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SIMULATION OF FLUID FLOW USING REDUCED-ORDER MODELING BY POD APPROACH APPLIED TO A FIXED TUBE BUNDLE SYSTEM

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ABSTRACT

Tube bundles in steam boilers of nuclear power plants and nuclear on-board stokehold are known to be exposed to high levels of vibrations. This coupled fluid-structure problem is very complex to numerically set up, because of its three-dimensional characteristics and because of the large number of degrees of freedom involved. A complete numerical resolution of such a problem is currently not viable, all the more so as a precise understanding of this system behaviour needs a large amount of data, obtained by very expensive calculations. We propose here to apply the now classical reduced order method called Proper Orthogonal Decomposition to a case of 2D flow around a tube bundle. Such a case is simpler than a complete steam generator tube bundle; however, it allows observing the POD projection behaviour in order to project its application on a more realistic case.

The choice of POD leads to reduced calculation times and could eventually allow parametrical investigations thanks to a low data quantity. But, it implies several challenges inherent to the fluid-structure characteristic of the problem. Previous works on the dynamic analysis of steam generator tube bundles already provided interesting results in the case of quiescent fluid [J.F. Sigrist, D. Broc; Dynamic Analysis of a Steam Generator Tube Bundle with Fluid-Structure Interaction; Pressure Vessel and Piping, July 27-31, 2008, Chicago]. Within the framework of the present study, the implementation of POD in academic cases (one-dimensional equations, 2D-single tube configuration) is presented. Then, firsts POD modes for a 2D tube bundle configuration is considered; the corresponding reduced model obtained thanks to a Galerkin projection on POD modes is finally presented. The fixed case is first studied; future work will concern the fluid-structure interaction problem. Present study recalls the efficiency of the reduced model to reproduce similar problems from a unique data set for various configurations as well as the efficiency of the reduction for simple cases. Results on the velocity flow-field obtained thanks to the reduced-order model computation are encouraging for future works of fluid-structure interaction and 3D cases.

INTRODUCTION

Nuclear power plants or on-board stokehold steam boilers functioning intrinsically induces several vibratory levels, especially concerning the tube bundle part of the boiler [5], [11], [31], [37]. It is shown that fluid-elastic instabilities can occur in such a configuration [9], [12], [13], leading to a certain destruction of tubes: this is why the study and a precise comprehension of this vibration mechanism are crucial. But, a good comprehension remains difficult because of the high number of parameters that play a role in the vibrations generation [32], [33]. Thus, only laboratory and/or on-site experiments are not sufficient (although very precise from the physical point of view) and it becomes necessary to develop accurate and robust CFD numerical codes [22], [40] in order to set up parametric studies that could help the understanding of violent phenomena like fluid-elastic instabilities.
Another constraint is the high resource level that is necessary to set up this fluid/structure interaction problem: to be as close as possible to real conditions, a fully 3D turbulent flow has to be computed by the CFD code [18], added to the cost of the coupling with a CSD code. In an industrial configuration, such a computation remains inconceivable, first because of the resource cost, second because of the CPU time involved.
We propose an alternative that could offer perspectives in the study of tube bundle vibrations, using reduced-order models.

Theses models are well-known and widely used in the field of fluid mechanics [10], [16] as well as structure field [2], but they still represent a challenge within the frame of fluid-structure interactions [14], [26], [42]. They however could give a better comprehension of the physics of this fluid/structure interaction problem, giving information on parameters which could no be easily obtained from experiments. The reduced-order model that we propose to set up in this paper is the Proper Orthogonal Decomposition (POD) [21], [24] which is now used in many fields [1], [21], [27].

The paper is organized as follows: a first part is dedicated to the main vibrations problems that can encounter a tube bundle in real conditions. Then, current numerical models that are used to solve and study such a problem are briefly presented in the second part. Proper Orthogonal Decomposition will be described as well as its potential contribution specifically for this crucial question of tube bundle vibrations. Finally, in the third part, first numerical results in the use of POD are proposed and perspectives for a future work are exposed.

1. HEAT EXCHANGER TUBE BUNDLE VIBRATIONS PROBLEMS

Figure 1 shows the general functioning of an on-board stokehold steam boiler of a water pressurized reactor (WPR). The functioning of a civil nuclear steam boiler is quite the same ; both are WPR. Water of the primary circuit feeds the tubes (in red) driven by a pump. Liquid water of the secondary circuit comes from the top of the tube bundle and vaporizes by ascension along the tubes.

