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SHORT-TERM INSTABILITY IN STOCHASTIC AEROELASTICITY

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ABSTRACT

Dynamic systems with lumped parameters are considered which interact with fluid flow with random temporal variations of speed. The variations may lead to "short-term" dynamic instability of a system - which is nominally stable in the classical sense - whereby occasional random excursions beyond neutral stability boundary result in rare short outbreaks in response. As long as it may be impractical to preclude completely such outbreaks for a designed system, subject to highly uncertain dynamic loads, the corresponding system's response should be analyzed to evaluate its reliability.

Linear models of the systems are studied to this end for the case of slow variations in the flow speed using parabolic approximation for the variations during the excursions together with Krylov-Bogoliubov (KB) averaging for the transient response. This results in a solution for probability density function (PDF) of the response in terms of PDF of the flow speed; the results may be of importance for predicting fatigue life. First-passage problem for the random response is also reduced to that for the flow speed. The analysis is used also to derive on-line identification procedure for the system from its observed intermittent response with set of rare outbreaks. Specific examples for analytical and numerical solutions for systems with random temporal variations of flow speed include: 1D and 2D galloping of elastically suspended rigid bodies in cross-flow; classical two-degrees-of-freedom flutter; bundles of heat exchanger tubes in cross-flow with potential for flutter-type instability.

1. INTRODUCTION AND METHOD OF ANALYSIS

Classical definitions of stability and instability deal with behavior of dynamic systems as time $t \to \infty$. They are known to be potentially inappropriate for systems with short service life where reliability should be evaluated from transient response analysis whereas attempts to secure complete stability may lead to impossible or impractical design. These classical definitions may also be not perfectly adequate for certain systems intended for long-term operation. Such systems are designed, as a rule, to operate within their stability domain in the classical sense as long as their "nominal" or expected values of parameters are considered. However, if the parameters experience random temporal variations with occasional crossings of the "classical" instability boundary high-amplitudes outbreaks in response may be observed during such "short-term instability" – see example in Fig. 1. Design of any system operating with such spontaneous outbreaks should rely upon its reliability analysis with respect to, say, low-cycle fatigue and/or first-passage-type failures. Basic procedure for such reliability analyses had been outlined in [1 – 3]. It relies upon the following approximation of a stationary zero-mean random process *g(t)* with unit standard deviation in the vicinity of its peak which exceeds a given level u [4, 5], that is after upcrossing level u at time instant $t = 0$

$$g\left(t/u\right) \cong u + \left(1/u\right)\left(\varsigma t - \lambda^2 t^2/2\right) \text{ so that } g\left(t\right) \cong u + \varsigma t - \left(u/2\right)\left(\lambda t\right)^2$$
$$\text{for } t \in \left[0, 2\varsigma\big/\lambda^2 u\right] \text{ and} \tag{1}$$
$$\max_t g\left(t\right) = g\left(\varsigma\big/\lambda^2 u\right) = g_p = u + \varsigma^2\big/2\lambda^2 u.$$

Here subscript "p" is used for peak values of random processes, ς is the random slope of *g(t)* at the instant of upcrossing and

$$\lambda^2 = \sigma_{\dot{g}}^2 = \int_{-\infty}^{\infty} \omega^2 \Phi_{gg}(\omega) d\omega \Big/ \int_{-\infty}^{\infty} \Phi_{gg}(\omega) d\omega$$

where $\Phi_{gg}(\omega)$ is power spectral density (PSD) of *g(t)* so that λ is a mean frequency of *g(t)*. Thus the parabolic approximation (1) implies that the random process *g(t)* is regarded as deterministic within the high-level excursion of duration $\tau_f = \lambda t_f = 2\varsigma/\lambda u$ above level *u;* during this time interval it depends just on its initial slope ς at upcrossing which is regarded as a random *variable* for the excursion. Furthermore, the instant of downcrossing τ_f is clearly obtained as a second root of equation $g(t) = u$, the first one being *t = 0*. This probabilistic description may be used together with the solution for the transient response within the instability domain to find probability density function (PDF) of the response peaks in terms of that of *g(t)* as will be illustrated in the following for several examples of aeroelastic response.

