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A DIMENSIONLESS REPRESENTATION OF THE TURBULENCE DRIVEN PANEL RESPONSE

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ABSTRACT

In this paper a summary is presented concerning several experiences in predicting and measuring the structural response under turbulent boundary layer excitations. The theoretical, numerical and experimental evaluations involved both wind tunnel and towing tank (water) tests in which a flow wetted a plane plate over one face. A critical review of all these sets is presented together with the possibility to adopt a dimensionless representation for the response. This is done in order to tentatively compare measurement sets and/or predictive results obtained in nominally different conditions. Specifically, the attention is devoted to the definition of the possible normalisation of the required axes: the excitation frequency and the response metric. To this aim relations suggested by the dimensional analysis are applied to four distinct data sets finding the best choice of dimensionless parameters that allow the collapse of the different curves in a single one. The functional relations between these parameters are discussed and an analytical expression for the dimensionless plate response is obtained.

INTRODUCTION

Turbulent boundary layer induced vibrations of elastic structures is one of the major noise source in naval, aerospace, and automotive engineering.

It is well known that the computational cost associated to the solution of the coupled problem is often unfeasible for most of the real applications.

The literature regarding this kind of problems is devoted mainly to

(i) *the analytical characterization of the pressure field, based on experimental data,*
(ii) *the definition of scaling laws for the power spectral density [1,2,3] and*
(iii) *the predictive models [3,4,5,6] for the cross spectral density, being these quantities the input for the structural analysis.*

These latter empirical representations of the pressure load avoid the use of heavy direct numerical simulations (DNS) to solve the Poisson equations governing the pressure field generated by the turbulent boundary layer. It is well known that the actual limit for DNS are Reynolds number values of the order of 1000.

On the other side, the numerical solution of the structural equations especially when dealing with complex and huge structures such as an aircraft fuselage or a ship hull deserves some attention. When the structural wavelengths are small if compared to the typical dimension of the problem, *i.e.* at *high frequency*, the number of degrees of freedom necessary to calculate accurately the structural response increases rapidly.

Moreover, even at low frequency the computational cost can be considerably high since, when the flow wavelengths are smaller than the structural ones: the discretization of the structural domain is dominated by the aero-fluid dynamic scale. This is the typical situation occurring in naval problems and in aeronautical applications above the coincidence frequency.

The energy methods such as the Statistical Energy Analysis, [7], can be invoked at high excitation frequencies *i.e.* for high values of the modal overlap factor, but they cannot fruitfully used in the low-mid frequency range. Furthermore, the definition of the input power starting from a general model for the pressure cross spectral density not using the separation of variables can be very complicated and time consuming.

A chance to drastically reduce the computational time can be the identification of suitable scaling laws for the structural response able to determine, at least for a certain class of problems, the collapse of different data sets in a unique curve.

To this aim, in the present work, four different experimental set-ups are considered and compared.

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Test conditions refer for all cases to a stationary turbulent boundary layer in an incompressible, zero pressure gradient flow over a thin, flat elastic plate with no pre-stresses.

Dimensional analysis is used to recover the dimensionless parameters that govern the response of the plate to the fluctuating pressure load and an analytical expression for the dimensionless plate response is found.

NOMENCLATURE

a stream-wise plate length
b cross-wise plate length
f frequency
E Young's modulus
h plate thickness
U free stream velocity
U_c convection velocity
u_τ friction velocity
S_w spectral density of the plate displacement
S_p spectral density of wall pressure fluctuations
X_{pp} Cross spectral density of wall pressure fluctuations
δ turbulent boundary layer thickness
δ^* displacement thickness
η cross-wise spatial separation
ξ stream-wise spatial separation
ρ_f fluid density
ρ_s material density
ω radian frequency

DESCRIPTION OF THE EXPERIMENTAL SETUP

The four data sets considered for this analysis regard incompressible and stationary turbulent boundary layers in almost zero pressure gradient flow acting on thin and flat plates.

Among the large amount of data available in the literature on wall pressure and induced structural vibration, these experimental setup are the only ones able to provide information on pressure fluctuations and on structural deformation acquired in the same facility and in the same nominal conditions. As it will be clear in the next section this fact is fundamental for the validation of the proposed scaling procedure.

