left(S^2+\dfrac{2Sb}{\left(\sqrt{\dfrac{1}{2}II_D}\right)^{\frac{1}{2}}}+\dfrac{b^2}{\sqrt{\dfrac{1}{2}II_D}}\right)\boldsymbol{D}, & \dfrac{1}{2}II_\tau>b^4 \\ \boldsymbol{D}=0, & \dfrac{1}{2}II_\tau<b^4\end{cases} \qquad (4)$$

where S is a constant related to the viscosity of fluid, the unit is [Pa.s]$^{1/2}$; b is a constant related to the fluid stress, the unit is [N/m^2]$^{1/2}$, $\boldsymbol{D}$ is the deformation rate tensor, the relationship between $\boldsymbol{D}$ and $\boldsymbol{v}$ is $\boldsymbol{D} = \nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^T$, the second order invariant $II_D = \boldsymbol{D} : \boldsymbol{D}$, $II_\tau = \boldsymbol{\tau} : \boldsymbol{\tau}$.

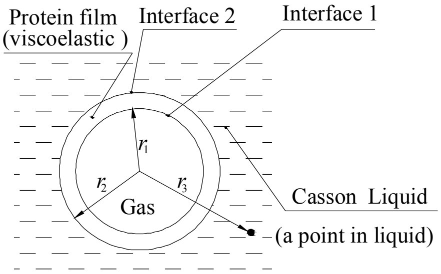

Figure 1. The protein bubble in Casson liquid

As shown in Fig.1, the gas and protein film of bubble form interface 1, protein film of bubbles and the liquid form interface 2, the radius of inner and outer bubble wall is r_1 and r_2 respectively, the r_3 represents the distance of a point in the liquid from the center of the bubble.

It is assumed that the liquid around the bubble is static before the deformation of bubble, and only the radial displacement occurs when the shape of bubble is deformed.

On the other hand, the flow velocity in the r direction v_r is determined by the continuity equation. It is assumed that the liquid is incompressible and homogeneous, the effects of gravity, gas diffusion, and heat conduction are neglected. Thus the velocity of the liquid outside the bubble is expressed as [11]

$$v_r = \left(\frac{r_2}{r}\right)^2 \dot{r}_2 \tag{5}$$

Substituting Eq. (5) into Eq. (4), the normal stresses in the directions of r,θ,φ can be obtained as follows

$$\tau_{L,rr} = M_0\left(\frac{\dot{r}_2 r_2^2}{r^3}\right) + M_1\left|\frac{\dot{r}_2 r_2^2}{r^3}\right|^{-\frac{1}{2}}\left(\frac{\dot{r}_2 r_2^2}{r^3}\right) + M_2\frac{\dot{r}_2}{|\dot{r}_2|}$$

$$M_0 = 4S^2, \qquad M_1 = \frac{8Sb}{\sqrt{2\sqrt{3}}}, \qquad M_2 = \frac{2b^2}{\sqrt{3}} \tag{6-a}$$

$$\tau_{L,\theta\theta} = \tau_{L,\varphi\varphi} = -\frac{M_0}{2}\left(\frac{\dot{r}_2 r_2^2}{r^3}\right) - \frac{M_1}{2}\left|\frac{\dot{r}_2 r_2^2}{r^3}\right|^{-\frac{1}{2}}\left(\frac{\dot{r}_2 r_2^2}{r^3}\right) - \frac{M_2}{2}\frac{\dot{r}_2}{|\dot{r}_2|} \tag{6-b}$$

where subscript L represents the liquid medium. It is assumed that the pressure of the liquid is $p_L(r)$, thus the total normal stresses of the liquid acting on the bubble wall in the directions of r,θ,φ can be obtained

$$\sigma_{L,rr} = -p_L(r) - M_0\left(\frac{\dot{r}_2 r_2^2}{r^3}\right) - M_1\left|\frac{\dot{r}_2 r_2^2}{r^3}\right|^{-\frac{1}{2}}\left(\frac{\dot{r}_2 r_2^2}{r^3}\right) - M_2\frac{\dot{r}_2}{|\dot{r}_2|} \tag{7-a}$$

$$\sigma_{L,\theta\theta} = \sigma_{L,\varphi\varphi} = -p_L(r) + \frac{M_0}{2}\left(\frac{\dot{r}_2 r_2^2}{r^3}\right) + \frac{M_1}{2}\left|\frac{\dot{r}_2 r_2^2}{r^3}\right|^{-\frac{1}{2}}\left(\frac{\dot{r}_2 r_2^2}{r^3}\right) + \frac{M_2}{2}\frac{\dot{r}_2}{|\dot{r}_2|} \tag{7-b}$$

