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INFLUENCE OF GEOMETRY ON THE NON-LINEAR VIBRATIONS OF CYLINDRICAL SHELLS WITH INTERNAL FLOWING FLUID

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ABSTRACT
In this work, the influence of the characteristic geometric parameters of a cylindrical shell, such as radius-to-thickness and radius-to-length ratios, on both the linear and non-linear vibrations of a fluid-filled cylindrical shell with internal flowing fluid is studied. The Donnell non-linear shallow shell equations are used to study a simply supported cylindrical shell subjected to both lateral and axial time-dependent loads with internal flowing fluid. The fluid is assumed to be inviscid and incompressible and the flow isentropic and irrotational. An expansion with eight degrees of freedom, containing the fundamental, companion, gyroscopic and five axisymmetric modes is used to describe the lateral displacement of the shell. The Galerkin method is used to obtain the nonlinear equations of motion which are, in turn, solved by the Runge-Kutta method. First, the parametric linear equations are used to study the influence of geometry and physical properties on the natural frequencies, critical flow and critical circumferential wavenumber. Secondly, numerical methods are used to describe the influence of geometric characteristics on the non-linear frequency-amplitude relations of the shell. The results obtained show the influence of the geometric parameters on the vibration characteristics of the shell and can be used as a basic tool for design of cylindrical shells in a dynamic environment.

1. INTRODUCTION
Cylindrical shells are widely used structures in several engineering areas such as civil, mechanical, and offshore among others. Their high efficiency as load carrying members for both axial and lateral loads makes cylindrical shells the most common shell geometry for industrial applications. In many of these applications the shells are used for the transportation of fluids. Hence, the analysis of the linear and non-linear vibrations of cylindrical shells with internal flowing fluid and under various loading conditions has thus become an important research area in applied mechanics. Also, the adequate selection of geometric characteristics is fundamental in designing against instability.

In [1] and [2], it is possible to find very extensive literature reviews related to the nonlinear dynamics of shells in vacuum, and shells filled with or surrounded by quiescent or flowing fluids. These topics are also presented in detail in the book by Paidoüssis [3] on fluid-structure interactions and the book by Amabili [4] on nonlinear vibrations and stability of shells and plates. Here only a few key contributions are mentioned.

The seminal works [5] and [6] gave the original idea for the modal expansions of the shell flexural displacement involving the symmetric and asymmetric modes; later, the studies [7] and [8] contributed to the understanding of the influence of the companion mode on the behavior of cylindrical shells. In the fundamental work [9] it was found that the presence of a dense fluid in the shell increases the softening characteristics of the frequency-amplitude relation when compared with the results for the same shell in vacuum. In a series of important papers [10] – [13] the nonlinear free and forced vibrations of a simply supported, circular cylindrical shell in contact with an incompressible and inviscid, quiescent or flowing dense fluid are studied using the Donnell's nonlinear shallow-shell theory. However most of these investigations are concerned with the analysis of elastic isotropic shells with fixed geometric characteristics and there are very little specific works related to the analysis of the effect of geometry on the non-linear vibrations of cylindrical shells. Other interesting works on nonlinear dynamics of cylindrical shells can be found in [14]-[17]. Recently, Karagiozis et al. [19] using Donnell's nonlinear theory studied the nonlinear stability of cylindrical shells subjected to internal fluid flow. The effect of varying the thickness-to-radius (*h*/*R*) and length-to-radius (*L*/*R*) was investigated. Results show that, depending on the radius and on the circumferential wavenumber, the shells show completely different behavior.

In this work, an eight-degree-of-freedom model is used to study the instabilities of perfect circular cylindrical shells with both axial and lateral loads and with internal flowing fluid. The fluid is assumed to be inviscid and incompressible and the flow isentropic and irrotacional. To discretize the shell, the Donnell shallow shell equations are used together with the Galerkin method to derive a set of coupled ordinary differential equations. In order to study the effect of the geometrical characteristics of the shell, several analyses are developed to understand their influence on the natural frequencies, critical loads, circumferential wavenumber and nonlinear frequency-amplitude relation. The results obtained could be used as a design tool by engineers and scientist to select adequate shell geometries.

NOMENCLATURE

c	damping coefficient
D	flexural rigidity
E	Young's modulus
f	amplitude of lateral load
F	stress function
F_o	lateral load parameter
I_n, I'_n	modified Bessel functions
L, R, h	length, radius and thickness
m	number of half-waves in the axial direction
n	number of waves in the circum. direction
$\overline{n}$	circumferential wave length parameter
$\tilde{N}_x$	axial load
P_e	compressive uniform static axial load
P_d	amplitude of dynamic axial load
P_h	perturbation pressure
t	time
u, w, θ	axial, lateral and circumf. displacements
U	flow velocity
$\overline{U}$, U_b	velocity parameter
U_{bcr}	critical velocity parameter
V_a	non-dimentionalizing factor
x, z, v	axial, lateral and circumferential coordinates
Z	Batdorf's parameter
Γ_o	static load parameter
Δ	fluid parameter
ν	Poisson ratio
$\xi_{i,j}$	time dependent modal amplitudes
ρ_s	mass density
ρ_F	internal fluid density
ω_L	frequency of axial load
ω_o	natural frequency
Ω	frequency parameter
∇	bi-harmonic operator

2. MATHEMATICAL FORMULATION

2.1 Shell equations

Consider a simply supported thin-walled circular cylindrical shell of radius *R*, length *L*, and thickness *h*. The shell is assumed to be made of an elastic, homogeneous and isotropic material with Young's modulus *E*, Poisson ratio ν, and mass density ρ_s. The axial, circumferential and radial co-ordinates are denoted by *x*, *y* and *z*, respectively, and the corresponding displacements on the shell surface are denoted by *u*, *v* and *w*, as shown in Fig. 1. In this work the mathematical formulation will follow that previously presented in references [10], [14], [15] and [17].

The shell is subjected to both a lateral pressure *f* and a distributed axial load along the edges *x*=0 and *L* given by

$$\tilde{N}_x(t) = -\frac{P_e}{2\pi R} - \frac{P_d}{2\pi R}\cos(\omega_L t) \tag{1}$$

where P_e is a compressive uniform static load.