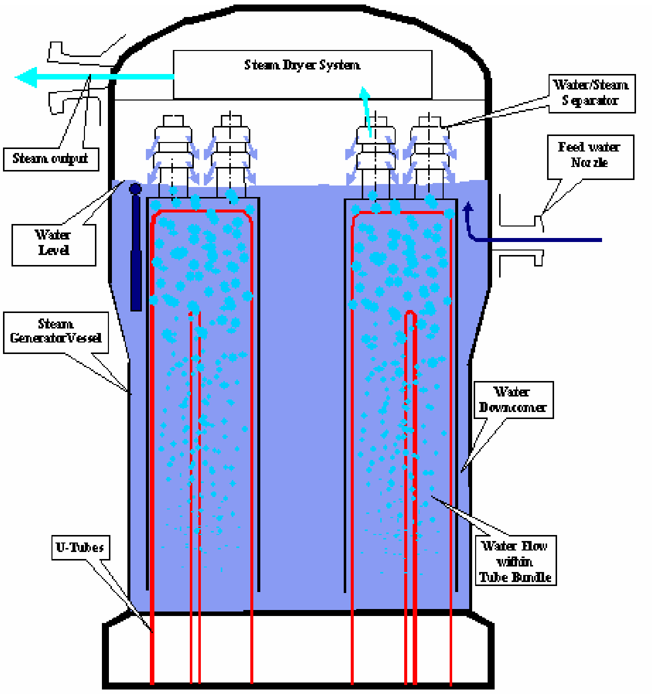


Fig. 1. Steam boiler system

Main variables used in this paper are presented in table 1:

Variable	Definition
ρ	Fluid density (kg.m $^{-3}$)
μ	Fluid dynamic viscosity (kg.m $^{-1}$.s $^{-1}$)
D	Diameter of one tube (m)
P	Step between two tube diameters (m)
$[M]$	Total mass matrix
$[C]$	Damping matrix
$[K]$	Stiffness matrix

Tab. 1. Main variables of the system

Classically, the equation of motion of one tube is the following:

$$[M]\{\ddot{Q}(t)\}+[C]\{\dot{Q}(t)\}+[K]\{Q(t)\}=\{F_{ext}\} \qquad (1)$$

where $\{Q(t)\}$ represents the motion generalized coordinates vector, $\{F_{ext}\}$ is the fluid forces vector to which the tube is subjected.

Definition of these parameters is of high importance and is far from easy. Various models have been proposed in order to take into account each coupling mechanism (for example, the contribution of several damping parameters have to be considered). Experimental data compiled by Pettigrew and Taylor [33], [34] have been used to define semi-empirical formulations for several damping components, according to the thickness of the grid supporting the tubes, natural frequency of the considered vibration mode of a tube, total mass, and considerations on tube supports. Moreover, these formulations are different when considering a single liquid phase, a single gas phase or a two-phase flow.

Before presenting different tubes excitation phenomena, it is necessary to redefine dimensionless numbers that govern the fluid flow, Reynolds number and Strouhal number. Table 2 gathers these dimensionless numbers.

Variable	Description
R_e	$\dfrac{\rho U_p D}{\mu}$
S_t	$\dfrac{f_s D}{U_p}$

Tab. 2. Dimensionless numbers for a fluid-structure interaction problem in a tube bundle configuration

The step fluid velocity U_p takes into account the tube confinement. It is defined as $U_p = U_\infty \dfrac{P}{P-D}$ where U_∞ is the equivalent mean flow velocity which would have been imposed in an infinite domain.

Four vibratory excitation mechanisms are likely to exist under cross-flow: turbulent excitation, vortex-induced vibrations, acoustic resonance and fluid-elastic instability. Figure 2 reminds the evolution of a tube response to a cross-flow against mean fluid flow velocity.

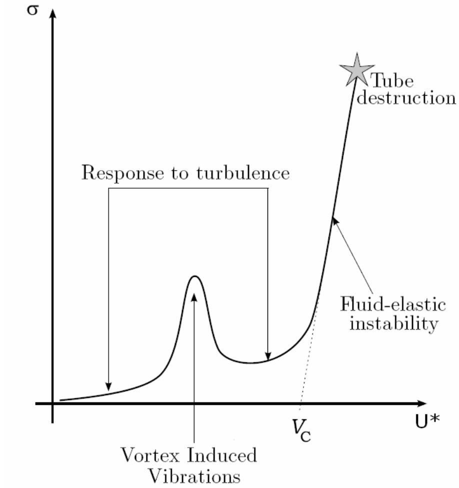


Fig. 2. Scheme of vibratory response of a cylinder in tube bundle

The apparition of each phenomenon depends on parameters that are not always known and/or easily observable. But researchers have collected information in order to predict and to prevent destructive mechanisms.

Turbulent excitation is unavoidable: Reynolds number of the flow regime is $R_e \in [10^4;10^7]$. Moreover, turbulence is recommended in order to produce a mixing as perfect as possible, to obtain good heat transfers. However, turbulence induces generally low structural motion amplitude [4] so this mechanism has to be taken into account in fretting-wear damage considerations [34].

Vortex-Induced Vibrations (VIV) are well known in the case of a single tube in infinite domain (see [11], [43] for example). In the present case of tube bundles, this phenomenon is more complex. The presence of a "lock-in" [20], [23] has been detected by Pettigrew and Gorman [31] in air, but Axisa [4] never encountered the set up of such a mechanism. Pettigrew and Taylor [33] explain that the presence of turbulence tends to reduce the possibility of vortices to set up, that reduces vortex-induced vibrations. Furthermore, Païdoussis [30] insists on the fact that the distinction between both mechanisms (turbulence and vortex shedding) is far from easy.