The first two sets of data are extracted from a database containing measurements of wall pressure fluctuations and structural response acquired in the INSEAN towing tank. The first one is relative to an experimental campaign performed on a 1:15 model of a catamaran hull (Figure 1). A Plexiglas plate was inserted in the bottom of the hull in correspondence of the stern region, pressure and acceleration measurements were performed for model speed of 3.3 m/s and 5.3 m/s respectively. A complete description of this experimental campaign can be found in [3].

The second set of data belongs to an experimental setup designed to measure wall pressure fluctuations and the response of elastic portions of a 1:8 scaled bulbous model (see figure 2). The considered data regards the stern measuring section where the flow has reached stationary conditions and where pressure gradient effects due both to water surface deformation and to structural curvature can be neglected.

The elastic element inserted in the model is a Plexiglas thin plate, the model velocity in this case ranged between 2.72 m/s and 6.36 m/s. A complete description of this experimental campaign can be found in [8].

Figure 1 Catamaran model

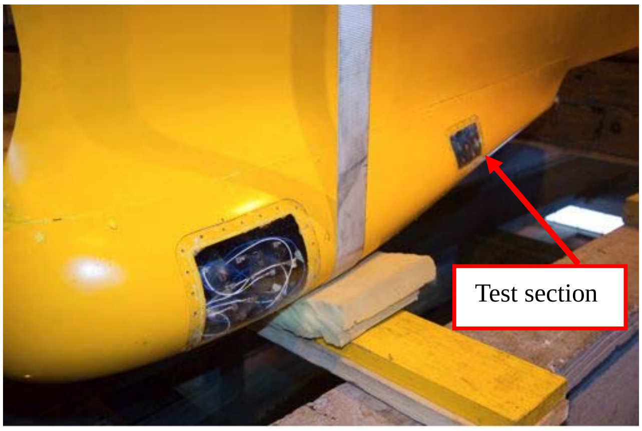

Figure 2 Scaled model (1:8) of a bulbous

The last two sets of data are obtained from measurements performed in aerodynamic tunnels. The former was designed by Finnveden et al. [9] in the frame of ENABLE project. It consists of an aluminum plate exposed to flow velocities of 80, 100 and 120 m/s respectively.

The latter is part of the experimental campaign performed by Totaro et al. [10] on four different plates. The data considered for this analysis regard the PVC plate for a flow velocity equal to 50 m/s.

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Table 1 presents the principal characteristics of the four plates. Table 2 lists the principal mean flow TBL parameters of the four experimental set-ups.

Table 1 Plates dimensions and material properties

	Plate 1	***Plate 2***	***Plate 3***	***Plate 4***
material	PVC	PVC	Aluminum	PVC
density [Kg/m^3]	1190	1190	2700	1400
Young's modulus	3.2E+9	3.2E+9	7.1E+10	4.5E+9
thickness [mm]	3	3.3	1.6	1
length [m]	0.6	0.242	0.768	0.6
width [m]	0.2	0.144	0.328	0.3

Table 2 TBL mean flow parameters

Plate	**Fluid**	***U* [m/s]**	**δ [mm]**	**u_τ [m/s]**
1	Water	3.30	120.0	0.110
		5.30	113.0	0.163
2		2.720	55.2	0.091
		3.64	51.0	0.102
		5.45	49.7	0.147
		6.36	48.0	0.171
3	Air	80.00	50.0	2.600
		100.00	50.0	3.100
		120.00	53.0	3.700
4		50.00	85.0	1.960

DIMENSIONAL ANALYSIS

The power spectral density of the plate velocity S_v was represented by the so called metric response R [9] defined as:

$$R = \frac{\omega^2 S_v (\rho_s h)^2}{S_p} \quad (1)$$

With this dimensionless representation the structural response was made independent of the power spectral density of wall pressure fluctuations however, the spatial characteristics of the fluid-structure interaction were not taken into account neither in the response axis nor in the frequency one. As it will be shown in the next section this representation does not lead to a collapse of spectra.

In order to find other dimensionless representation for the structural response an approach based on dimensional analysis is used: within the present approach, the power spectral density of the plate displacement is considered as the output variable.

Furthermore, the cross spectral density of wall pressure fluctuations can be written as the product of the single point spectral density $S_p(\omega)$ and of a function F that depends on the spatial coordinates, on frequency and on flow parameters [4,5,6] as:

$$X_{pp} = S_p(\omega)\ F(\xi, \eta, \delta, U_c, \omega)$$

where ξ, η represent the distance between different points of the plate in stream-wise and cross-wise direction respectively.