The nonlinear equation of motion, based on the von Kármán-Donnell shallow shell theory, in terms of a stress function *F* and the lateral displacement *w*, is given by

$$D\nabla^4 w + c\, h\, \dot{w} + \rho_s\, h\, \ddot{w} = f + \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + \frac{1}{R}\left[\frac{\partial^2 F}{\partial \theta^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x\,\partial\theta}\frac{\partial^2 w}{\partial x\,\partial\theta} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial \theta^2}\right], \tag{2}$$

where $D = Eh^3/[12(1-\nu^2)]$ is the flexural rigidity, *c* (kg/m^3 s) is the damping coefficient, and *f* and P_h are the radial pressures applied to the surface of the shell.

The compatibility equation is given by

$$\frac{1}{Eh}\nabla^4 F = -\frac{1}{R}\frac{\partial^2 w}{\partial x^2} - \frac{1}{R^2}\left[-\left(\frac{\partial w}{\partial x\,\partial\theta}\right)^2 + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial \theta^2}\right], \tag{3}$$

In Eqs. (2) and (3) the bi-harmonic operator is defined as $\nabla^4 = [\partial^2/\partial x^2 + \partial^2/(R^2\partial\theta^2)]^2$.

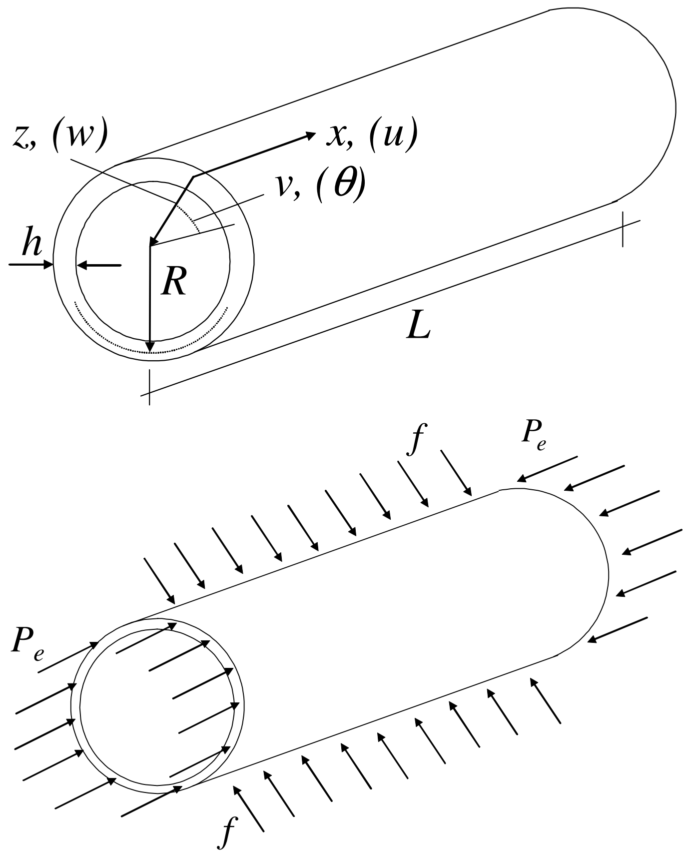


Figure 1: Shell geometry and loads

2.2 Solution expansion for the transverse displacement

The numerical model is developed by expanding the transverse displacement component w in series form in the circumferential and axial variables. From previous investigations on modal solutions for the nonlinear analysis of cylindrical shells under axial loads [10, 17] it is clear that, in order to obtain a consistent modeling with a limited number of modes, the sum of shape functions for the displacements must (i) express the most important nonlinear coupling between the modes and (ii) describe consistently the correct frequency-amplitude relation. Here, the following modal expansion is adopted [10]:

$$\begin{aligned} w(x,\theta,t) &= \xi_{1,1}(t)\,h\,\sin(q)\cos(n\theta)+\xi_{1,1c}(t)\,h\,\sin(q)\sin(n\theta) \\ &+\xi_{1,2}(t)\,h\,\sin(2q)\cos(n\theta)+\xi_{1,2c}(t)\,h\,\sin(2q)\sin(n\theta) \\ &+\xi_{0,1}(t)h\sin(q)+\xi_{0,3}(t)h\sin(3q)+\xi_{0,5}(t)h\sin(5q) \\ &+\xi_{0,7}(t)h\sin(7q), \end{aligned} \tag{4}$$

where $\xi_{1,1}(t)$, $\xi_{1,1c}(t)$, $\xi_{1,2}(t)$, $\xi_{1,2c}(t)$, $\xi_{0,1}(t)$, $\xi_{0,3}(t)$, $\xi_{0,5}(t)$ and $\xi_{0,7}(t)$ are the time dependent modal amplitudes, $q = m\pi x/L$ and m and n are, respectively, the number of half-waves in the axial direction and the number of waves in the radial direction. This leads to an eight-degree-of-freedom reduced order model. This model includes the basic vibration mode, the companion mode, gyroscopic modes and four axi-symmetric modes. These modes are enough to describe the basic nonlinear interactions responsible for the characteristic softening behavior exhibited by most cylindrical shells, the in-out asymmetry of the displacement field, the internal resonance 1:1 and the symmetry-breaking effect of the axial fluid flow in the axial direction..

2.3 Fluid loading

The shell is assumed to be filled with a flowing fluid with velocity U. Using the linear potential flow theory, the perturbation pressure on the shell wall is given by [3, 4]

$$P_h = \rho_F \frac{L}{m\pi}\frac{\mathrm{I_n}}{\mathrm{I_n'}}\left(\frac{\partial^2 w}{\partial t^2}+2U\frac{\partial^2 w}{\partial t\,\partial x}+U^2\frac{\partial^2 w}{\partial x^2}\right), \tag{5}$$

where $\mathrm{I_n}$ is the modified Bessel functions of order n and I_n' its derivative with respect to the argument; ρ_F is the fluid density.