Eqs. (7-a) and (7-b) denote the relationship between total normal stresses of Casson liquid acting on the bubble wall in the directions of r,θ,φ and bubble deformation in the protein bubble deformation process.

THE DYNAMIC EQUATION OF PROTEIN BUBBLE

According to the stress of protein bubble caused by deformation, and to the relationship between the stress acting on the bubble wall around the liquid and the deformation of bubble, the dynamic equation of protein bubble can be derived.

It is also has assumed that the effects of gravity, the diffusion of the gas through the protein film, the heat conduction between protein film, gas, and the surrounding liquid are neglected, under the above conditions, the finite deformation of protein bubble should satisfy the following dynamic equation [7]:

$$\rho_M\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r}\right) = \frac{\partial \tau_{rr}}{\partial r} + \frac{2\tau_{rr} - \tau_{\theta\theta} - \tau_{\varphi\varphi}}{r} \tag{8}$$

Where ρ_M is the density of protein film, τ_{rr}, $\tau_{\theta\theta}$ and $\tau_{\varphi\varphi}$ are the normal stress components in the r, θ, and φ directions respectively. The velocity components in the r-directions v_r is determined by

$$v_r = \left(\frac{r_1}{r}\right)^2 \dot{r}_1, \qquad (r_1 \le r \le r_2) \tag{9}$$

By using Eq. (9), the Eq. (8) can be integrated from r_1 to r_2 to obtain

$$J_1 = \rho_M \int_{r_1}^{r_2}\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r}\right)dr \tag{10}$$

$$J_2 = \int_{r_1}^{r_2}\left(\frac{\partial \tau_{M,rr}}{\partial r} + \frac{2}{r}\left(\tau_{M,rr}(r,t) - \tau_{M,\theta\theta}(r,t)\right)\right)dr \tag{11}$$

J_2 can be expressed as [12]

$$J_2 = \tau_{M,rr}(r_1,t) - \tau_{M,rr}(r_2,t) + 4\left(M(x) - \frac{1}{\tau_0}\int_0^t e^{-(t-\tau)/\tau_0} M(x)\,d\tau\right) \tag{12}$$

where

$$M(x) = \frac{C_1 + 2C_2x^2 + 4C_1x^3 - 4C_2x^5}{4x^4} - \frac{C_1 + 2C_2A^2(x) + 4C_1A^3(x) - 4C_2A^5(x)}{4A^4(x)} \tag{13}$$

and

$$x(t) = \frac{r_1(t)}{r_{01}}, \quad \delta = \frac{r_{02}^3}{r_{01}^3} - 1, \quad A(x) = \left((\delta + x^3)/(\delta + 1)\right)^{\frac{1}{3}} \tag{14}$$

According to the force balance at the interface 1 and interface 2 shown in Fig. 1, the stresses should satisfy the following boundary conditions, the variants σ_1 and σ_2 denote the coefficient of surface tension.

$$p_G(r_1) = \tau_{M,rr}(r_1,t) + \frac{2\sigma_1}{r_1} \tag{15}$$

$$p_L(r_2)+\tau_{L,rr}(r_2,t)=\tau_{M,rr}(r_2,t)-\frac{2\sigma_2}{r_2} \tag{16}$$

So the liquid pressure in the interface 2 should satisfy

$$p_L(r_2)=p_G(r_1)-\left(\tau_{M,rr}(r_1)-\tau_{M,rr}(r_2)\right)-\tau_{L,rr}(r_2)-\frac{2\sigma_2}{r_2}-\frac{2\sigma_1}{r_1} \tag{17}$$