2. GALLOPING OF A RIGID BODY UNDER RANDOMLY VARYING WIND SPEED

1D galloping. According to the basic model due to Den-Hartog [6], transverse motion of a SDOF system exposed to the fluid flow may be excited because of instability due to aerodynamic damping which can be negative; this damping is proportional to square of windspeed and to slope of the curve of lift force vs. angle of attack. It will be assumed that the squared windspeed is a stationary random process so that the equation of motion may be written as

$$\ddot{X} + 2(\alpha - q(t))\dot{X} + \Omega^2 X = 0 \qquad (2)$$

where *q(t)* is a zero-mean stationary random process whereas mean coefficient of aerodynamic damping is just deducted from the coefficient of structural damping. The total mean damping coefficient α is assumed to be positive so that the system is dynamically stable (asymptotically) in the mean and its response should be zero as long as the total damping coefficient $\alpha - q(t)$ remains positive. However, if this randomly varying damping coefficient may occasionally cross zero level, the outbreaks in response would be observed within finite time intervals – see one such outbreak ("puff") in Fig. 1. Now we denote

$$q(t) = \sigma_q \cdot g(t) \quad \text{and} \quad u = \alpha/\sigma_q \qquad (3)$$

where σ_q is standard deviation of *q(t)* and substitute the parabolic approximation (1) into the stochastic equation of motion (2) thereby reducing it to an ordinary differential equation (ODE) with a single random parameter ς. This ODE for any crossing should be integrated starting from the instant of upcrossing t_u to the final instant of the peak of the response *X(t)* for a given outbreak.

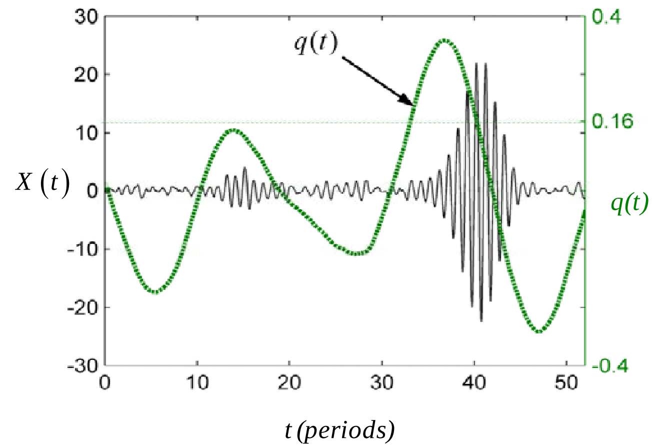

Fig. 1 Response sample with "outbreak" (solid line) of a SDOF system with apparent viscous damping factor *0.16 – q(t)***; sample of** *q(t)* **is shown by dash-dot line.**

The problem of transient response can be solved analytically for the present case using KB averaging over the response period for the case $|\alpha - q(t)| << \Omega$, $\lambda << \Omega$ [1] upon introducing slowly varying amplitude and phase $A(t), \varphi(t)$ as $X(t) = A\sin\psi, \dot{X}(t) = \Omega A\cos\psi, \psi = \Omega t + \varphi$. This results in a first-order ODE for *A(t)* which has a solution

$$A(\tau) = A_0 \exp\left\{(\sigma_q/\lambda)\left[(\varsigma/\lambda)\left(\tau^2/2\right) - u\tau^3/6\right]\right\} \qquad (4)$$

where $\tau = \lambda(t - t_u)$ and A_0 is response amplitude at the instant of upcrossing. Thus, predicting response amplitude requires that this value be estimated somehow from a subcritical response analysis. On the other hand, whenever the inverse problem of interpreting measured response is considered, a simple formula as presented later may be used for estimating A_0 from a signal like the one shown in Fig. 1.

