## 

## study on evaluation of sound speed in duct with tube banks

Kunihiko ISHIHARA<br>The University of Tokushima, Institute of<br>Technology and Science<br>Tokushima-City, Tokushima-Pref, JAPAN


#### Abstract

As tube banks are set in a duct in a boiler and a heat exchanger, the resonance phenomenon or the self sustained tone are generated due to the interference between vortex shedding and the acoustic characteristics of the duct. It is necessary to know the resonance frequency of the duct, namely sound speed, for avoiding any trouble that may arise. In general, it is said that the sound speed decreases in the duct with tube banks and an evaluation formula is given. However, this formula is often used for the perpendicular direction of the flow. We wanted to know whether this formula would be able to be used for the flow direction and for various arrays of patterns or not. In this paper, the applicability of this expression is discussed by using FEM analysis and experiments.


## INTRODUCTION

In boilers and heat exchangers, the tube bank is set in a duct and the heat exchange is executed between the water inside the tube and the high temperature gases outside the tube. On this occasion, the Karman vortex with the frequency proportional to the velocity is generated behind the tubes and it excites the acoustic field in the duct. On the other hand, inside the duct forms an acoustic field and has the acoustic natural frequency determined by the sound speed and the duct size. Then, when the shedding frequency of the Karman vortex coincides with the acoustic natural frequency of the duct, the resonant phenomenon arises and the self-sustained tone is generated especially when the duct has a small acoustic damping. When these phenomena arise, the factory is forced to stop due to the complaint of the neighbor or tremendous amount of money is required. Consequently, it is necessary to examine the possibility of the resonance and the self-sustained tone in the design stage. It is necessary to know the exact sound speed in the tube bank in the design stage to examine these possibilities.

Parker [1] and Blevins [2] proposed the sound speed in a tube bank $c_{\mathrm{e}}$ as follows:

$$
c_{e} / c_{0}=1 / \sqrt{1+\sigma}
$$

Where $\sigma$ is the ratio of the volume occupied by tubes to the total duct volume and $c_{0}$ is the sound speed in the ambient fluid. This expression shows that the sound speed $c_{\mathrm{e}}$ is dependent on only $\sigma$. We questioned whether this expression is applicable to any array patterns of tube banks if only $\sigma$ is the same.

In this paper, the applicability of this expression will be examined by experiments and by FEM analysis.

## MAIN NOMENCLATURES

$a$ : Added mass coefficient
$\sigma:$ Filling fraction or Volume fraction (the ratio of the volume occupied by tubes to the total duct volume)
$c_{0}$ : Sound speed when tube bank does not exist ( $\sigma=0$ )
$c_{e}$ : Sound speed when tube bank exists $(\sigma \neq 0)$
$d$ : Outer diameter of tube
$f$ : Force vector per unit volume
$\rho$ : Fluid density
$p$ : Pressure
$Q$ : Ratio of mass flux per unit volume
$\boldsymbol{U}$ : Flow velocity vector
$\boldsymbol{u}(t)$ : Fluctuating flow velocity vector
$u$ : Flow velocity of x direction
$V$ : Volume occupied by tubes
$\lambda$ : Wave length of sound
$t$ : Time
$\nabla$ : Differentiating operator
Suffix of $U_{0}, \rho_{0}, p_{0}$ show the mean amount

## GENERAL DESCRIPTION OF THEORY

The continuity and the momentum equations in non viscous fluid are given as follows.
$\frac{\partial \rho}{\partial t}+\nabla \cdot(\rho \boldsymbol{U})=Q$
$\rho \frac{\partial \boldsymbol{U}}{\partial t}+(\rho \boldsymbol{U} \cdot \nabla) \boldsymbol{U}=-\nabla p+\boldsymbol{f}$
The velocity, the fluid density and the pressure are assumed to be the sum of the steady and the unsteady components.
$\boldsymbol{U}=\boldsymbol{U}_{0}+\boldsymbol{u}(t), \quad \rho=\rho_{0}+\rho(t), \quad p=p_{0}+p(t) \quad \cdots \quad$ (3)
Substituting Eq. (3) to Equations (1) and (2) and neglecting higher order terms and subtracting the mean flow component, the following equations can be obtained.
$\frac{\partial \rho}{\partial t}+\rho_{0} \nabla \cdot \boldsymbol{u}+\boldsymbol{U}_{0} \cdot \nabla \rho=Q$
$\rho_{0} \frac{\partial \boldsymbol{u}}{\partial t}+\rho_{0} \boldsymbol{U}_{\boldsymbol{0}} \cdot \nabla \boldsymbol{u}+\rho \boldsymbol{U}_{\boldsymbol{0}} \cdot \nabla \boldsymbol{U}_{\boldsymbol{0}}+\rho_{0} \boldsymbol{u} \cdot \nabla \boldsymbol{U}_{0}+\nabla p=\boldsymbol{f}$