2.4 Linear analysis

Substituting the fundamental mode in Eq. (3), obtaining the stress function, applying the Galerkin method and considering only one half-longitudinal wave in the axial direction (m=1), it is possible to obtain the expressions for the lowest natural frequency. Using the circumferential wave length parameter ($\bar{n}$) and Batdorf's parameter (Z) given respectively by [18]

$$\bar{n} = \frac{nL}{\pi R}, \quad Z = \frac{L^2}{Rh}\left(1-\nu^2\right)^{1/2}, \tag{6}$$

and the following non-dimensional parameters

$$\begin{gathered} \Omega^2 = \frac{R^2\rho_s}{\pi^4 E}\omega^2, \quad \Gamma_0 = \frac{R}{2\pi^2 E L^2 h}P_e, \quad F_0 = \frac{L^2 R}{\pi^2 D}f, \\ \Delta = \frac{L}{\pi h}\frac{\rho_F}{\rho_s}\frac{\mathrm{I_n}}{\mathrm{I_n'}}, \quad \bar{U} = \frac{U}{V_a}, \quad V_a = \frac{\pi^2}{L}\sqrt{\frac{Eh^2}{12\left(1-\nu^2\right)\rho_s}}, \end{gathered} \tag{7}$$

the lowest non-dimensional frequency is given by

$$\begin{aligned} \Omega^2 = \frac{1}{1+\Delta}\Bigg[&-\frac{\pi^2}{12}\frac{\Delta}{Z^2}\bar{U}^2 - \frac{\Gamma_o}{\pi} - \frac{\bar{n}^2}{12Z^2}F_o \\ &+\left\{\frac{\left(1+\bar{n}^2\right)^2}{12Z^2}+\frac{1}{\pi^4\left(1+\bar{n}^2\right)^2}\right\}\Bigg]. \end{aligned} \tag{8}$$

3. NUMERICAL RESULTS

3.1 Linear analysis

Consider a simply supported cylindrical shell loaded with both an axial load and a lateral pressure and with the following physical properties ν = 0.3 and E = 2.1x10^{11} N/m^2. As a first analysis, Fig. 2 shows the influence of the *L*/*R* and *R*/*h* ratios on the lowest natural frequency of an empty cylindrical shell and the associated number of circumferential waves (n). As can be observed, the *L*/*R* and *R*/*h* ratios influence directly the natural frequencies values and the number of circumferential waves. Shells with the same *L*/*R* and *R*/*h* ratios (same *Z*) have the same lowest non-dimensional natural frequency and wavelength. This figure shows that most shell geometries can be analyzed using Donnell's shallow shell theory ($n \geq 5$). For a given *R/h* ratio, Ω and n decrease as *L/R* increases, and the shell tends to a long tube. For a given *L/R*, Ω decreases as *R/h* increases.

Figure 3 shows the influence of the *L*/*R* and *R*/*h* ratios on the critical flow velocity and the associated circumferential wavenumber. As the *L*/*R* and *R*/*h* ratios increase, the critical flow velocity also increases. However, for certain values of *L*/*R* ratio the critical flow suddenly decreases and turns to increase again, creating a kind of saw tooth curve. These discontinuities are due to the change in circumferential wavenumber (n) associated with the critical flow velocity.

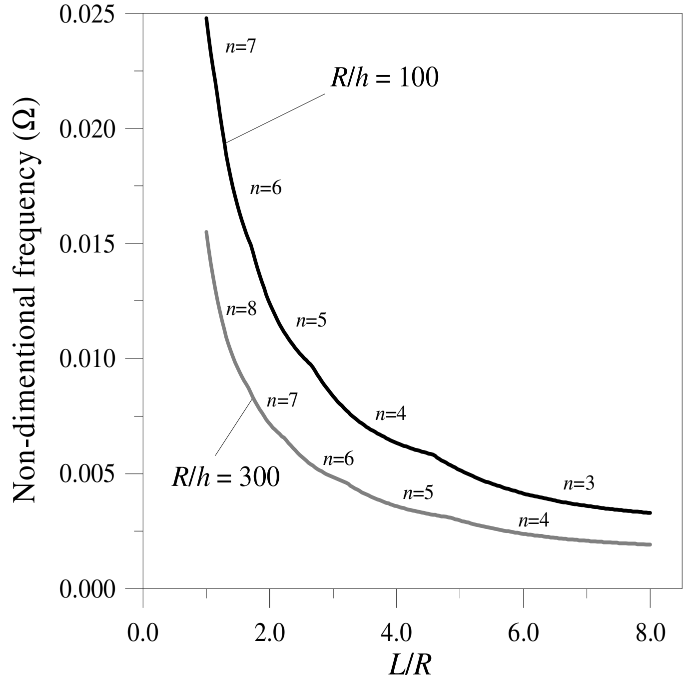
Figure 4 depicts the level curves of the lowest natural frequency parameter Ω as a function of the shell geometric parameters *L*/*R* and *R*/*h*. The blank region represents the geometries for which Donnell's theory cannot be applied (n<5). It is possible to observe that, for low values of the geometric relations, the lowest natural frequency displays both high values and strong gradients. The kinks in each level curve correspond to changes in the circumferential wavenumber associated with the lowest natural frequency.

3.2 Nonlinear analysis

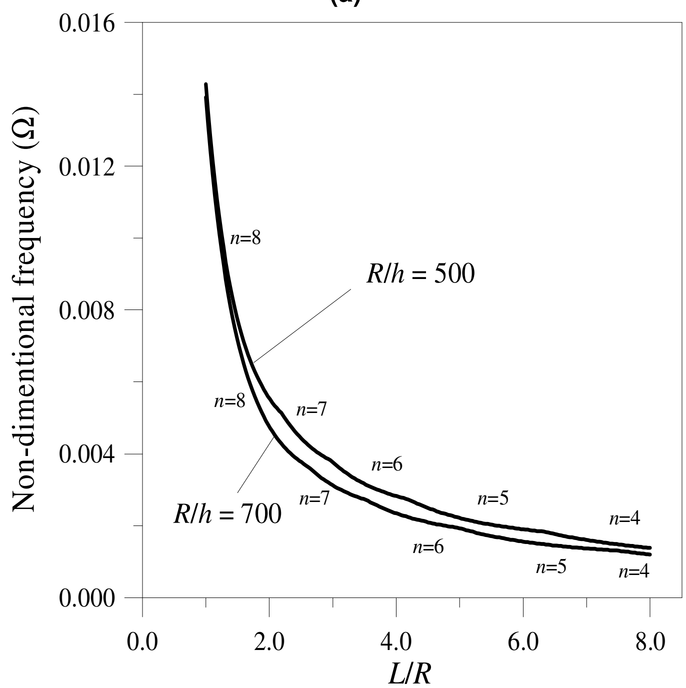
Now we study the influence of the geometric parameters on the nonlinear frequency-amplitude relation. For this, consider a thin-walled cylindrical shell with h=0.002 m, E=2.1x10^{11} N/m^2, ν=0.3, ρ_S=7850 kg/m^3 and ρ_F=1000 kg/m^3.

For the parametric analysis, nineteen different shell geometries with increasing *L/R* ratios and different *R*/*h* relations are adopted. Table 1 shows the *L/R* and *R/h* ratios, circumferential wavenumber associated with the lowest natural frequency, natural frequencies of the empty and fluid-filled shell in rad/s and the critical flow velocity.