The peak value of the response amplitude is seen to be attained precisely at the final instant of the excursion into the instability domain $\tau_f = 2\varsigma/\lambda u$; from the eq. (4) it is

$$A_p = A(\tau_f) = A_0 \exp(2\delta) \text{ where } \delta = \left(\sigma_q / 3\lambda u^2\right)(\varsigma/\lambda)^3. \qquad (5)$$

Thus, the eqs. (1) and (5) define implicitly relation between $\bar{A}_p = A_p/A_0$ and g_p - that is between peak values of the amplitude ratio and of *g(t)*. The explicit relation can be simply derived by excluding ς/λ . Let $\bar{A}_p = h(g_p)$ for $g_p \ge u$. Then the function inverse to *h* (denoted by superscript "-1") can be obtained as

$$g_p = u + (\varsigma/\lambda)^2 (1/2u) = h^{-1}(\bar{A}_p) = \\ = u + (1/2u)\left[\left(3\lambda u^2/2\sigma_q\right)\ln \bar{A}_p\right]^{2/3} \tag{6}$$

These relations open way to predicting reliability for the system (2) from relevant statistics of *g(t).* Thus, the first-passage problem for *A(t)* with barrier A_* is reduced to that for *g(t)* with barrier $g_* = h^{-1}(\bar{A}_*)$ as evaluated by using the relation (6). Furthermore, the PDF of *g(t)* can be used to obtain the PDF of $\bar{A}_p$; this may be of importance for evaluating low-cycle fatigue life for a system subject to the short-term dynamic instability. The derivation includes two steps. First the pdf $p_g(g_p)$ of *peaks* of *g(t)* is obtained from that of the *g(t)* itself as described in [4, 5]; then the basic relation for the PDF of a nonlinear function of a random variable is applied:

$$p(\bar{A}_p) = p_g\left(h^{-1}(\bar{A}_p)\right) \cdot \left|dh^{-1}/d\bar{A}_p\right|. \tag{7}$$

It should be just kept in mind that this PDF is non-zero for $\bar{A}_p \ge 1$ rather than for $\bar{A}_p \ge 0$ as long as the "fixed" subcritical response amplitude has been introduced. Furthermore, $p(\bar{A}_p)$ has a singularity at $\bar{A}_p = 1$ and it goes without saying that this *unconditional* PDF is normalized not to unity but to *Prob(g(t)>u)*, that is to the total probability for dynamic instability. Its use for predicting reliability in engineering applications is possible as long as some information on most probable actual subcritical response amplitude A_0 is available. Moreover, if the latter is a random variable its PDF may be used together with eq. (7) to find PDF of the actual (nonscaled) response amplitude and/or its peaks [3]. The solution (6), (7) is illustrated in Fig. 2 for Gaussian *q(t)* and compared with results of direct numerical simulation of the eq. (2) (zero-mean part of squared Gaussian wind speed would be approximately Gaussian if it is small compared with the mean value)

The eq. (5) is convenient for evaluating the system's properties from its measured (on-line!) intermittent response with outbreaks as one shown in Fig. 1. To this end one can use peak amplitudes A_p , which are attained at instants $\tau_f = 2\varsigma/\lambda u$ in the local time frames and corresponding amplitudes A_i at inflexion points of the curve $\ln A(\tau)$. From the eq. (5) $A_i = A(\tau_i) = A_0 \exp\delta,$ so that $A_p/A_i = \exp\delta;$ also $A_0 = A_i^2/A_p$. Thus, for each one of the observed response outbreaks one can identify in a global time frame the instants $t_f = t_u + \tau_f/\lambda$ and $t_i = t_u + \tau_i/\lambda$ which correspond to peak amplitude A_p and inflexion-point amplitude A_i respectively; the instants of upcrossings can also be identified as $t_u = t_f - 2(t_f - t_i) = 2t_i - t_f$. The frequency λ may now be obtained by averaging time difference $t_f - t_i$ over all observed outbreaks of response. This averaging which is equivalent to probabilistic averaging for ergodic *g(t)* will be denoted by angular brackets.