Fig. 1 Model of Tube
$\boldsymbol{U}_{0}$ in a cell is assumed to be symmetrical above and below as shown in Fig. 1 and $\boldsymbol{U}$ is assumed to vary slowly in the cell. This means that $\nabla \boldsymbol{U}_{0}$ is anti-symmetric over the cell and $\boldsymbol{u} \cdot \nabla \boldsymbol{U}_{0}$ is small enough to be neglected. The variation of $\rho$ becomes the variation of pressure from the relation $p=\rho c_{0}^{2}$.
As a result, Equations (4) and (5) become the following equations:
$\frac{1}{c_{0}^{2}} \frac{\partial p}{\partial t}+\rho_{0} \nabla \cdot \boldsymbol{u}+\frac{1}{c_{0}^{2}} \boldsymbol{U}_{\boldsymbol{0}} \cdot \nabla p=Q$
$\rho_{0} \frac{\partial \boldsymbol{u}}{\partial t}+\rho_{0} \boldsymbol{U}_{\boldsymbol{0}} \cdot \nabla \boldsymbol{u}+\nabla p=\boldsymbol{f}$
Considering only the effect of the tubes on the sound speed at low Mach numbers; $U_{0} / c_{0} \ll 1$ here, the effect of the mean flow is neglected. Then Eq. (6) and Eq. (7) become as follows:
$\frac{1}{c_{0}^{2}} \frac{\partial p}{\partial t}+\rho_{0} \nabla \cdot \boldsymbol{u}=Q$
$\rho_{0} \frac{\partial \boldsymbol{u}}{\partial t}+\nabla p=\boldsymbol{f}$
These equations are the equations of motion describing the
sound propagation passing through the incompressible tubes arrayed regularly with small $d / \lambda$. The assumption of small $d / \lambda$ means the acoustically compact which is ordinarily used in an aero-acoustic analysis [7].

First, the force acting between the tube and the fluid is calculated. The physical tube can be replaced by the force that the tube exerts on the fluid. The part occupied by the tube in the unit cell shown in Fig. 1 (b) is replaced by the fluid density $\rho$ and the compressible fluid with the sound speed $c_{0}$. To simulate the real tube, the body force $f$ and the mass flux $Q$ are assumed to act toward the virtual tube region as follows:
The fluid region of the virtual tube is
(1) Not compressed
(2) Maintained steadily (Fixed at the same place and not deformed)
(3) Given the force related to the shedding vortex and damping The increase of pressure $p$ compresses the volume $V$ which is the region occupied by the virtual tube, by $\delta V$.
$\frac{\delta V}{V}=-\frac{\delta \rho}{\rho}=-\frac{\delta p}{\rho c_{0}^{2}}$
Where iso-thermal change is assumed. If the mass flux would be injected to the region occupied by the virtual tube, the volume of the virtual tube is not compressed. The mass flux became the following:

$$
\begin{equation*}
Q=-\left(\frac{d}{d t}\right)\left(\frac{\sigma \delta m}{V}\right)=\left(\frac{\sigma}{c_{0}^{2}}\right) \frac{\partial(\delta p)}{\partial t} \tag{10}
\end{equation*}
$$

This is the average mass flux which must be injected to the fluid cell to simulate the effect of an incompressible tube.