To get the normal stress of interface 2, let r=r_2 in Eq. (9-a)

$$\tau_{L,rr}(r_2)=M_0\left(\frac{\dot r_2}{r_2}\right)+M_1\left|\frac{\dot r_2}{r_2}\right|^{-\frac{1}{2}}\left(\frac{\dot r_2}{r_2}\right)+M_2\frac{\dot r_2}{|\dot r_2|} \tag{18}$$

It is assumed that the pressure of gas in the bubble is p_0 and the deformation ratio x (0) =1 at the instant t=0, the gas in the bubble undergoes an isothermal process when the volume of the bubble oscillates under the load, that is, the gas pressure in the bubble $p_G(r_1)$ obeys the following equation:

$$p_G(r_1)=p_0(r_{01}/r_1)^3=p_0x^{-3} \tag{19}$$

The dynamic equation of flowing fluid must be used to get the pressure $p_L(r_2)$ of interface 2. The dynamic equation can be expressed as

$$\rho_L\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}\right)=\frac{\partial\sigma_{rr}}{\partial r}+\frac{2\sigma_{rr}-\sigma_{\theta\theta}-\sigma_{\varphi\varphi}}{r} \tag{20}$$

ρ_L is the density of liquid, integrating above equation at the interval [r_2, r_3], we obtain

$$J_3=\rho_L\int_{r_2}^{r_3}\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}\right)dr \tag{21}$$

$$J_4=\int_{r_2}^{r_3}\left(\frac{\partial\sigma_{L,rr}}{\partial r}+\frac{2\sigma_{L,rr}-\sigma_{L,\theta\theta}-\sigma_{L,\varphi\varphi}}{r}\right)dr \tag{22}$$

According J_1+J_3=J_2+J_4 , and using the dimensionless parameter of Eq. (14), we obtain the dynamic equation for the protein bubble in Casson fluid.

$$K_1(x)\ddot x+K_2(x)\dot x^2+K_3(x)\dot x+K_4(x)\dot x^{\frac{1}{2}}=K_5(x) \tag{23}$$

where

$$K_1(x)=\rho_M r_{01}^2 x\left(1+\frac{\rho_L-\rho_M}{\rho_M}\left(\frac{x^3}{x^3+\delta}\right)^{\frac{1}{3}}-\frac{\rho_L}{\rho_M}x\frac{r_{01}}{r_3}\right) \tag{24-a}$$

$$K_2(x)=\rho_M r_{01}^2\left(\frac{3}{2}+\frac{\rho_L-\rho_M}{\rho_M}\left(\frac{x^3}{x^3+\delta}\right)^{\frac{1}{3}}\left(2-\frac{1}{2}\frac{x^3}{x^3+\delta}\right)\right)$$
$$+\rho_L r_{01}^2\left(\frac{1}{2}\left(x\frac{r_{01}}{r_3}\right)^4-2\frac{r_{01}}{r_3}x\right) \tag{24-b}$$

$$K_3(x)=M_0\frac{x^2}{\delta+x^3} \tag{24-c}$$

When $\dot x<0$,

$$K_4(x)=M_1\left(\frac{x^2}{\delta+x^3}\right)^{\frac{1}{2}}\left(\left(\frac{r_{02}}{r_3}\right)^{\frac{3}{2}}\left(\frac{\delta+x^3}{1+\delta}\right)^{\frac{1}{2}}-2\right) \tag{24-d}$$

When $\dot x>0$,

$$K_4(x)=M_1\left(\frac{x^2}{\delta+x^3}\right)^{\frac{1}{2}}\left(2-\left(\frac{r_{02}}{r_3}\right)^{\frac{3}{2}}\left(\frac{\delta+x^3}{1+\delta}\right)^{\frac{1}{2}}\right) \tag{24-e}$$

$$K_5(x)=p_0x^{-3}-p_L(r_3)$$
$$+4\left(M(x)-\frac{1}{\tau_0}\int_0^t e^{-(t-\tau)/\tau_0}M(x)d\tau\right)-\frac{2\sigma_1}{r_{01}x}$$
$$-\frac{2\sigma_2}{r_{02}}\left(\frac{1+\delta}{\delta+x^3}\right)^{\frac{1}{3}}-M_2\beta+3M_2\beta\ln\left(\left(\frac{r_{02}}{r_3}\right)\left(\frac{\delta+x^3}{1+\delta}\right)^{\frac{1}{3}}\right) \tag{24-f}$$

In Eq. (24-f), $\beta=\frac{\dot x}{|\dot x|}$, so if $\dot x>0$, $\beta=1$, otherwise $\beta=-1$.