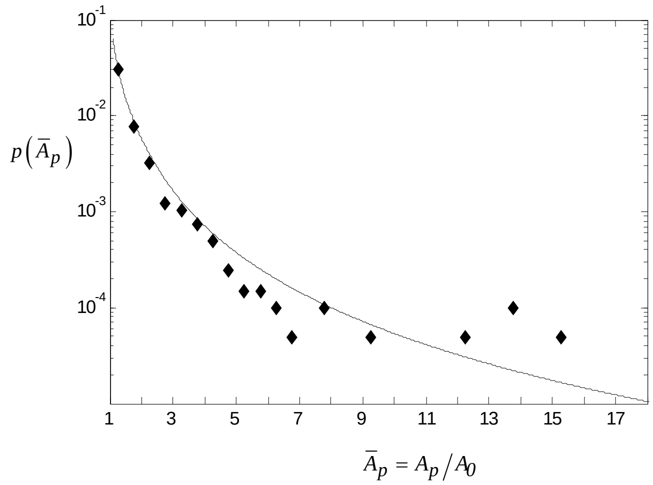

Fig. 2 Probability density function $p(\bar{A}_p)$ of the peaks of relative amplitude $\bar{A}_p = A_p/A_0$: theory (solid curve) vs. numerical simulation of the eq. (2) for the case $\alpha = 0.16, \Omega = 2.0$, $\lambda = 0.1$ and $u = 2$.

Let the process *g(t)* be Gaussian, so that ς has the Raleigh PDF [4, 5]

$$p(\varsigma) = \left(\varsigma/\lambda^2\right)\exp\left(-\varsigma^2/2\lambda^2\right) \text{ and} \\ n_u = (\lambda/2\pi)\exp\left(-u^2/2\right) \tag{8}$$

where n_u is mean number of upcrossings of the level *u* per unit time by *g(t)* with its reciprocal being mean time interval between two consecutive upcrossings. Then

$$\langle t_f - t_i \rangle = \langle \tau_i \rangle/\lambda = \left(1/\lambda^2 u\right) \cdot \int_0^\infty \varsigma\, p(\varsigma) d\varsigma = \sqrt{\pi/2}/\lambda u \tag{9}$$

therefore scaled total mean damping factor *u* can be identified from observed response sample together with the frequency λ using eqs. (8) and (9). Finally, formula

$$\left\langle \ln\left(A_p / A_i\right)\right\rangle = \langle\delta\rangle = \left(\sigma_q \big/ 3\lambda u^2\right)\cdot \int_0^\infty (\varsigma/\lambda)^3\, p(\varsigma)\, d\varsigma = \left(\sigma_q \big/ \lambda u^2\right)\sqrt{\pi/2} \quad (10)$$

can be used to calculate σ_q as long as the quantity in its LHS is estimated by averaging over all observed outbreaks in the intermittent response. Thus, the above procedure provides on-line estimates both for the mean apparent damping coefficient – which may be regarded as a nominal stability margin – and for standard deviation and mean frequency of its random temporal variations.

2D galloping. Consider an infinite rigid horizontal cylinder with blunt cross-section mounted on elastic suspension springs and subject to a fluid cross flow. Recent studies indicate however that horizontal vibrations (along flow) very often may also be present which are coupled with the vertical ones [7]. Thus, the same TDOF model as in [7] will be considered with full two-by-two aerodynamic damping matrix **B** but for special case of identical stiffnesses of suspension springs in directions *x* (horizontal) and *y* (vertical); two differential equations for displacements in these directions may be written as

$$\begin{aligned} &\ddot{x} + 2\alpha\dot{x} + \Omega^2 x + \mu(t)\left(\beta_{xx}\dot{x} + \beta_{xy}\dot{y}\right) = 0, \\ &\ddot{y} + 2\alpha\dot{y} + \Omega^2 y + \mu(t)\left(\beta_{yx}\dot{x} + \beta_{yy}\dot{y}\right) = 0 \end{aligned} \quad (11)$$

Here four coefficients β which are elements of the aerodynamic damping matrix **B** depend on lift and drag factors and their derivatives over angle of attack according to the relations that are presented in [7] together with expression for scaled flow speed μ. The latter is assumed here to experience slow temporal variations. The system (11) has only one natural frequency due to the assumption of equal stiffnesses in *x*- and *y*-directions whereas α is an equivalent viscous damping of the cylinder assumed to be identical in two directions.