First, in Fig. 5 are shown the normalized frequency-amplitude relations ω/ω_0 of fluid filled shells. Shells with the same *L/R* ratio but different *R/h* ratios are displayed together in each one of the sub-figures. All curves show a softening behavior, but the degree of non-linearity and the folding (bending back) point in each curve vary strongly with the shell geometry. For a given *L/R* ratio, the initial nonlinearity of the curve increases as *R/h* decreases. However, for a fixed *R/h* ratio, the nonlinearity decreases as the *L/R* ratio increases. For a given *L/R* ratio, the vibration amplitude at which the folding occurs increases with the *R/h* ratio. The initial curvature of the frequency-amplitude relation can be used to measure the degree of non-linearity of the shell, since in most practical applications, due to the presence of damping, the initial part of the frequency-amplitude relation will dictate the behavior of the resonance curves and the jumps at the saddle-node bifurcations.



(a)



(b)

Figure 2: Variation of the lowest frequency parameter Ω as a function of the shell geometric parameters *L/R* and *R/h*.

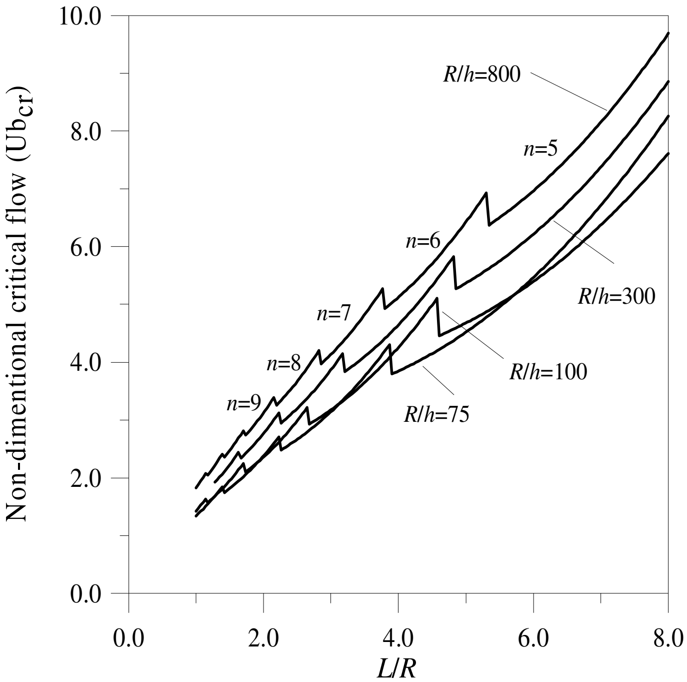


Figure 3: Variation of critical flow velocity parameter U_{bcr} as a function of the shell geometric parameters *L/R* and *R/h*.

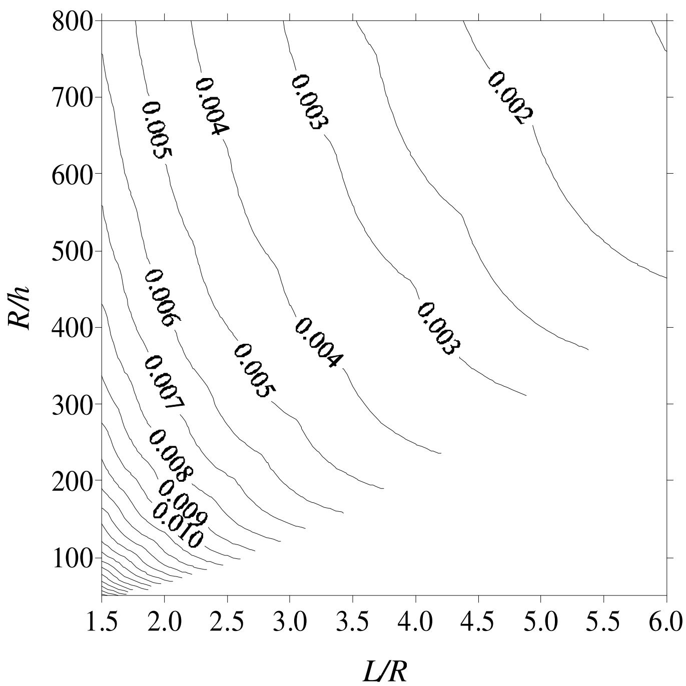


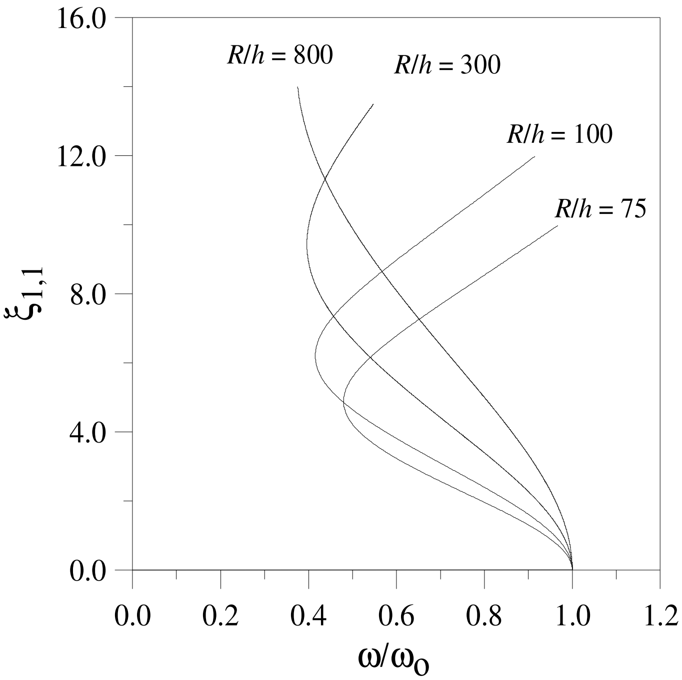
Figure 4: Level curves of non-dimensional frequency parameter Ω as a function of *L/R* and *R/h*.