Eq. (23) is a nonlinear vibration equation, it describes bubble radius changing with time for finite deformation in Casson liquid. By analyzing the load shown in Eq. (24-f), we can find that $2\sigma_1/(r_{01}x)$, $(2\sigma_2/r_{02})\left[(1+\delta)/(\delta+x^3)\right]^{\frac{1}{3}}$ denote the surface tension, $p_0x^{-3}-p_L(r_3)^{\frac{1}{3}}$ represents the pressure difference between the interface 1 and 2 which is changed with the deformation rate x(t) of bubble inner diameter; the component M(x) denotes the stress of protein film caused by the elastic deformation; and the remainder reflect the stress caused by viscosity of the Casson liquid and the viscoelasticity of protein film respectively. As the various parameters in the expressions are complex, it is difficult to obtain the analytical solution to study the influence of main parameters on the vibration of bubble, a numerical method is used to analyze the vibration of bubble wall in the next section.

RESULTS AND DISCUSSIONS

From Eqs. (24-a)~(24-d), it is shown that several important factors will affect the vibration of bubble . The main factors include the viscosity of Casson liquid, the viscoelasticity of protein film, and the surface tension of the bubble. In this section, we use the fourth-order Runge–Kutta numerical method and a MATLAB program to solve the differential Eq. (23), also we analyze and discuss the results.

The viscous stress is related to the variation of velocity. If r_3 in Figure 1 is large enough, the fluid velocity there will approach to zero, and the viscous stress will approach to zero too. The fluid domain for simulation is very important. We have studied how the ratio r_3/r_{01} affects the numerical results and found that when $r_3/r_{01}\geq 100$, the effect of r_3 on the results is very small. The pressure at this point we defined as liquid static pressure p_L, and let $p_L(r_3) = p_L$. In this paper, we choose $r_3 / r_{01} = 100$ as the fluid domain.

The following parameters will be used in the process of simulation. ρ_L=1.054×10^{3}kg/m^3, ρ_M =0.9×10^3 kg/m^3 , σ_1=6×10^{-2} N/m, σ_2=4×10^{-2} N/m.

A. The Effect of viscosity of Casson liquid on the vibration of protein bubble

The viscosity of blood is relevant to its components, such as the number percent of red blood cells, white blood cells and macromolecules in plasma. The viscosity of blood is also relevant to the intrinsic physical and chemical conditions, which include the temperature, PH value and osmotic pressure.

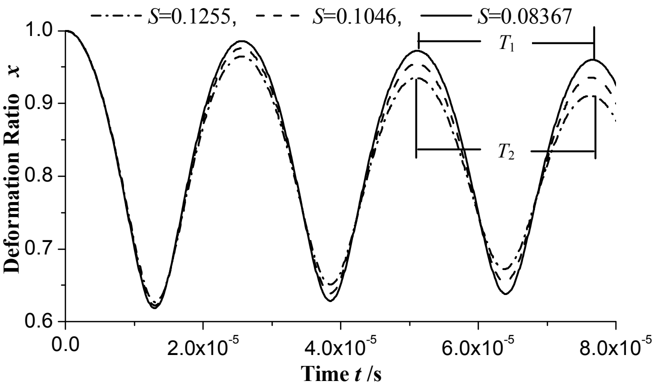

Figure 2. The effect of viscosity of Casson liquid on the vibration of protein bubble

When the Casson model is used to describe the constitutive relation of blood, the parameter S which characterizes the viscosity of blood will be affected by many factors, when S is changed, the vibration properties of protein bubble immersed in blood will be changed too. Fig. 2 simulates the vibration properties of protein bubble in the states of S=0.08367，0.1044 and 0.1255 respectively. The initial radius of protein bubble r_{01}=1.0×10^{-4}m, film thickness Δ=1.0×10^{-5}m, the initial pressure in the inner part of protein bubble p_0=0.1MPa, the surrounding static pressure p_L=0.2MPa.