Acoustic resonance is susceptible to appear in the context of a single-phase gas exchanger. It strongly depends on the tubes arrangement. This phenomenon is not taken into account within the framework of this study since exchangers are fluid-fluid exchangers. Works on this mechanism have been led in several configurations [9], [44].

Finally, fluid-elastic instability is the most spectacular vibratory excitation phenomenon [8], [36]: it leads to a very quick destruction of the tubes that have been excited. For theses grounds, researchers particularly focused on this mechanism in order to avoid it at all costs. When flow velocity reaches a certain critical threshold V_C, structural motion produces a fluid force with the same orientation as structure motion direction: this leads to vibratory amplitudes much larger than those usually observed. Only the tube breaking, caused by repeated impacts between the tube and its support, will stop the excitation. This is precisely an interaction between fluid and elastic efforts, the first feeds the second and conversely: on that point, this mechanism differs from VIV, whose amplitude is auto-limited. A very large number of models, empirical or semi-empirical, have been proposed in the hope of avoiding such a situation. A very widespread model is the Connors model [15], who proposed to express the critical mean fluid velocity V_C as:

$$V_C = K\sqrt{A_R} \tag{2}$$

V_C is a function of the Scruton number $A_R = \frac{2\pi\zeta m}{\rho D^2}$ where ζ is the global damping for the considered tube mode and m is the total mass per unit length. This dimensionless number measures the energy proportion that the system can dissipate thanks to its proper damping, compared to the energy proportion that the fluid provides to the structure through the fluid-elastic coupling force. A constant K weights this number; it is defined as $K = \sqrt{\frac{\pi}{k_1 k_2}}$ where k_1 and k_2 are stiffness constants of two neighbouring tubes. When the global damping becomes negative, the system becomes instable. This model has been enriched by many authors [19], [30], [32]. The notion of a delay between structure solicitations and the flow reaction is also introduced. This delay has a big influence on the velocity stability threshold. Price [36] shows mathematically that a fluid-elastic instability phenomenon is set up by a negative work of fluid efforts. Price highlights three mechanisms that can explain this energy extraction to the fluid by the structure: first, the discrepancy between structure displacement and fluid forces, which presupposes that the damping is governing the phenomenon, the physics being related to structure displacement. When this damping becomes negative, instabilities appear. The second mechanism takes place when at least two degrees of freedom are involved and when there is a phase discrepancy between them. Structure displacement is impacted, that is why the mechanism is described as leaded by stiffness. The third mechanism is the apparition of hysteresis in fluid forces evolution because of non-linearities. Here, efforts amplitude depends on the structure motion direction.

But, flow passing a tube bundle is a system containing a very high number of degrees of freedom, so a precise analytical description of exciting efforts is not possible; moreover, several modes can be excited, considering relatives cylinders motions.

For each vibration mechanism, experimental data have been collected and exploited by various authors in order to define criteria to respect [12], [25], [34]. Sometimes, semi-analytical models have been developed, particularly in the case of the fluid-elastic instability phenomenon; see [36] for example.

CFD models have been set up in order to avoid experimental costs and to observe a large number of parameters. Vortex-Induced Vibrations have been numerically studied with high precision and most of their mechanisms are now well understood. However, when turbulence is present in the flow, fluid-structure interactions are more difficult to represent, notably because of the three dimensional nature of the turbulent flow. Fluid-elastic instability is also very hard to model for the same reason and because of the number of parameters that are involved.

A constant challenge in numerical modelling is based on interactions between fluid and structure motions. In order to solve these interactions, two classes of approaches exist [28]: the first is called monolithic approach, and consist in the use of a unique formulation for fluid and structure modelling. This approach is theoretically optimal, but very costly and only adapted to simple geometries. The second is a partitioned approach: fluid and structure equations are resolved separately, with information communication between both of them. A good description of these approaches can be found in [39].

However, in both cases, a complete numerical resolution of the fluid-structure interaction in a tube bundle in running rate cannot be carried out. In this context, the use of Reduced Order Models (ROM) can be a solution to achieve the realization of such a study. A ROM allows solving a problem which formulation contains the bulk of the system information with a reduced number of degrees of freedom.

In the framework of fluid dynamics studies, the criterion that ensures the fact that "the bulk of the system information" is kept can be an energetic criterion. Using this criterion, the optimal approach is the well known Proper Orthogonal Decomposition (POD). This method is the subject of next section; its potential application for the study of fluid-structure interactions in tube bundles is also developed.