Figure 6 shows the level curves of the initial curvature as a function of *L/R* and *R/h*. For all cases analyzed here the curvature is always negative. This means that at the main resonance region cylindrical shells usually display a softening behavior. As shown in [17], the main factor responsible for this behavior is the coupling between the fundamental vibration mode (linear mode) with the axisymmetric modes (see Eq. 4). This modal coupling leads to dominant negative quadratic terms in the discretized nonlinear equation of motion which dominates the initial behavior of the frequency-amplitude relation. As in Figure 4, the blank region corresponds to geometries with $n<5$. The results of this detailed parametric analysis show clearly that the initial curvature increases as the *L/R* and *R/h* ratios decrease, as illustrated in Figure 5. Long and thin shells display an almost linear initial behavior. As observed in Table 1, the circumferential wavenumber of associated with the lowest natural frequency decreases as *R/h* decreases and as *L/R* increases.

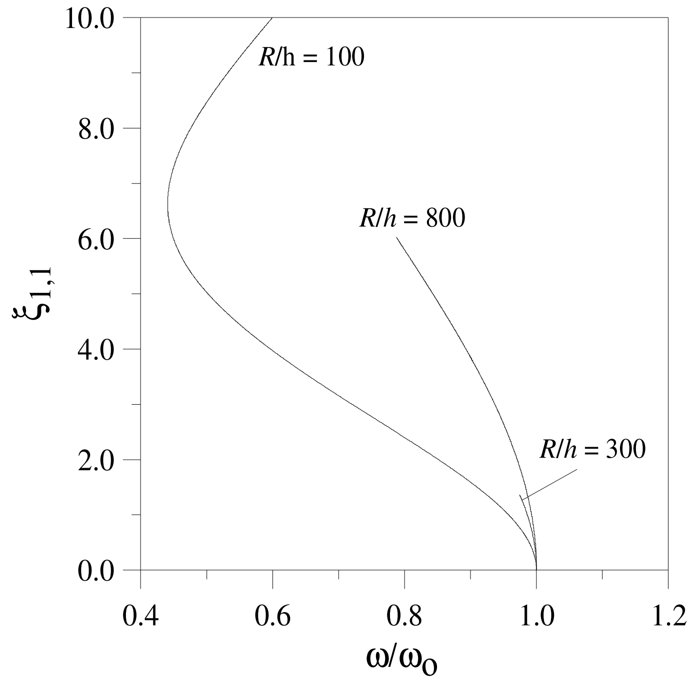
Table 1: Selected geometric relations, natural frequencies and critical flow velocities.

				Empty	Fluid Filled	
Case	*L/R*	*R/h*	*n*	ω_o	ω_o	U_{cr}
1	1.75	800	9	160.96	46.22	43.01
2		300	7	699.43	278.47	101.46
3		100	5	3687.64	1993.99	264.08
4		75	5	5550.06	3307.66	344.83
5	2.00	800	9	139.82	40.07	42.61
6		300	7	609.16	241.87	100.66
7		100	5	3165.03	1704.36	257.53
8		75	5	4919.12	2920.81	346.64
9	2.25	800	8	125.04	33.94	40.42
10		300	6	552.27	205.45	95.11
11		100	5	2819.21	1513.76	257.03
12	2.50	800	8	111.76	30.30	40.09
13		300	6	487.85	181.15	93.15
14		100	5	2585.99	1385.64	261.29
15	3.00	800	7	93.59	23.84	37.66
16		300	6	411.08	152.28	93.93
17	4.00	800	6	70.50	16.68	34.98
18		300	5	304.48	104.03	84.56
19	5.00	800	6	56.66	13.39	35.09

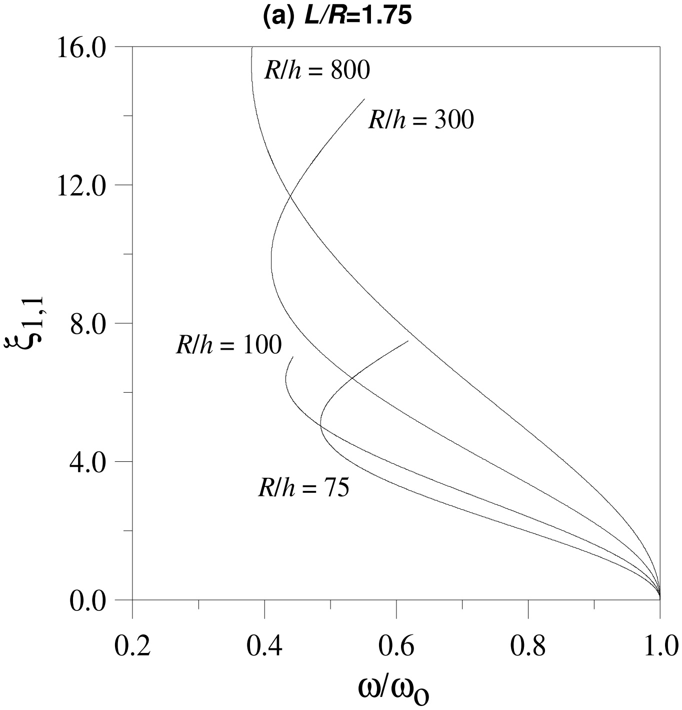
The influence of the number of circumferential waves *n* on the non-linear frequency-amplitude relation is illustrated in Fig. 7 where the frequency-amplitude relations for fluid-filled shells with the same number of circumferential waves are compared. Based on these results, one can conclude that shells with the same *R/h* ratio and the same circumferential wave number *n* but different *L/R* ratios have very similar nonlinear behavior up to very large deflections. This means that the degree of non-linearity of the shell is a function of *R/h* and *n*.