Next, when the uniform velocity $U_{0}$ is accelerated and decelerated due to the enlargement and contraction of the flow pass width based on the existence of the tube, the flow field gives the circular cylinder the added mass.

$$
f=\rho_{0}(1+a) \frac{\partial u}{\partial t}
$$

This is the force added to the cylinder per unit volume. The first term of the right side shows the buoyant force due to the pressure gradient and the second term shows the virtual mass force.
$a$ is the added mass coefficient and becomes 1.0 theoretically in the case of an isolated cylinder. The force corresponding to the force per unit volume in the cell becomes $\sigma f$. Then the force that the cylinder gives to the fluid occupied volume ratio $1-\sigma$ is as follows:

$$
\begin{equation*}
f=-\left[\rho_{0} \sigma(1+a) /(1-\sigma)\right] \frac{\partial u}{\partial t} \tag{11}
\end{equation*}
$$

For simplicity, we considered the case of low solidity and a
one-direction propagation.
Substituting Eq. (10) and Eq. (11) to Eq. (8) and Eq. (9), we can obtain the following equations.
$\frac{1-\sigma}{c_{0}^{2}} \frac{\partial p}{\partial t}+\rho_{0} \frac{\partial u}{\partial x}=0$
$\rho_{0}\left[1+\left\{\frac{\sigma}{1-\sigma}\right\}(1+a)\right] \frac{\partial u}{\partial t}+\frac{\partial p}{\partial x}=0$
Where $u$ is the particle velocity concerning the x direction propagation and the pressure $p$ is the time variance $\delta p$.

Eliminating $u$ from Eq. (12) and Eq. (13), we can obtain the following equation.
$\frac{1}{c_{e}^{2}} \frac{\partial^{2} p}{\partial t^{2}}-\frac{\partial^{2} p}{\partial x^{2}}=0$
Where
$c_{e}=\frac{c_{0}}{\sqrt{1+\sigma a}}$
Consequently, the equivalent sound speed $c_{\mathrm{e}}$ becomes small due to the existence of tubes.

The sound speed is reduced by (a) the increase of effective density due to the existence of tubes and (b) the decrease of the effective volume elasticity.
That is to say,
(a)Increase of the pressure $\rightarrow$ Decreasing the volume of the virtual tube $\rightarrow$ But the tube is not compressed $\rightarrow Q$ must be injected to the tube region by not decreasing the volume $\rightarrow Q$ increases $\rightarrow$ Increase of apparent density $\rho$.
(b)Accelerated flow $\rightarrow$ the flow gives an added mass force to the cylinder $\rightarrow$ But the cylinder does not move $\rightarrow$ the cylinder must give the force to the fluid inversely $\rightarrow f$ arises $\rightarrow$ Increase of apparent compressibility $\rightarrow$ Decrease of the effective volume elasticity $K$.Definition : $K=d p /(-d v / v)$
In the case of comparatively small $\sigma$, Eq. (15) becomes the Parker's Equation (16) when $a \approx 1$.
$c_{e}=\frac{c_{0}}{\sqrt{1+\sigma}}$
When $\sigma$ increases $a$ becomes larger than 1.0. However, the reference [2] does not show the value and it is described in the reference that Eq. (15) and Eq. (16) are independent of the array pattern. Then we will examine these results in the next chapters. The effect of the mean flow $U_{0}$ is ignored in the theory described here. According to the reference [6], the effect of mean flow can be ignored when the reduced frequency $\omega d / U_{0}$ is greater than 5.0. As most existing boilers satisfies with this criterion it is considered that the effect of the mean flow can be ignored.

## ANALYSIS

Here we will examine the validity of Eq. (16) and applicability by comparing Eq. (16) and the analytical result by FEM in two cases of existence and non existence of the tube array in the duct.

## FE analytical model

Fig. 2 and Fig. 3 show the analytical models of two and three dimensional systems respectively. The size of the twodimensional model is 2 m in x direction (longitudinal) and 0.6 m in y direction (width). On the other hand, the size of the threedimensional model is 2 m in x direction (longitudinal), 0.6 m in y direction (width) and the height ( z direction) is varied 0.2 m , 0.4 m and 0.6 m . The models of circular cylinders are made in the duct and the radius $R$, the number of tubes $N$ and the filling fraction $\sigma$ are parameters in this analysis. The boundary conditions are that the pressure is zero at the duct inlet and outlet, and the particle velocity of normal direction to the duct wall is zero.


Fig. 2 Two-dimensional Model ( $\sigma=0.3,48$ tubes)


Fig . 3 Three-dimensional Model ( $\sigma=0.3,48$ tubes)

## Analytical cases

The analysis will be executed by giving the filling fraction $\sigma$, the tube radius $R$ and the number of tubes $N$ in the twodimensional model (2D model hereafter) and three-dimensional model (3D model hereafter). The analysis was done with the models having the same filling fraction and the different number of tube by varying the $R$. The analytical cases are shown in table 1 .