2. PROPER ORTHOGONAL DECOMPOSITION (POD)

In classic Computational Fluid Dynamics studies, approximated Navier-Stokes equations are computed on a three-dimensional domain Ω for a time interval $[0;T]$. In the case of a large three-dimensional domain, and if the flow is turbulent, calculation times can be very long. Moreover, if a parametric study has to be set up, it is necessary to lead as many calculations as there are values of the parameter. Proper Orthogonal Decomposition allows saving calculation time on computations, and provides a projection basis that can be reused in parametric studies: this technique allows avoiding rerun calculations. In an industrial context, theses advantages have to be taken into account.

Proper Orthogonal Decomposition has notably been introduced by Lumley [29] within the framework of coherent structures extraction of turbulent flows. A rigorous description of POD can be found in [21] for example; a large amount of domains is now using POD techniques, what leads to variant methods, see [14] or [38]. Here we briefly present the POD formulation.

Let us consider a domain Ω of the set of all real numbers and a time interval $[0;T]$. Spatial and time variables are respectively $x \in \Omega$ and $t \in [0;T]$.

Let $u(x,t)$ be the unknown field, for example the velocity field (which is the unknown of Navier-Stokes equations), with $u(x,t) \in H(\Omega,T)$, H denoting a Hilbert space. Proper Orthogonal Decomposition consists in determining a determinist basis $\{\Phi_n\}_{n=1,\dots,N}$ of functions which give the optimum representation of the field $u(x,t)$. N is the size of the POD basis.

A practical approach of POD has been proposed by Sirovich [41], it is called Snapshot POD: this method is based on making the most of samples of experimental or numerical data. Let consider M snapshots of the velocity field $u(x,t)$ (these snapshots can be equally taken from an experimental or numerical set), these snapshot have been sampled during a time period T. Snapshot POD consists in solving the following eigenvalue problem:

$$\sum_{k=1}^{M} \frac{1}{M} \left(u(t_i), u(t_k)\right)_{L^2(\Omega)} A_k = \lambda A_i \qquad (3)$$

for each $i = 1,\dots,M$, where λ represents eigenvalues. Each element of the POD basis is a linear combination of snapshots, coefficients are A_n^k, $n = 1,\dots,N$:

$$\Phi_n(x) = \sum_{n=1}^{M} A_n^k u(x,t_n) \qquad n = 1,\dots,N \qquad (4)$$

The POD basis $\{\Phi_n\}_{n=1,\dots,N}$ has the following property: it is orthonormal and for an incompressible flow, each element of the basis (*i.e.* each POD mode) satisfies the incompressibility condition as well as boundary conditions of the problem. For a given $n \in [1,2,..,N]$, the energetic contribution of the POD

mode Φ_n is captured by the corresponding eigenvalue λ_n and eigenvalues are ranked in descending order ($\lambda_1 > \lambda_2 > ... > \lambda_N$). Thus, Proper Orthogonal Decomposition is optimal in an energetic sense: using the first POD modes means keeping the most part of the system energy. Thus, as for a spectral method, the POD basis is truncated to N^* modes, where N^* is less or equal to the POD basis size. To determine N^*, an energetic criterion is used.
As the Proper Orthogonal Basis is fully spatial and based on time snapshots, its use within a fluid-structure interaction resolution is not immediate. Indeed, if the numerical sample from which the snapshots are extracted has been obtained thanks to a moving mesh technique, the construction of a POD basis has no sense, since the POD modes are not time-dependants. Thus, in the case of fluid-structure interaction problems, an extension of the Snapshot POD is necessary. It has been presented precisely by Liberge [26] who proposes to work on a static spatial domain using a projection of snapshots taken from a moving domain study. Here, we first present some POD properties working on very simple cases; then, first applications on the case we are interested in will be proposed.
When POD modes $\{\Phi_n\}_{n=1,...,N}$ are determined, a low order dynamical system can be solved. For that, a partial differential equations system is projected on the POD basis constructed for the field $u(x,t)$. Then, a system of ordinary differential equations, which size is N^*, is obtained.
For example, in the very simple case of the one-dimensional heat transfer equation, written as:

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} - \dfrac{\partial^2 u(x,t)}{\partial x^2} = 0 \qquad t \in [0,T], x \in \Omega \\ + BC \\ + IC \end{cases} \tag{5}$$

where BC and IC respectively stand for boundary conditions and initial conditions; the dynamical system after projection on POD modes reads:

$$\left(\frac{\partial u(x,t)}{\partial t}, \Phi_i\right) + \left(\frac{\partial u(x,t)}{\partial x}, \frac{\partial \Phi_i}{\partial x}\right) = 0 \tag{6}$$

with Φ_i the ith POD mode, assuming homogeneous boundary conditions. It is shown that the field $u(x,t)$ can be decomposed along variables x and t as:

$$u(x,t) = \sum_{n=1}^{N^*} a_n(t)\Phi_n(x) \tag{7}$$

Where $\{\Phi_n\}_{n=1,...,N^*}$ are elements of the POD basis and $a_n(t)$ are time coefficients.
Thus the low order dynamical system becomes:

$$\frac{da_i}{dt} = -\sum_{n=1}^{N^*} a_n \left(\frac{d\Phi_n}{dx}, \frac{d\Phi_i}{dx}\right) \tag{8}$$