The assumption of small damping ratios is adopted for analysis by KB-averaging using transformation $x = x_c \cos\Omega t + x_s \sin\Omega t, \dot{x} = \Omega\left(-x_c \sin\Omega t + x_s \cos\Omega t\right)$ together with similar change of variables for $y, \dot{y}$. Resolving these relations for new "slow" state variables and differentiating yields four first-order ODEs with small parameter in their RHSs. Thus averaging over the response period $2\pi/\Omega$ in "rapid" time can be applied, once again with fixed bifurcation parameter μ in "rapid" time. This results in two identical uncoupled equations for 2D-vectors $\mathbf{z_c} = \left[x_c, y_c\right]^T$ and $\mathbf{z_s} = \left[x_s, y_s\right]^T$ where superscript "T" denotes transposed vector. As a result

$$\dot{\mathbf{z}}_\mathbf{c} = \left\{-\alpha\mathbf{I} - \frac{1}{2}\mu(t)\mathbf{B}\right\}\mathbf{z_c} \text{ and } \dot{\mathbf{z}}_\mathbf{s} = \left\{-\alpha\mathbf{I} - \frac{1}{2}\mu(t)\mathbf{B}\right\}\mathbf{z_s} \quad (12)$$

where **I** is identity matrix.

Condition for neutral stability of zero solution to any one of the ODE's (12) is that of zero determinant of the matrix in braces. It leads to quadratic equation for critical value of the bifurcation parameter which will be denoted by a star subscript. For the example to be presented $Tr\mathbf{B} \equiv \beta_{xx} + \beta_{yy} > 0$, $Det\mathbf{B} \equiv \beta_{xx}\beta_{yy} - \beta_{xy}\beta_{yx} < 0$. Then

$$\mu_* = -\frac{\alpha}{Det\mathbf{B}}\left[Tr\mathbf{B} + \sqrt{\left(Tr\mathbf{B}\right)^2 - 4Det\mathbf{B}}\right] \quad (13)$$

The parabolic approximation for *g(t)* may be applied once again thereby reducing the problem to solution of any one of the vector ODEs (12) with $\mu(t) \equiv \langle\mu\rangle + \sigma_q g(t) =$ $= \mu_* - \sigma_q\left(u - g(t)\right) = \mu_* + \sigma_q\left[\varsigma t - (u/2)(\lambda t)^2\right]$ where $u = \left(\mu_* - \langle\mu\rangle\right)/\sigma_q$ and μ_* as defined by the eq. (14). Numerical integration has been performed for a case $\Omega = 1s^{-1}, \alpha = 0.007s^{-1}, \lambda = 0.01s^{-1}$ and same aerodynamic damping matrix as in one of the examples in [7]: $\beta_{xx} = 2.14,\ \beta_{xy} = 0.46,\ \beta_{yx} = 1.2,\ \beta_{yy} = -0.32$ so that $\mu_* = 0.0266$.

Figure 3 shows PDF $p\left(\bar{A}_p\right)$ of $\bar{A}_p = A_p / A_0$. Here A_p is peak value of $y_c(\tau)$ whereas ratio of the initial values $A_0 = y_c(0)$ and $x_c(0)$ for the first eq. (12) has been assigned the same as the eigenvector of the matrix in the RHS of the ODEs (12).

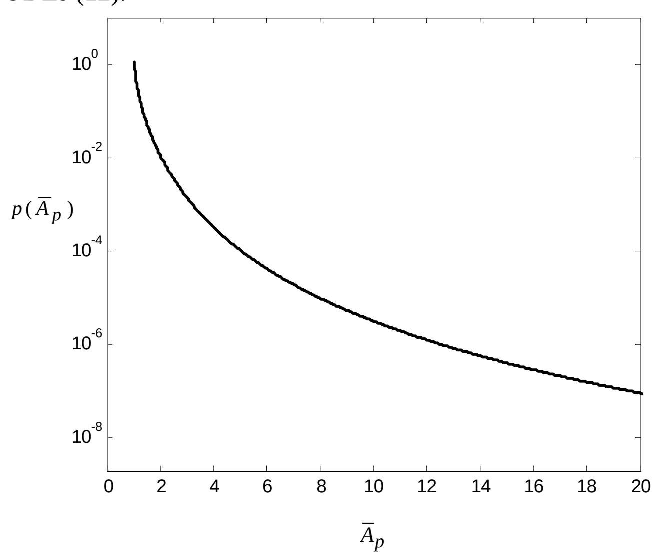

Fig. 3 Theoretical PDF of scaled peak vertical response of a rigid body in a 2D galloping.