## Analytical results

First, the divided number in height of the three-dimensional model is decided. Fig. 4 shows the result of natural frequencies

Table1 Analysis Cases

| $\sigma$ | $R[\mathrm{~m}]$ | $N$ | $\sigma$ | $R[\mathrm{~m}]$ | $N$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.201 | 0.04 | 48 | 0.531 | 0.13 | 12 |
| 0.201 | 0.08 | 12 | 0.101 | 0.04 | 24 |
| 0.314 | 0.05 | 48 | 0.226 | 0.06 | 24 |
| 0.314 | 0.1 | 12 | 0.307 | 0.07 | 24 |
| 0.452 | 0.06 | 48 | 0.402 | 0.08 | 24 |
| 0.452 | 0.12 | 12 | 0.509 | 0.09 | 24 |
| 0.531 | 0.065 | 48 | 0.113 | 0.06 | 12 |



Fig . 4 Comparison of Resonant Frequency by Division Number in Analysis
of each mode in cases of some divided numbers under the analytical condition of $h=0.6 \mathrm{~m}, \quad \sigma=0.3$ and $N=12$. As can be seen from Fig.4, the effect of divided number on the natural frequency does not appear. Then the analysis is executed by three divided numbers in height.

## Comparison between 2D model and 3D model

Fig. 5 shows the comparison of analytical results of 2D and 3D models (symbols) and the theoretical result (solid line) of the x direction mode in the case of $\sigma=0.3$ and $N=48$. On the other hand, Fig. 6 shows the comparison of analytical results of 2D and 3D models (symbols) and theoretical result (solid line) of $x-y$ combined mode in the same case.

As can be seen from Fig. 5 and Fig.6, the resonant frequencies obtained by the analysis (2D and 3D) are in good agreement with the theoretical result. As can be seen from Fig.5, the analytical results are in good agreement with the theoretical results of especially below the 3rd order mode in the case of x - direction mode. In addition, when $\sigma$ is less than 0.2 , the natural frequencies by analysis which are not shown here are in good agreement with the theoretical results. Contrast to this, the tendency can be seen that the analytical results become smaller than the theoretical values with increasing $\sigma$.

As can be seen from Fig.6, the analytical results are in good agreement with the theoretical results in also the case of x -y combined mode. The numbers $m$ and $n$ in the parentheses
show the mode order of $x$-direction $m$ and the mode order of $y$ direction $n$.


Fig . 5 Comparison of Analysis Value with Theoretical Value of Resonant Frequency of x -direction-mode $\quad(\sigma=0.3,48$ tubes)


Fig . 6 Comparison of Analysis Value with Theoretical Value of Resonant Frequency of $\mathrm{x}-\mathrm{y}$ combined mode $\quad(\sigma=0.3,48$ tubes)

## Effect of filling fraction and number of tubes on resonance frequency (x-direction)

Fig. 7 shows the effect of tube numbers on the resonance frequency in the case of $\sigma=0.3$ and Fig. 8 shows the effect of $\sigma$ on the resonance frequency in the case of $N=24$. From Fig. 7 and Fig.8, it can be seen that the effect of $\sigma$ on the resonant frequency is large and the effect of $N$ is small. The tendency can be seen that the analytical results becomes smaller than the theoretical results with increasing $\sigma$ ( See Fig.9). Figures are not shown here, it can be seen that the analytical results become larger than the theoretical results in low modes and approaches the theoretical results by becoming a higher mode in the case of $x-y$ combined mode. This is the inverse tendency of $x$ direction mode, and the dispersion of the resonant frequency becomes large in a low mode with an increasing $\sigma$. As well, Eq. (16) is used for the theory in Fig. 4 to Fig. 9.

## Amendment of relation between sound speed and filling fraction

When analytical results of the resonant frequency are compared with the theoretical value obtained by Parker's

Equation (16), it is clarified that the analytical value of higher mode becomes smaller than the theoretical value with increasing $\sigma$ (greater than 0.2 ). Then when $\sigma$ is large we proposed the modified expression of Eq. (16) by giving an appropriate value to the correction coefficient $a$. As a result, we obtained the following expressions.