As the numerical simulation results shown in Fig. 2, increasing the viscosity of Casson liquid will accelerate the amplitude decaying of protein bubble wall. In addition, the period T_1=2.376×10^{-5} s and T_2=2.412×10^{-5} s in Fig. 2, it shows that the period becomes longer when the viscosity of liquid is increasing. From Eqs. (24-c)~(24-e), with the viscosity increases, the vibration damping system will increase, thereby leading the above results.

B. The effect of viscoelasticity of protein film on vibration of protein bubble

Adjusting the geometric parameters of bubbles, viscoelastic parameters of protein film as well as the viscosity of the Casson liquid, the numerical simulation results shows that, when the viscosity of the protein film is greater than that of Casson liquid, the influence of viscosity of protein film on bubble deformation will be larger.

In Fig. 3, we choose the same radius of protein bubble r_{01}=5.0×10^{-5}m, film thickness Δ=1.0×10^{-5}m, the surrounding fluid parameters: S=8.367×10^{-2} [Pa.s]$^{1/2}$, b=7.273×10^{-2} [N/m^2]$^{1/2}$. One group of protein film parameters are as follows: C_1=8Pa，C_2=2Pa, μ (0) =10Pa，. η_m=0.1Pa.s，the other group is C_1=80Pa，C_2=20Pa，μ (0)=100Pa，η_m=1Pa.s. η_m is the apparent viscosity of protein film. Initial pressure inside the protein bubble is p_0=0.1MPa，static pressure of surrounding fluid is p_L=0.2MPa。

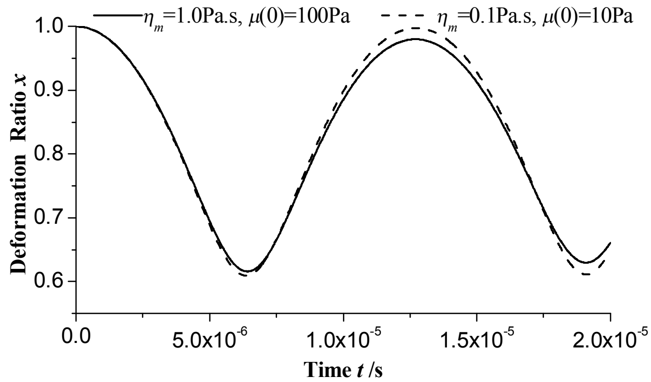

Figure 3. The effect of viscoelastic parameters of protein film on the vibration of protein bubble

As shown in Fig. 3, if the viscosity of protein film is greater than the that of Casson, the results indicate that increasing the viscosity of protein film will accelerate the amplitude decaying of protein bubble wall. In Fig. 3 the balance location x_1=0.71 for η_m=0.1Pa.s, μ (0) =10Pa，so the relative deformation of protein bubble is Δx_1=1-0.71=0.29, but the other one is Δx_2=0.27 for η_m=1.0Pa.s, μ (0) =100Pa. it denotes that under the condition of above liquid parameters and geometry parameters, the greater the viscoelasticity of protein film is, the stronger its load bearing capacity is.

C. Effect of surface tension on the vibration of protein bubble

As shown in Fig. 4, the geometric dimensions of protein bubbles are radius r_{01}=1.0×10^{-4}m, film thickness Δ=1.0×10^{-5}m. Two cases are simulated respectively. Those are case 1: without considering the surface tension; case 2: considering the surface tension at the interface 1 and 2 shown in Fig. 1. The surface tension coefficients σ_1 =0.06N/m, σ_2 =0.04N/m.

The results show that under the consideration of surface tension, the bubble wall will vibrate with higher decaying velocity of amplitude. In Fig. 4, relative deformation of protein bubble Δx_1=0.21 for case 1, Δx_2=0.19 case 2, it denotes that without considering the surface tension will lead to a greater relative deformation of protein bubble.