3. TDOF FLUTTER DUE TO FLOW WITH RANDOMLY VARYING SPEED

Damped tuned system: **tube row in a cross-flow of fluid.** A flutter-type mechanism of dynamic instability due to nonconservative fluid forces has been proposed by Connors [8, 9]. It is related to proximity effect and corresponding cross-stiffnesses of two neighbouring circular cylinders (tubes). Each tube is assumed to have two DOFs corresponding to motions along and across the flow with displacements x_j and y_j respectively where *j* is number of tube (single spanwise mode may be considered for each direction in case of elastic tubes). General equations of motion of the row of identical tubes accounting for linearized fluid forces are presented in [9] and their stability analysis is presented for two neighbouring tubes under assumption that only two certain cross-stiffness coefficients are involved in dynamic instability. Thus, it is the case of a TDOF flutter indeed and the equations of coupled motion of tube *j* along the flow and tube *j + 1* perpendicular to flow are

$$\begin{aligned} &\ddot{x}_j + 2\alpha\dot{x}_j + \Omega^2 x_j = \left(\gamma C_x/m\right) y_{j+1} \text{ and} \\ &\ddot{y}_{j+1} + 2\alpha y_{j+1} + \Omega^2 y_{j+1} = -\left(\gamma K_x/m\right) x_j . \end{aligned} \tag{14}$$

Here $\gamma = \rho U^2/2$ and $\Omega = \sqrt{k/m}$ where *U* is flow speed, *m* is mass of tube and *k* is its stiffness; the latter is assumed here to be the same for both directions *x* and *y* as well as the structural damping factor α ; C_x, K_y are fluidelastic coefficients which are involved in the TDOF flutter that leads to whirling motion that is observed in tests [9], whereas two other fluidelastic coefficients C_y, K_x are neglected.

Assuming now that flow speed *U* is subject to relatively slow temporal random variations we may apply the basic procedure for stochastic analysis. Introducing complex coordinate $z = x_j + \mathrm{i}y_{j+1}, i = \sqrt{-1}$, we may replace two real equations (14) by a single complex equation

$$\begin{aligned} &\ddot{z} + 2\alpha\dot{z} + \Omega^2 z + i\gamma\Lambda^2\left(z + \sigma\bar{z}\right) = 0 \\ &\text{where } K_y + C_x = 2\Lambda^2, K_y - C_x = 2\sigma\Lambda^2 . \end{aligned} \tag{15}$$

where bar denotes complex conjugate quantity. To apply KB-averaging for the case of lightly damped system introduce change of variables

$$\begin{aligned} &z = z_+ \exp\left(i\Omega t\right) + z_- \exp\left(-i\Omega t\right), \\ &\dot{z} = i\Omega[z_+ \exp\left(i\Omega t\right) - z_- \exp\left(-i\Omega t\right)] \end{aligned} \tag{16}$$

resolve relations (16) for $z_+(t)$ and $z_-(t)$ and differentiate the resulting expressions Subsequent application of the KB-averaging yields then

$$\begin{aligned} &\dot{z}_+ = -\left(\alpha + \gamma\Lambda^2/2\Omega\right) z_+ - \left(\gamma\,\sigma\Lambda^2/2\Omega\right) z_-, \\ &\dot{z}_- = \left(\gamma\,\sigma\Lambda^2/2\Omega\right) z_+ + \left[-\alpha + \gamma\Lambda^2/2\Omega\right] z_- \end{aligned} \tag{17}$$

Condition for neutral stability of this system is that of vanishing determinant of the RHS of this ODE set; it yields the corresponding critical value γ_* of γ as

$$\gamma_* = 2\alpha\Omega\Big/\left(\Lambda^2\sqrt{1-\sigma^2}\right) = 2\alpha\Omega\Big/\sqrt{K_y C_x} \tag{18}$$

and it clearly coincides with the exact value as obtained in [9] by direct application of the Routh-Hurwitz criterion to the original equations (14).

Direct numerical integration of the ODEs (17) may now be applied for the case where parabolic approximation (1) is used for zero-mean part of $\gamma(t)$. It may happen however, that data on fluidelastic coefficients C_x, K_y are available only from stability tests. As can be seen from formula (18) critical flow speed depends only on product of these coefficients. As long as individual values of C_x and K_y aren't known it would be reasonable to consider "the worst case" – one with smallest critical speed. As can be seen from eq. (18), it is the case where $C_x = K_y$ or $\sigma = 0$. And in this special case two ODEs (17) are uncoupled so that we may consider only the second of these, which is prone to short-term instability and apply complex version of the above analytical solution for the SDOF case [2].