Fig . 7 Resonant Frequency of the Effect on Tube Number

$$
(\sigma=0.3)
$$



Fig . 8 Resonant Frequency of the Effect on Occupied Area Ratio (24Tubes, 3D-height 0.4 m )


Fig . 9 Comparison of Analysis Value with Theoretical Value of Resonant Frequency $\quad(\sigma=0.2, \sigma=0.3, \quad \sigma=0.5)$

$$
\begin{align*}
& \frac{f_{n}}{f_{n 0}}=\frac{1}{\sqrt{1+a \sigma}} \frac{c}{c_{0}}  \tag{17}\\
& a=(10 / 3) \sigma+1 / 3 \tag{18}
\end{align*}
$$

Where $f_{\mathrm{n}}, f_{\mathrm{n} 0}$ are the resonant frequencies of the duct with and without a tube bank respectively.

Fig. 10 shows the comparison between the experimental result and two predicted results such as $a=1.0$ and $a=1.35$ of correction factor. The modified result $(a=1.35)$ is good agreement with the experimental result. Fig. 11 shows the relation between the correction factor $a$ and the filling fraction $\sigma$. The correction factor $a$ described here is the same as the added mass coefficient $a$ described in "GENERAL DESCRIPTION OF THEORY".


Fig . 10 Compensation of theoretical formula $(\sigma=0.3)$


Fig. $11 \quad a$ vs. $\sigma$

## EXPERIMENT

## Experimental setup

Fig. 12 shows the experimental setup. The duct is made of acrylic plastic and its length is 1000 mm and the size of cross section is $200 \mathrm{~mm} \times 250 \mathrm{~mm}$. Many tubes are inserted in the duct and various array patterns are composed.

## Experimental cases

The experimental cases are shown in Table2. Where (1), (2), (3) are square arrays, (4) and (5), (6) and (7), (8) and (9) are the cases of the same filling fraction and different array patterns, respectively. In addition, the four staggered array patterns as shown in Fig. 13 are also used in this experiment.

These patterns have the same filling fraction $\sigma=0.229$ and are constructed by the same $L / d$.


Fig. 12 Experimental Apparatus
Table2 Occupied Area Ratio of Various Cases

| $L / d$ | 1.5 | 2.0 | 2.5 |
| :--- | :---: | :---: | :---: |
| 1.5 | $(1) 0.306$ | $(5) 0.255$ | $(7) 0.204$ |
| 2.0 | $(4) 0.255$ | $(2) 0.212$ | $(9) 0.170$ |
| 2.5 | $(6) 0.204$ | $(8) 0.170$ | $(3) 0.136$ |



Fig. 13 Tube Array and Size
( $\mathrm{T} / \mathrm{d}=1.5, \quad \sigma=0.229$ )


Fig. 14 Tube Array and Size

## Measuring method

The tubes are inserted in the duct as shown in Fig. 14 and the pure tone is given from the speaker set upstream to the duct. The loudness the sound from speaker is determined as when the peak value becomes 100 dB after confirming the frequency
characteristic of the sound from the speaker being flat. The frequency is varied by every 1 Hz and the sound pressure level is measured at the position 100 mm away from the duct at the right end. From this result, we find some peak frequencies and defined them the resonant frequency of each mode. The resonant frequencies of the first and the second modes obtained by the experiment are compared with those obtained by the theory. The theoretical value is called the predicted value here and is described by $f_{\mathrm{n} \text { pre. }}$

The filling fraction $\sigma$ is obtained by changing the pitch in the flow direction $L$ and the pitch perpendicular to flow direction $T$ (See Fig.13). How to obtain the predicted value of resonant frequency $f_{n}$ pre in each array is as follows.

The effective sound speed of each array $c_{e}$ is first obtained. Next, the resonant frequency $f_{\mathrm{n} 0}$ when $\sigma=0$ is obtained by the experiment and the resonant frequency at any $\sigma$ is calculated by the following equation:

$$
\begin{equation*}
f_{n \text { pre }}=f_{n 0} \times\left(c_{e} / c_{0}\right) \tag{19}
\end{equation*}
$$

Where $c_{0}$ at $\sigma=0$ is given by $331.5 \sqrt{(273+t) / 273}$. Where $t$ is the temperature $\left({ }^{\circ} \mathrm{C}\right)$.