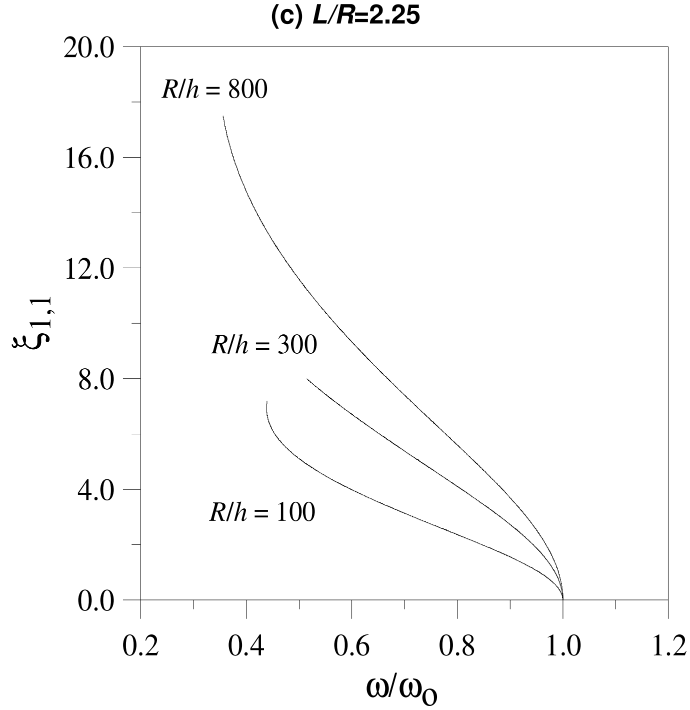
(a) *L/R*=1.75



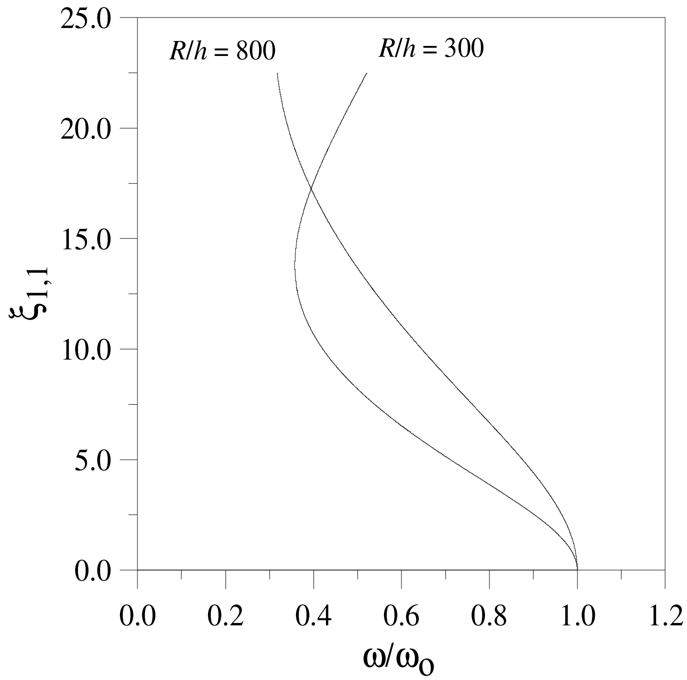
(c) *L/R*=2.25



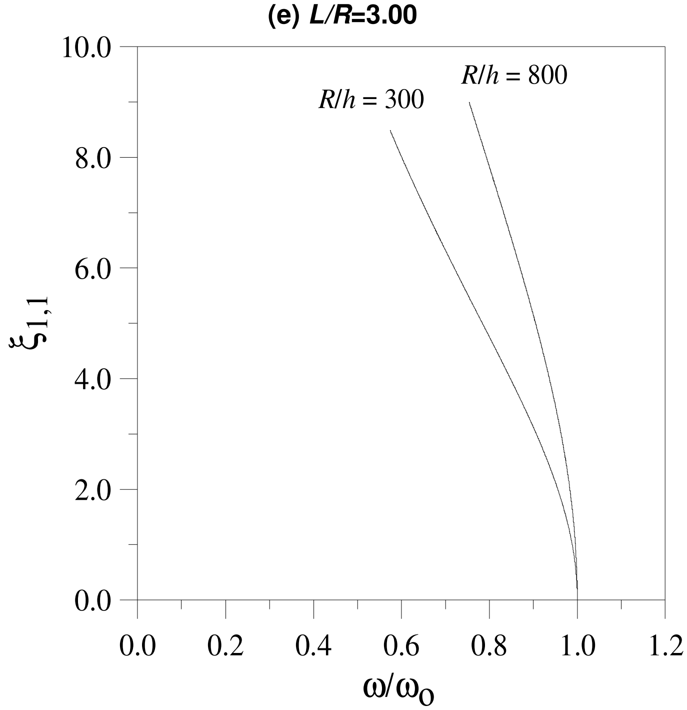
(b) *L/R*=2.00



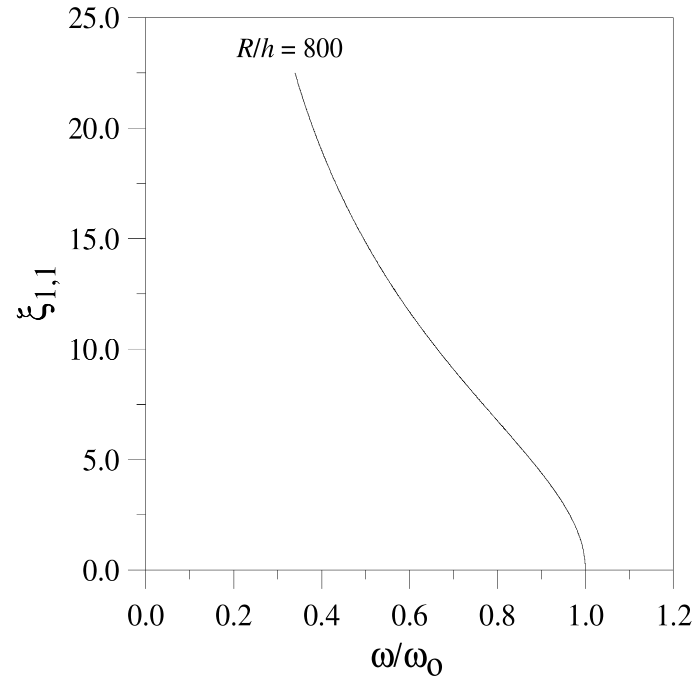
(d) *L/R*=2.50



(e) *L/R*=3.00



(f) *L/R*=4.00



(g) *L/R*=5.00

Figure 5: Frequency amplitude relations for fluid filled shells for geometries with same *L*/*R* ratios and different values of *R/h*. A) *L*/*R* = 1.75, b) *L*/*R* = 2.00, c) *L*/*R* = 2.25, d) *L*/*R* = 2.50, e) *L*/*R* = 3.00, f) *L*/*R* = 4.00, g) *L*/*R* = 5.00.