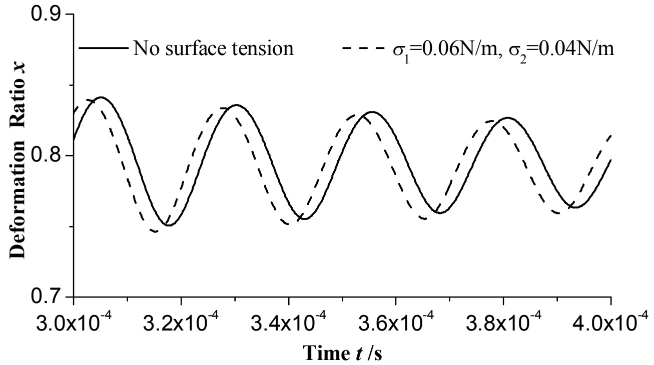

Figure 4. The effect of surface tension on the vibration of protein bubble in Casson liquid

CONCLUSIONS

In this paper, we chose a Casson model to describe the constitutive relation of blood in the process of flowing. The forces of liquid acting on the protein bubble surface are analyzed, the finite deformation properties of viscoelastic protein film under the acting of pressure difference are studied, and vibration equation of protein bubble in blood liquid is built. Numerical simulation of this equation is presented to study the vibration properties of protein bubble in blood. Increasing the viscosity of Casson liquid will accelerate the amplitude decaying of protein bubble wall; the period becomes longer. In the condition that the viscosity of protein film is greater than that of Casson fluid, increasing the viscoelasticity of protein film will also accelerate the amplitude decaying of protein bubble wall. Further more, the greater the viscoelasticity of protein film is, the stronger its load bearing capacity is. Under the consideration of surface tension, the bubble wall will vibrate with higher decaying velocity of amplitude. Without considering the surface tension will lead to a greater relative deformation of protein bubble.

ACKNOWLEDGMENTS

Project is supported by the National Natural Science Foundation of China (Grant No. 10902070) and the Leading Academic Discipline Project of Shanghai Municipal Education Commission (Grant No. J50501).

REFERENCES

1 Cavalieri et al., 2008, "Polymer microbubbles as diagnostic and therapeutic gas delivery device," *Chemistry of Materials, Vol. 20,* pp. 3254-3258.

2 Kobayashi et al., 2002, "Endothelial cell injury in venue and capillary induced by contrast ultrasonography," *Ultrasound in Medicine & Biology, Vol. 28,* pp. 949-956.

3 Bekeredjian, R., Graybum, P. A., Shohet, R., 2005, "Use of ultrasound contrast agents for gene or drug delivery in cardiovascular medicine". *Journal of the American College of Cardiology, Vol. 45,* pp.329-335.

4 Kondo et al., 2004, "Treatment of acute myocardial infraction by hepatocyte growth factor gene transfer of a 'functional' gene using ultrasonic microbubble destruction," *Journal of the American College of Cardiology, Vol. 44,* pp. 644-653.

5 Charles, C. C., 1995, "The effects of an elastic solid surface layer on the radial pulsations of gas bubbles," *Journal of the Acoustical Society of America, Vol. 97,* pp. 1510-1521.

6 Frinking, P. J. A., and Jong. N., 1998a, "Acoustic modeling of shell-encapsulated gas bubbles," *Ultrasound in Medicine and Biology, Vol. 24,* pp.523-533.

7 Frinking et al., 1999b, "Scattering properties of encapsulated gas bubbles at high ultrasound pressures," *Journal of the Acoustical Society of America, Vol.105,* pp. 1989-1996.

8 Plesset, M. S., and Hsieh, D. Y., 1960, "Theory of gas bubble dynamics in oscil-lating pressure fields," *Physics of Fluids, Vol. 3,* pp. 882-892.

9 Simo, J. C., 1987, "On a fully three-dimensional finite-strain viscoelastic damage model: formulation and computational aspects," *Computer Methods in Applied Mechanics and Engineering, Vol. 60,* pp.153-173.

10 Wang, H. M., Ma, J. M., Zhang, W., 2007, "Effect of film viscoelasticity on the finite deformation of a spherical bubble," *Journal of Applied Physics, Vol.102,* pp.074919.

11 Shima, T. A., and Tsujino, T., 1978, "The behavior of gas bubble in the Casson fluid," *Journal of Applied Mechanics, Transactions ASME, Vol. 45,* pp. 37-42.

12 Wang et al., 2009, "Vibration of a single protein bubble in Bingham liquid," *Journal of Hydrodynamics, Vol. 21,* pp. 658-668.