with orthonormal property of POD modes.
A very interesting characteristic of the POD basis is its ability to represent a solution different from the solution of the problem from which the basis is computed. Of course, the new problem has to be similar to the first one, which is precisely the case in the framework of a parametric study. An example on the one-dimensional heat transfer equation with two different boundary conditions is proposed, based on the work of Chinesta [14]: we define a first field with a heat flux step function for boundary condition, and a second field with a heat flux ramp function for boundary condition.
A POD basis is computed from the first problem and both reduced dynamical systems are computed by projection on this unique basis. The reconstruction gives good results, see figure 2.
In order to check POD characteristics for a flow past an obstacle, a POD basis is computed for the problem of a single circular cylinder in cross-flow at $R_e = 100$ (the problem of lock-in for such a configuration have been previously studied, see [35]; here the cylinder is fixed, calculations have been run using the CFD code *Code_Saturne* [3]). Figure 3 shows first and second velocity components of the flow at a date t in the sampling period $[0;T]$. Then, figure 4 shows the streamwise component of the two firsts POD modes obtained from this sample. The sampling period $[0;T]$ corresponds to one lift force fluctuations period (i.e. around 6 s) with a time step of $\Delta t = 0.025$ s. 150 snapshots have been taken into account to constitute the sample. In the case of a low Reynolds number flow around a circular cylinder, just one or two fluctuations periods are necessary to make a good sample. In the case of a turbulent flow, more pseudo-cycles are needed to take into account most of the energy of the flow.
If we remember that the first mode energetic contribution is preponderant, it is easy to understand that the first POD mode is linked to the mean flow, while next modes (from Φ_2 to Φ_{N^*}) contain the energy of flow fluctuations. As the vortex shedding phenomenon comes from these fluctuations, they are visible only from the second mode.

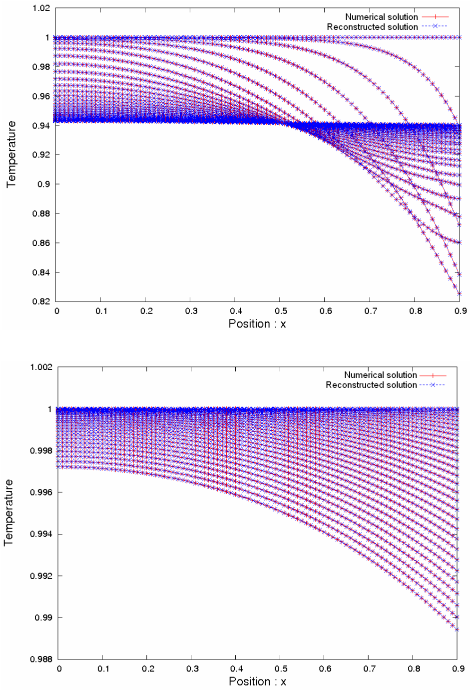


Fig. 2. POD reconstruction of two similar problems by projection on a unique POD basis

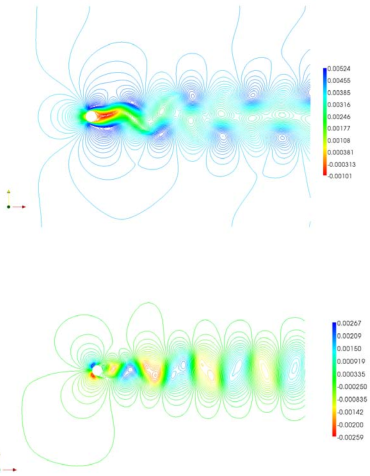
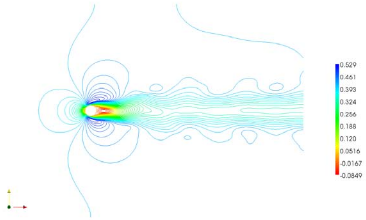
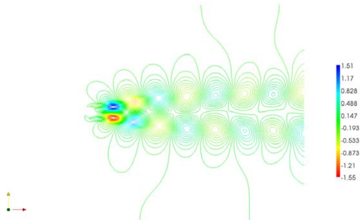


Fig. 3. Streamwise and cross-stream velocity components at time t for a fixed cylinder in cross-flow at $\mathrm{Re} = 100$



4a: 1st mode, streamwise component



4b: 2nd mode, streamwise component

Fig. 4. First component of the two firsts POD modes computed from the instantaneous velocity field for a fixed cylinder in cross-flow at $\mathrm{Re} = 100$