Undamped mistuned system. For significantly mistuned systems classical TDOF flutter is controlled mainly by condition for coalescing of natural frequencies whereas influence of damping may be of secondary importance. Thus, consider the undamped model as governed by equations of motion

$$\ddot{X}_1 + \Omega_1^2 X_1 + \gamma X_2 = 0,\ \ \ddot{X}_2 + \Omega_2^2 X_2 - \gamma X_1 = 0 . \tag{19}$$

It will be assumed that the mean value of the bifurcation parameter γ belongs to stability domain whereas its random temporal variations are sufficiently slow. Denoting

$$X_\pm = X_1 \pm X_2, \Lambda^2 = \frac{1}{2}\left(\Omega_2^2 + \Omega_1^2\right), \sigma = \frac{\Omega_2^2 - \Omega_1^2}{\Omega_2^2 + \Omega_1^2} \tag{20}$$

(it will be assumed for definiteness that $\Omega_2 > \Omega_1$) the eqs. (20) may be transformed to

$$\begin{aligned} &\ddot{X}_+ + \Lambda^2 X_+ = \left(\sigma\Lambda^2 + \gamma\right) X_-, \\ &\ddot{X}_- + \Lambda^2 X_- = \left(\sigma\Lambda^2 - \gamma\right) X_+ . \end{aligned} \tag{21}$$

Assuming that $\sigma << 1, \gamma << \Lambda^2$ and $\lambda << \Lambda$ we may denote

$$X_+(t) = X_{+c}(t)\cos\Lambda t, \dot{X}_+(t) = -\Lambda X_{+c}(t)\sin\Lambda t,$$
$$X_-(t) = X_{-s}(t)\sin\Lambda t, \dot{X}_-(t) = \Lambda X_{-s}(t)\cos\Lambda t$$

and apply KB-averaging for new, slowly varying variables. This results in two ODEs

$$\dot{X}_{+c} = -(\sigma\Lambda/2 + \gamma/2\Lambda)X_{-s} \text{ and } \dot{X}_{-s} = (\sigma\Lambda/2 - \gamma/2\Lambda)X_{+c} \quad (22\text{ a,b})$$

which may be transformed to an equivalent single second-order ODE for any one of the two amplitudes. Thus

$$\ddot{X}_{+c} + \left[(\sigma\Lambda/2)^2 - (\gamma/2\Lambda)^2\right]X_{+c} = 0 \quad . \quad (23)$$

It is clearly seen that the equilibrium solution $X_{+c} \equiv 0$ to the Eq. (23) is unstable *statically* if $\gamma > \gamma_* = \sigma\Lambda^2$. This critical value of the bifurcation parameter as obtained by asymptotic analysis clearly coincides with the exact condition for *dynamic* instability of the original system (19) which corresponds to coalescing of the system's natural frequencies as obtained from the corresponding characteristic equation [10].

Now for time-variant γ we separate mean value and zero-mean part of its square denoted by angular brackets and subscript "zero" respectively. This results in

$$\ddot{X}_{+c} + \left[\Delta^2 - q(t)\right]X_{+c} = 0$$
$$\text{where } \Delta^2 = (\sigma\Lambda/2)^2 - (\gamma/2\Lambda)^2, q(t) = \gamma_0^2(t)/(2\Lambda)^2 \quad (24)$$

where $\Delta^2 > 0$ as long as the mean system is stable. The zero-mean process *q(t)* may once again be scaled to its standard deviation σ_q by introducing process $g(t) = q(t)/\sigma_q$. The parabolic approximation (1) may be used for this process, with scaled instability threshold $u = \Delta^2/\sigma_q$. Upon introducing transformed local time $\tau = \lambda(t - t_u)$ with origin at the instant of upcrossing t_u the eq. (24) is transformed to

$$X_{+c}'' + (\Delta/\lambda)^2\left[-\frac{\varsigma\tau}{\lambda u} + \tau^2/2\right]X_{+c} = 0, \quad (25)$$

where primes denote differentiation over τ . Fig. 4a illustrates relation between peak values $\bar{X}_{+c,p}$ and g_p of $\bar{X}_{+c}(t)$ and *g(t)* respectively, where $\bar{X}_{+c}(t)$ is the ratio between $X_{+c}(t)$ obtained by numerical integration of the eq. (25) and the assigned initial value, X_{+c0} . The integration was performed for various values of ς and $\Lambda = 1,\ \lambda = 0.1,\ \sigma = 0.1,$ so that $\Delta/\lambda = 0.3$.