## Experimental results and consideration

(1) Resonant frequency of duct without tube bank


Fig. 15 Comparison of Experimental Value with Analysis Value of Resonant Frequency without Tubes

Fig. 15 shows the frequency response of the sound pressure level in the duct without a tube bank $(\sigma=0)$. The solid line shows the experimental result and the dotted line shows the calculation result obtained by BEM. This calculation was executed by the commercial software (WAON) which can calculate the acoustic field including the end correction. The peak of the first mode appears clearly by the experiment and its resonance frequency is in good agreement with the analytical result. In the second mode, the experimental result was a little different with the analytical result. As a result, 144 Hz and 308 Hz are determined to be the experimental values of the first and the second resonant frequencies where $\sigma=0$. These values
are described by $f_{0}$ and are used in obtaining the predicted resonant frequency by use of Eq. (19).
(2) Resonant frequency in each tube array

1) Comparison between experimental and predicted values
Table3 shows the experimental value and the predicted value of the first and the second mode resonant frequencies at each case. And the value in parentheses is the ratio of the experimental result to the predicted value. The solid line and the dotted line in Fig. 16 to Fig. 19 show the position at experimental and predicted resonant frequencies, respectively. The values shown above in each figure are resonant frequencies.

As can be seen from Fig.16, 17 and (1), (2), (3) in Table3 which are results of the square arrays, the difference between the experimental and the predicted values is large ( $1^{\text {stt }}: 6 \%, 2^{\text {nd }}$ : $3 \%$ ) for a large filling fraction ( $\sigma=0.306$ ) and small for small filling fraction $(\sigma=0.212,0.136)$ in the cases of first and second mode. Furthermore, as shown in Table 4, when the staggered array with the same filling fraction and the different array are compared with each other, these have about the same resonant frequency. From this result, it can be said that the effect of the array pattern on the resonant frequency is a little in the staggered array.

Table 3 Comparison of Experimental Value with Predictive Value of Resonant Frequency

| $\sigma=0$ | $\begin{aligned} & 144 / 147 \\ & (0.98) \\ & \hline \end{aligned}$ | $\begin{aligned} & 308 / 294 \\ & (0.95) \end{aligned}$ |
| :---: | :---: | :---: |
| $\begin{aligned} & \hline \text { (1) } \sigma=0.306 \\ & L / d=\pi d=1.5 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 118 / 126 \\ & (0.94) \end{aligned}$ | $\begin{aligned} & \hline \hline 262 / 270 \\ & (0.97) \end{aligned}$ |
| $\begin{aligned} & \text { (2) } \sigma=0.306 \\ & L d=T d=1.5 \end{aligned}$ | $\begin{aligned} & \hline 130 / 131 \\ & (0.99) \end{aligned}$ | $\begin{aligned} & \hline 277 / 280 \\ & (0.97) \end{aligned}$ |
| $\begin{aligned} & \text { (3) } \sigma=0.212 \\ & L / d=\pi d=2.0 \end{aligned}$ | $\begin{aligned} & \hline 132 / 135 \\ & (0.98) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 279 / 289 \\ & (0.97) \end{aligned}$ |
| $\begin{aligned} & \text { (4) } \sigma=0.136 \\ & L d=T d=2.5 \end{aligned}$ | $\begin{aligned} & 129 / 129 \\ & (1.00) \end{aligned}$ | $\begin{aligned} & 275 / 275 \\ & (1.00) \end{aligned}$ |
| $\begin{aligned} & \text { (5) } \sigma=0.255 \\ & L / d=1.5, T d=2.0 \end{aligned}$ | $\begin{aligned} & 125 / 129 \\ & (0.97) \end{aligned}$ | $\begin{aligned} & 270 / 275 \\ & (0.94) \end{aligned}$ |
| $\begin{aligned} & \text { (6) } \sigma=0.255 \\ & L / d=2.0, T / d=1.5 \end{aligned}$ | $\begin{aligned} & \hline 128 / 131 \\ & (0.98) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 281 / 281 \\ & (1.00) \end{aligned}$ |
| $\begin{aligned} & \text { (7) } \sigma=0.204 \\ & L / d=1.5, T I d=2.5 \end{aligned}$ | $\begin{aligned} & \hline 127 / 131 \\ & (0.97) \end{aligned}$ | $\begin{aligned} & \hline 273 / 281 \\ & (0.97) \end{aligned}$ |
| $\begin{aligned} & 88 \sigma=0.204 \\ & L / d=2.5, T \\| d=1.5 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 131 / 137 \\ & (0.96) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 280 / 293 \\ & (0.976) \end{aligned}$ |
| $\begin{aligned} & \text { (9) } \sigma=0.107 \\ & L / d=2.0, T \\|=2.5 \end{aligned}$ | $\begin{aligned} & \hline 128 / 137 \\ & (0.93) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 278 / 293 \\ & (0.95) \end{aligned}$ |