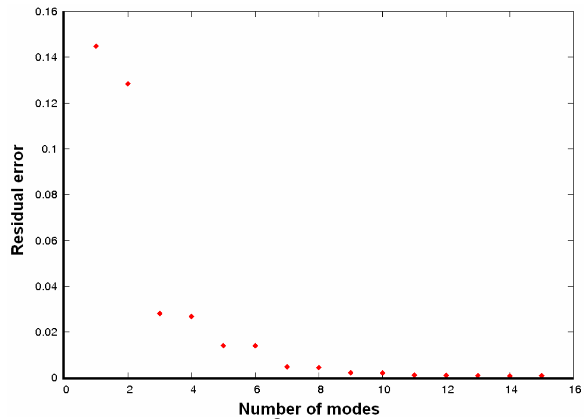


Fig. 5. Residual error (in L^2 norm) between instantaneous velocity field and its POD reconstruction for a fixed cylinder in cross-flow at $\mathrm{Re} = 100$

The real interest of extracting a POD basis from a set of snapshots of a fluid velocity field is the construction of a reduced-order model, by projection of Navier-Stokes equations on POD modes. It has been led for the single fixed circular cylinder, but what we are really interested in here is the tube bundle configuration: the reduced-order model construction is detailed in the following part.

3. TUBE BUNDLE CONFIGURATION

A tube bundle configuration is then proposed in order to be closer to the problem we are interested in. However, the chosen configuration remains simple: a 2D domain and only one tube and its neighbors are considered, with periodic boundary conditions. Thus, the domain is representing an infinite regular tube bundle. First, the central tube remains fixed. The mesh used is presented on figure 5.

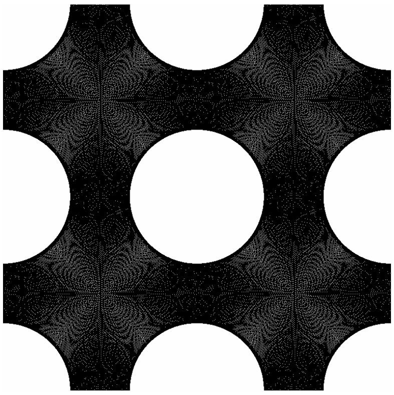


Fig. 5. Mesh of the pattern used for the POD study of a tube bundle configuration

The periodicity on all boundaries of this domain is a problem to generate a flow and to observe a non stationary flow. As explained in Longatte et *al.* [28], it is thus necessary to add a source term (here it is a mass flow rate) to Navier-Stokes equations and to generate numerical fluctuations with, for example, a velocity step function on a given time interval. When the source term is time-converged, a calculation with this term can be set up. With this method, a non-stationary flow is observed for a Reynolds number $\mathrm{Re} \approx 2600$.
The goal here is to be able to observe non stationary flow for the lower Reynolds number, because the 2-dimensional configuration at high Reynolds number would imply bad hydrodynamic efforts estimations. A comparison of these first results with a three-dimensional calculation at the same Reynolds number would be done in order to compare hydrodynamic efforts estimations and Strouhal number.
The POD method has been implemented on such a configuration; thus firsts POD modes can be observed on figure 6. This basis is composed of 4 modes obtained from the fluctuating velocity field: it means that the first of these modes does not represent the mean flow but fluctuations of the flow.
When POD modes are obtained, the following step is to project Navier-Stokes equations on these POD modes for the present tube bundle configuration and compare the fluid flow field obtained after projection to the original set of snapshots. From now on, the fluid flow configuration obtained thanks to the complete calculation is considered as a reference, even if it is not representing a real configuration. The important part of the present work is to check the POD reduced order model efficiency to reproduce a velocity field. A future work will consist in working on cases that can be compared to literature, in order to check if the complete calculation of the flow is correct. To the authors' knowledge, present configuration has not been numerically studied (inline 2D square array of fixed tubes at Reynolds number $\mathrm{Re} \approx 2000$). However, the CFD code used in this study, *Code_Saturne* [3], has been validated for fluid-structure interactions in various tube bundle configurations; see [6], [22], [28].

Time coefficients obtained thanks to a direct POD computation are constructed knowing POD modes and velocity field from the complete calculation:

$$a_n(t) = (u(x,t), \Phi_n(x))_{L^2(\Omega)} \ ; \ \forall \ n = 1,\dots,N^* \quad (9)$$

Using incompressible Navier-Stokes equations:

$$\begin{cases} \dfrac{\partial}{\partial t}u + (u \cdot \nabla)u = -\dfrac{1}{\rho}\nabla p + \nu \Delta u \\ \nabla \cdot u = 0 \end{cases} \quad (10)$$

where p is the pressure field and ρ, ν respectively flow density and cinematic viscosity. Low-order dynamical system obtained from the projection on the POD basis is:

$$\frac{da_i}{dt} = -\sum_{n=1}^{N^*}\sum_{m=1}^{N^*} a_n(t)a_m(t)B_{nmi}(x) - \nu\sum_{n=1}^{N^*} C_{ni}(x) - D_i(x) \quad (11)$$