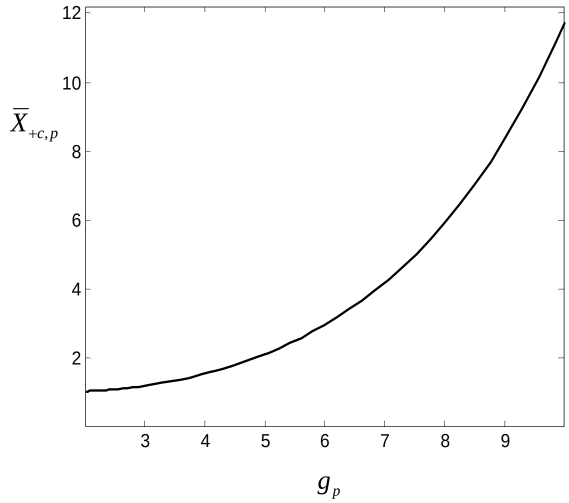

Fig. 4a Relation between peak values $\bar{X}_{+c,p}$ and g_p of $\bar{X}_{+c}(t)$ and g(t) respectively as obtained by numerical integration of the eq. (25).

Fig. 4b shows the PDF of $\bar{X}_{+c,p}$ as calculated according to the eq. (7) for the case of Gaussian variations of flow speed using the above numerically generated relation between $\bar{X}_{+c,p}$ and g_p .

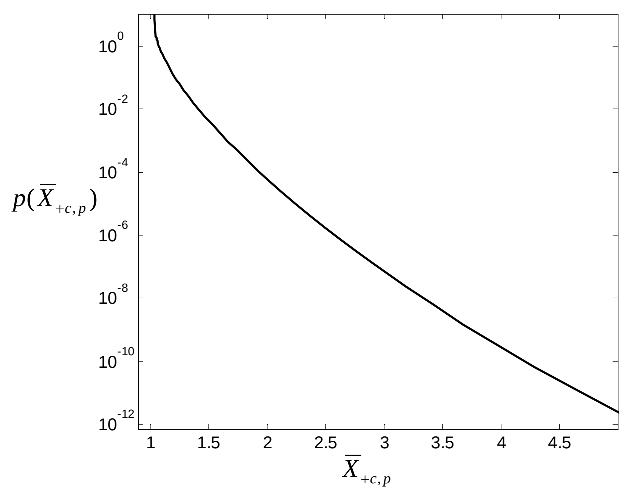

Fig. 4b Probability density function $p(\bar{X}_{+c,p}(t))$ of the peaks of relative amplitude as calculated using eq. (7) .

4. CONCLUSIONS

Dynamic systems with lumped parameters which interact with fluid flow with random temporal variations of speed have been considered in this paper. These variations may occasionally bring the system - which is nominally stable in the classical sense - into the domain of dynamic instability for brief periods. To evaluate the system reliability, linear models of the systems have been studied for the case of slow variations in the flow speed using parabolic approximation of the variation process during the excursions together with Krylov-Bogoliubov averaging for the transient response. This analysis results in a solution for probability density function of the response in terms of probability density function of the flow speed. Analytical and numerical solutions have been obtained for systems with random temporal variations of flow speed, including 1D and 2D galloping of elastically suspended rigid bodies in cross-flow and two-degrees-of-freedom flutter of bundle of tubes in cross-flow with potential for flutter-type instability.

The results may be used for predicting fatigue life in marginally unstable structures, i.e. those with relatively rare and brief potential excursions into domain of dynamic instability, and in on-line identification of a system from its observed intermittent response with set of rare outbreaks.

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