Table 4 Comparison of Experimental Value with Predictive Value of Resonant Frequency at Staggered Tube Array

| $(\mathrm{T} / \mathrm{d}=1.5, \quad \sigma=0.229)$ |  |  |
| :---: | :---: | :---: |
| Experimental value (Hz) <br> /predictive value (Hz) | 1st | 2nd |
| (i)-a $\sigma=0.229$ | $126 / 130(0.97)$ | $275 / 278(0.99)$ |
| (i)-b $\sigma=0.229$ | $125 / 130(0.96)$ | $269 / 278(0.99)$ |
| (ii)-a $\sigma=0.229$ | $125 / 130(0.96)$ | $269 / 278(0.99)$ |
| (ii)-b $\sigma=0.229$ | $124 / 130(0.95)$ | $269 / 278(0.99)$ |



Fig. 16 Experimental Result at $\mathrm{L} / \mathrm{d}=\mathrm{T} / \mathrm{d}=1.5$


Fig. 17 Experimental Result at $\mathrm{L} / \mathrm{d}=\mathrm{T} / \mathrm{d}=2.0$
In Eq. (16) which shows the relation between the sound speed and the filling fraction, we applied the present
experimental condition of $\sigma=0.306$ to the equations (17) and (18). Then, in the case of no corrections ( $a=1$ ), the $6 \%$ difference between the experimental and the predicted values can be seen for the first mode and $3 \%$ for the second mode. On the other hand, in the case of taking into account the correction, the difference becomes small as $2.5 \%$ and $1 \%$ for the first and the second mode, respectively. From this result, equations (17) and (18) mentioned in Chapter 3 are validated.

## 2) Relation between resonant frequency and filling fraction

It can be seen from Fig. 18 that the resonant frequencies of both the first and the second mode become small with increasing $\sigma$ in the square array as arrows show. This can be said to be reasonable by the theoretical equation (16). Fig. 19 and Fig. 20 show the comparison of resonant frequencies in different arrays with the filling fraction 0.255 and 0.107 , respectively. For high filling fractions ( $\sigma=0.255$ ), the difference between the predicted and the experimental speed of sound is large when high filling fraction $(\sigma=0.255)$. That is to say, the rate of decreasing is large ( $\sigma=0.255: 1^{\text {st }}-3.2 \%, 2^{\text {nd }}-1.9 \%, \sigma$ $\left.=0.107: 1^{\text {st }}-2.3 \%, 2^{\text {nd }}-0.7 \%\right)$. Table 5 shows the comparison of resonant frequency between the experiment and the prediction on the tube bank with the different filling fraction and the same array. On the other hand, Table 6 shows the comparisons of resonant frequency between the experiment and the prediction on the tube bank with the different array and the same filling fraction ( $\sigma=0.204$ ). Table 7, as Table 6, also shows the case of $\sigma=0.107$. From these results, it was found that the
values obtained by the FEM analysis are different from the experimental ones in these cases, but ratios of the experimental values and the predicted values of resonant frequencies are almost the same as shown in the bottom row of each table.


Fig. 18 Comparison of R. Frequency at Square Array


Fig. 19 Comparison of Resonant Frequency at $\mathrm{L} / \mathrm{d}=1.5, \mathrm{~T} / \mathrm{d}=2.0$ with $\mathrm{L} / \mathrm{d}=2.0, \mathrm{~T} / \mathrm{d}=1.5$ in $\sigma=0.255$


Fig. 20 Comparison of Resonant Frequency at $\mathrm{L} / \mathrm{d}=2.0, \mathrm{~T} / \mathrm{d}=2.5$ with $\mathrm{L} / \mathrm{d}=2.5, \mathrm{~T} / \mathrm{d}=2.0$ in $\sigma=0.107$

Table 5 Comparison of Resonant Frequency between Analysis and Measurement ( $\sigma=0.136$ and 0.212)

|  | Analysis value(Hz)/Exp. value(Hz) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1st |  | 2nd |  |
| (3) $L / d=T / d=2.5(\sigma=0.136)$ | 152 | 132 | 304 | 279 |
| $(4) L / d=T / d=2.0(\sigma=0.212)$ | 150 | 130 | 300 | 277 |
| (4)/3 | 0.99 | 0.98 | 0.99 | 0.99 |