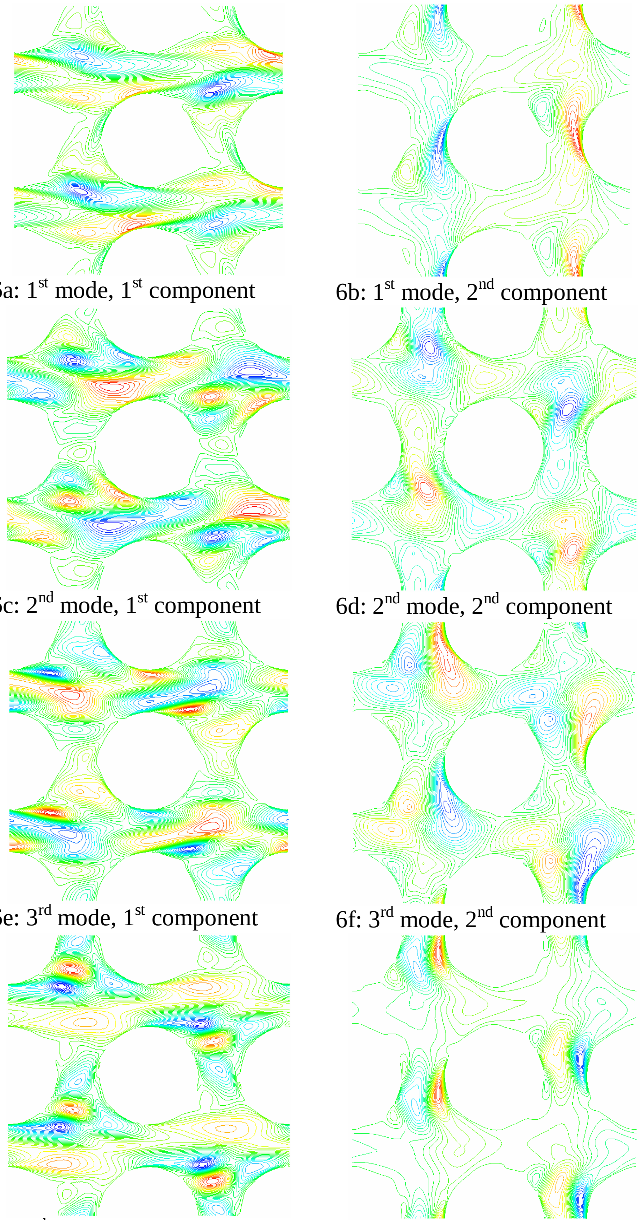
for each $i = 1,\dots,N^*$, with spatial coefficients B, C, D defined in table 3.

Coefficient	Definition
B_{nmi}	$\left((\Phi_n(x)\cdot\nabla)\Phi_m(x), \Phi_i(x)\right)_{L^2(\Omega)}$
C_{ni}	$\left(\nabla\Phi_n(x), \nabla\Phi_i(x)\right)_{L^2(\Omega)}$
D_i	$\left(\dfrac{1}{\rho}\nabla p, \Phi_i(x)\right)_{l^2(\Omega)}$

Tab. 3. Spatial coefficients of the POD reduced model for incompressible Navier-Stokes equations

Coefficient D cannot be calculated through the reduced-order model, since information is only given for the velocity field. Thus, it is necessary to model this coefficient. Various solutions exist to obtain the new pressure field. In this paper, coefficient D has not been modelled yet; it is a short-term perspective for our future work.
Figure 7 represents the comparison between the fluid velocity field from the complete calculation on an interval of 5 seconds (which correspond to 3 pseudo-periods of lift coefficient) and the fluid velocity field obtained after the projection of Navier-Stokes equations on the POD basis on the same time period. 100 snapshots have been taken to constitute the data sample. The POD basis is constituted of $N^* = 4$ modes.
On figure 7, time coefficients $\{a_n\}_{n=1,\dots,N^*}$ (see formula (7)) are plotted. Time coefficients directly constructed with the complete calculation velocity field are called "direct coefficients" on figure 7. Time coefficients obtained thanks to