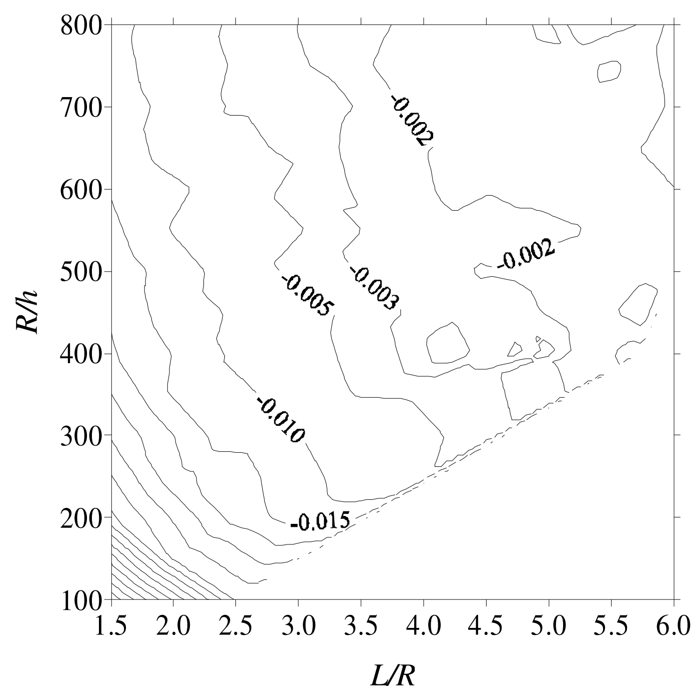
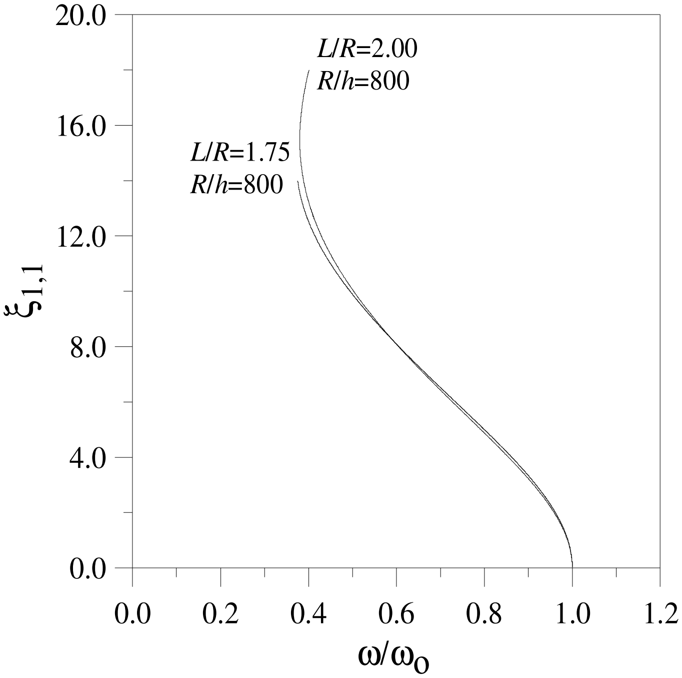
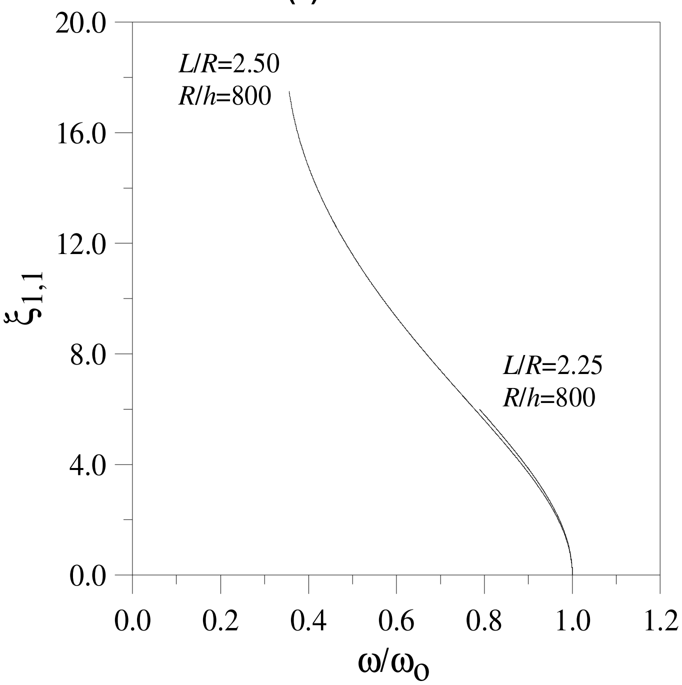


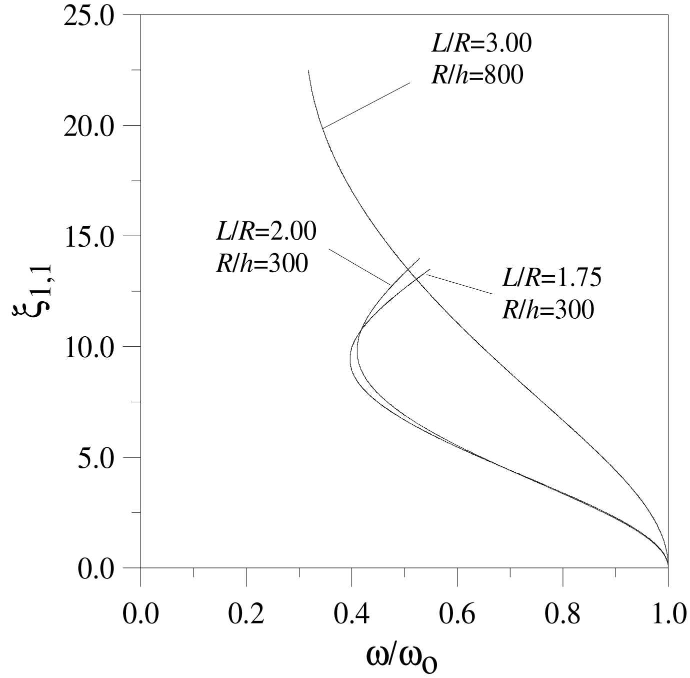
Figure 6: Level curves of the initial curvature of the frequency-amplitude relation as a function of *L*/*R* and *R*/*h*.