Table 6 Comparison of Resonant Frequency between Analysis and Measurement $(\mathrm{L} / \mathrm{d}=1.5, \mathrm{~T} / \mathrm{d}=2.5$ and $\mathrm{L} / \mathrm{d}=2.5, \mathrm{~L} / \mathrm{d}=1.5$ at

|  | Analysis value(Hz)/Exp. value(Hz) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 st |  | 2 nd |  |
| (6 $L / d=1.5, T / d=2.5(\sigma=0.204)$ | 151 | 128 | 303 | 281 |
| 77 $L / d=2.5, T / d=1.5(\sigma=0.204)$ | 148 | 127 | 297 | 273 |
| 77/6 | 0.98 | 0.99 | 0.98 | 0.99 |

Table 7 Comparison of Resonant Frequency between Analysis and Measurement $(\mathrm{L} / \mathrm{d}=2.0, \mathrm{~T} / \mathrm{d}=2.5$ and $\mathrm{L} / \mathrm{d}=2.5, \mathrm{~L} / \mathrm{d}=2.0$ at

|  | Analysis value(Hz)/Exp. value(Hz) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 st |  | 2nd |  |
| $(8) L / d=2.0, T / d=2.5(\sigma=0.107)$ | 152 | 131 | 303 | 280 |
| $(9 L / d=2.5, T / d=2.0(\sigma=0.107)$ | 150 | 128 | 301 | 278 |
| $99 / 8$ | 0.99 | 0.98 | 0.99 | 0.98 |

## CONCLUSIONS

In order to evaluate the sound speed in the duct with the tube bank, the analysis by FEM and the experiment were carried out and examined the validity of the theoretical expression. As a result, the following findings could be obtained.
(1) In examination due to this experiment and the analysis, the theoretical expression which shows the relation between the sound speed $c$ and the filling fraction $\sigma$ proposed by Parker and Blevins

$$
\frac{f_{n}}{f_{n 0}}=\frac{1}{\sqrt{1+\sigma}} \frac{c}{c_{0}}
$$

has a large error in high filling fraction and the error can be deduced by introducing the correcting factor. And in square array, when the same filling fraction, it was confirmed that this expression is comparatively in agreement with the experimental result with enlarging the pitch of flow direction
(2) It is required to correct the expression for $\sigma>0.2$. The correcting expression becomes as follows.

$$
\frac{f_{n}}{f_{n 0}}=\frac{1}{\sqrt{1+a \sigma}} \frac{c}{c_{0}}
$$

Where
$a=1.0 \quad$ for $\quad \sigma \leq 0.2 ; a=(10 / 3) \sigma+1 / 3 \quad$ for $\quad \sigma>0.2$
(3) In square arrays with the same filling fraction, it could be obtained by the experiment that the sound speed becomes relatively small with a small $T / d$. Where $T$ is the pitch of perpendicular to the flow direction and $d$ is the diameter of tube.
(4) In the staggered arrays with the same filling fraction, the effect of array patterns on the value of resonant frequency is relatively small.

## REFERENCES

[1] R. Parker, Acoustic Resonances in Passages Containing Banks of Heat Exchanger Tubes, Journal of Sound and Vibration, 57, (1979), pp.245-260
[2] R.D.Blevins, The Effect of Sound on Vortex Shedding from Circular Cylinders, Journal of Fluid Mechanics, 161, (1985), pp.217-237
[3] R.D.Blevins, Acoustic Modes of Heat Exchanger Tube Bundles, Journal of Sound and Vibration, 109(1), (1986), pp.19-31
[4] K. Ishihara, Study on High SPL Sound of Gas Heater Composed of Two Parallel Located Ducts with Tube

Bundles (1st Report, Understanding of Phenomenon) , Transaction of the JSME, 70-689, B (2004.1), pp.126-132
[5] H.Shiraki, Noise Protection Design and Simulation, Ohyo Gizyutu Syuppan, 198
[6] M.C.QUINN and M.S.HOWE, The influence of mean flow on the acoustic properties of a tube bank, Proceedings of the Royal Society of London, A396, (1984), pp.383-400
[7] M.S.Howe, Theory of Vortx Sound, CAMBRIDGE UNIVERSITY PRESS, (2003), p. 18