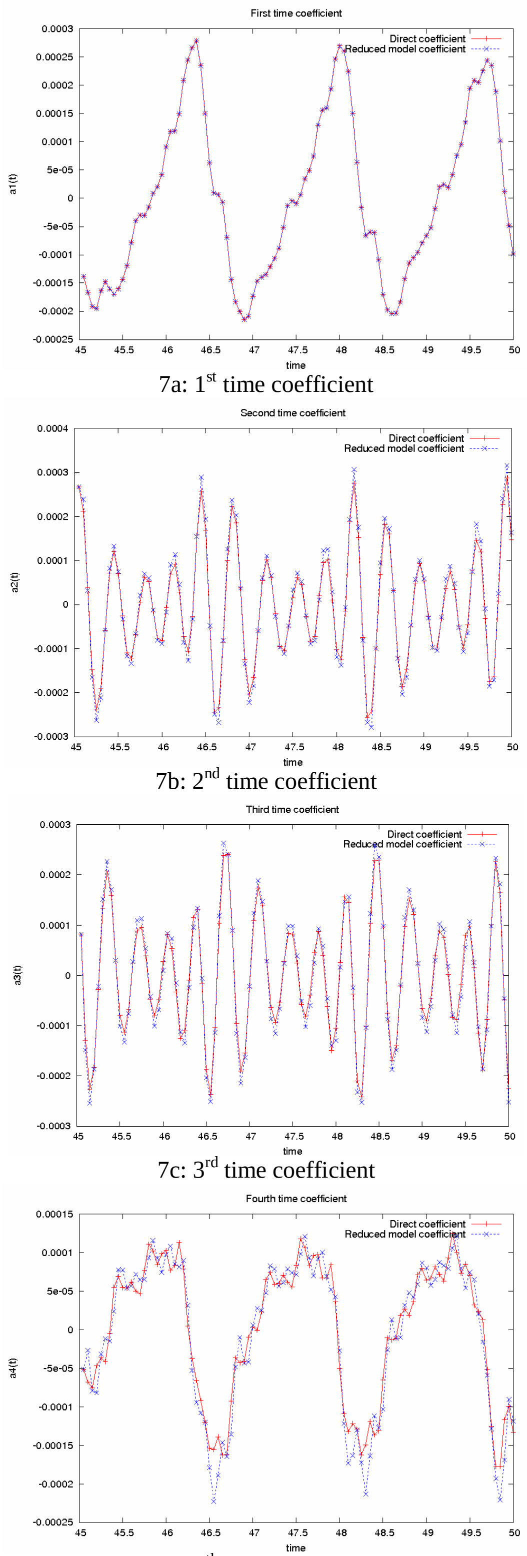
the dynamical system resolution are called "reduced model coefficients".



6a: 1st mode, 1st component 6b: 1st mode, 2nd component
6c: 2nd mode, 1st component 6d: 2nd mode, 2nd component
6e: 3rd mode, 1st component 6f: 3rd mode, 2nd component
6e: 4th mode, 1st component 6f: 4th mode, 2nd component

Fig. 6: Components of the first four POD modes of a periodic 2D fluid flow around a 9-tube bundle at $\mathrm{Re} \approx 2600$**. 1st component: streamwise component, 2nd component: cross-flow component.**

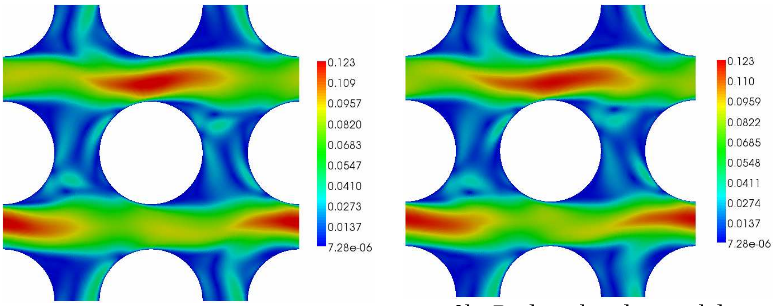
The complete calculation has been led for a time interval of 5 seconds with a time step $\Delta t = 0.001$ s.; a snapshot has been saved every 50 Δt. Results of the complete calculation versus the resolution of the reduced-order model for a date $t \in [0, T]$ are presented on figure 8.



7a: 1st time coefficient
7b: 2nd time coefficient
7c: 3rd time coefficient
7d: 4th time coefficient

Fig. 7: Comparison between time coefficients for the case of a periodic 2D flow around a 9-tube bundle at $\mathrm{Re} \approx 2600$**.**

The CPU time for a serial calculation have been estimated to 6 hours, for a mesh of around 60 000 cells. The resolution of the reduced-order model for the same time interval, computed with one processor, last 65 s of CPU time: the reduced-order model saves a considerable amount of time for this simple configuration. Working with the 3D configuration would certainly not give such a time saving because of the calculation of spatial coefficients of the dynamical system which can become very expensive.



8a: Complete calculation

8b: Reduced-order model resolution

Fig. 8: Comparison between the complete calculation and the reduced-order model resolution for a periodic 2D fluid flow around a 9-tube bundle at $Re \approx 2600$.

To represent a tube vibration in the bundle, a computation with moving mesh technique will be run. As mentioned earlier, the POD study of a fluid-structure interaction is a challenge because of the total spatial characteristic of the POD modes. A classic ALE method [17] to compute the field from which POD modes are computed would not be correct. That is why techniques with non-moving mesh have to be implemented, see [26]. Such methods will be set up in a future work and applied to this tube bundle configuration.

CONCLUSION

In this paper, the crucial problematic of vibratory excitation of a heat exchanger tube bundle is presented. Fluid-elastic instability is one of the most violent vibration mechanisms and a lot of studies have been led in order to define the critical fluid velocity and avoid such a phenomenon. This problematic is well known but not well understood. A way to improve our comprehension of tube bundle vibrations is to work with reduced order models (ROM). The most widespread ROM method, called Proper Orthogonal Decomposition (POD) and its properties are briefly presented. First applications to the tube bundle are proposed; future work will consist of taking into account the fluid-structure interaction thanks to Liberge works [26] and modelling the pressure term.

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