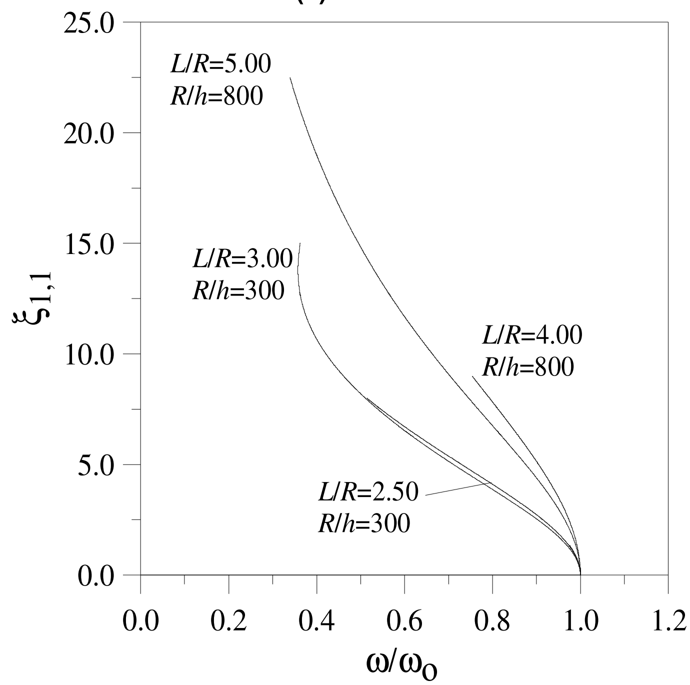
(a) *n* = 9



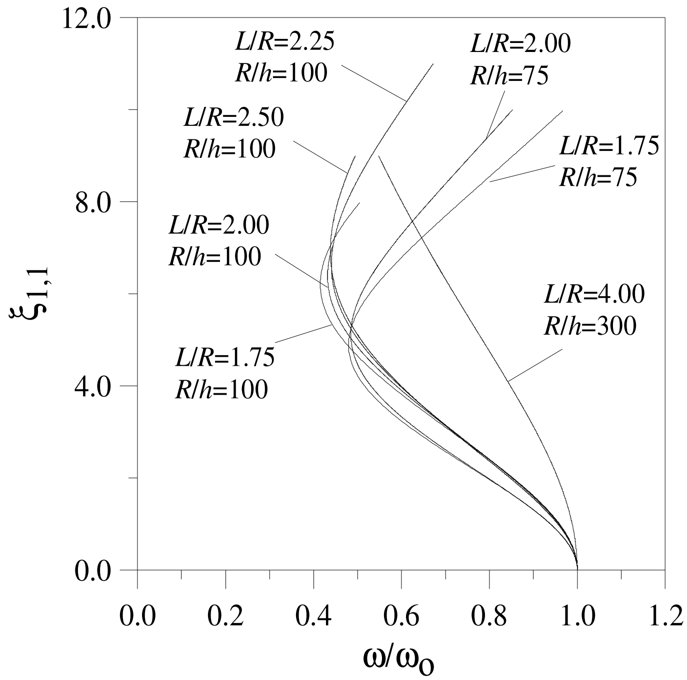
(b) *n* = 8



(c) *n* = 7



(d) *n* = 6



(e) *n* = 5

Figure 7: Frequency amplitude relations for fluid filled shells for geometries with same number of circumferential waves. a) *n* = 9, b) *n* = 8, c) *n* = 7, d) *n* = 6, e) *n* = 5.

To observe how the frequency-amplitude relation governs the non-linear behavior of the shell, Fig. 8 depicts the normalized bifurcation diagrams for varying fluid flow velocity, considering Cases 4, 11 and 18. These shells have different *L*/*R* and *R*/*h* ratios but same number of circumferential waves. As the fluid flow velocity increases, the shell response is trivial up to a critical value at which the trivial response becomes unstable, displaying a softening behavior (sub-critical bifurcation). Again, as observed in Fig. 6(e), the degree of the softening is related to the geometric ratios. Shells with a low *R*/*h* ratios (Case 4) display a higher non-linearity than shells with high *R*/*h* ratios (Case 18).

The influence of the shell geometric parameters on the frequency-amplitude relation for an empty shell is similar to the behavior observed here for a fluid-filled shell. However, as already shown in [9], independent of the shell geometry the softening behavior of the shell increases with the consideration of an internal fluid.

Finally a static compressive pre-stress state due to the lateral and axial load, as observed in Eq. (8), decreases the lowest natural frequency. They also increase the softening behavior of the cylindrical shell.

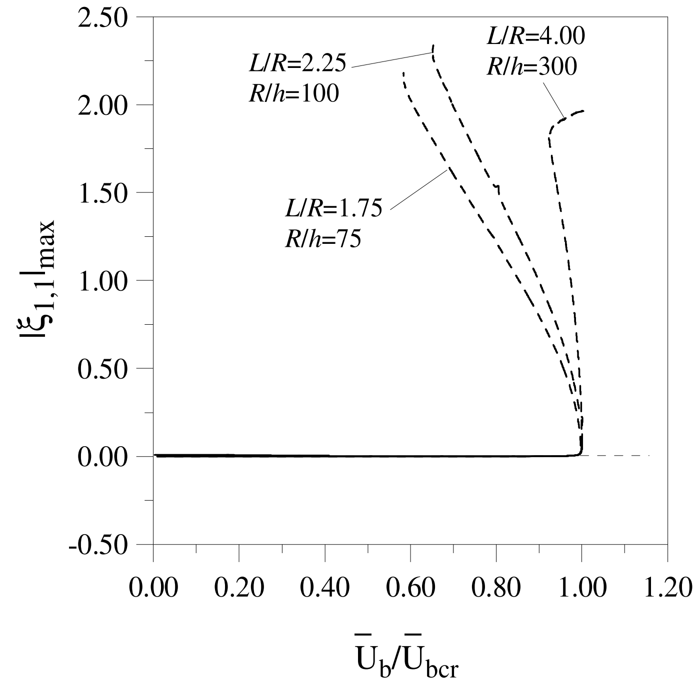


Figure 8: Bifurcation diagrams for fluid flow variation

4. CONCLUSION

In this work, the influence of geometric characteristics on the natural frequencies and on the non-linear frequency-amplitude relations of simply supported cylindrical shell subjected to both axial and lateral pressure loads is analyzed. To model the shell, the Donnell shallow shell theory is used together with an expansion of eight degrees of freedom to describe the lateral displacements of the shell. As observed, the non-linear frequency-amplitude relation of the shell is basically governed by the *L*/*R* and *R/h* ratios and shells with same *L*/*R* and *R/h* ratios display similar non-linear behavior. Also, shells with the *R*/*h* ration and same wave numbers, but different *L*/*R* ratios display similar non-linear behavior. All geometries analyzed in the paper display a softening behavior. The softening of the initial frequency-amplitude relation increases as the *R*/*h* and *L*/*R* ratios decrease. The softening characteristics also increase with the consideration of an internal fluid and of a compressive stress state. The knowledge of the non-linear frequency-amplitude relation is rather important since it governs the bifurcations and instabilities of the shell under external forcing. These results could serve as a design basis for engineers interested in choosing optimal geometries of cylindrical shells.

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