In the previous expression, according to [4,5] the convection velocity was chosen as the only representative velocity of the phenomenon assuming that, in the case of stationary turbulent boundary layer over a flat plate, it is possible to find general relations between the free stream velocity U and the convection velocity U_c typically assumed constant and equal to 0.7-0.8U.

Moreover, more sophisticated models [6,3] stated the dependence of the cross spectral density on the friction velocity u_τ however, under the above mentioned conditions, a simple relation with U [11] and thus with U_c exists. Similarly as the representative length for the fluid domain the boundary layer thickness was chosen being well known its relation with the other two typical lengths of the TBL *i.e.* the displacement and the momentum thickness.

With these statements the plate response to the pressure field induced by a turbulent boundary layer can be represented as a generic function g depending on the following dimensional fluid dynamic and structural variables:

$$g\ \left(S_w, S_p, \omega, \delta, U_c, \rho_s, E, h, a, b, \xi, \eta\right) = 0 \quad (2)$$

In Eq.(2) appear N=12 dimensional parameters thus, according to the Buckingham theorem [12], there are 9 dimensionless parameters governing the problem. The identification of these last is not unique but one set is given by:

$$\frac{S_w U_c}{h^3};\ \rho_s \sqrt{\frac{U_c^{\ 3} h}{S_p}};\ E\sqrt{\frac{h}{S_p U_c}};\ \frac{\omega h}{U_c};$$
$$\frac{a}{h};\ \frac{b}{h};\ \frac{\delta}{h};\ \frac{\xi}{h};\ \frac{\eta}{h}$$

Thus, the power spectral density of the plate displacement can be rewritten in the following form:

$$S_w = \frac{h^3}{U_c}\ g\ \left(\rho_s \sqrt{\frac{U_c^3 h}{S_p}}, E\sqrt{\frac{h}{S_p U_c}}, \frac{\omega h}{U_c}, \frac{a}{h}, \frac{b}{h}, \frac{\delta}{h}, \frac{\xi}{h}, \frac{\eta}{h}\right)$$

From the analysis of these parameters it is straightforward to define a dimensionless frequency: $\omega^* = \frac{\omega h}{U_c}$.

At this stage a critical analysis of the identified parameters is fundamental.

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Dimensionless ratios $\frac{a}{h}$ and $\frac{b}{h}$ are strictly related to the structural model used to describe the plate motion. In the present analysis only *thin* plates (i.e. governed by the Kirchoff equation) [13] thus, the value of these two parameters can be considered definitely large and their influence on the plate response as negligible.

The dimensionless parameter $\frac{\delta}{h}$ gives the fluid-structure degree of coupling. In particular, it is a measure of the influence of the structural deformation on the flow field.

From previous considerations h is small but comparing values of h given in Table 1 with values of δ provided in Table 2, it can be concluded that is always $\frac{\delta}{h} >> 1$ large; then, the fluid domain is not affected by the structural deformation.

Furthermore, if it is assumed that the major contribution to the plate response is due to diagonal terms of the cross spectral density matrix, it follows that ξ=0 and η=0. Under this condition $\frac{\xi}{h}$ and $\frac{\eta}{h}$ can be neglected too.

The only two dimensionless parameters that seem important for the present problem are those involving the pressure power spectral density.

Without any other considerations on the physics of the problem, the plate response can be dependent on one of them or on a combination of the two. Nevertheless, in order to make the plate response independent of the input it seems convenient to consider the ratio $\frac{S_w}{S_p}$ as done already in [9].

With this position, the three possible functional dependences are:

$$S_w = \frac{h^3}{U_c}\left(E\sqrt{\frac{h}{S_p U_c}}\right)^{-2} g\left(\omega^*\right) \tag{3}$$

$$S_w = \frac{h^3}{U_c}\left(\rho_s\sqrt{\frac{U_c^3 h}{S_p}}\right)^{-2} g\left(\omega^*\right) \tag{4}$$

$$S_w = \frac{h^3}{U_c}\left(\rho_s E\sqrt{\frac{h}{S_p U_c}}\sqrt{\frac{U_c^3 h}{S_p}}\right)^{-1} g\left(\omega^*\right) \tag{5}$$

In this way it is possible to accordingly define dimensionless displacement functions of the dimensionless frequency ω^* only *i.e.*:

$$\frac{S_w}{S_p}\left(\frac{E}{h}\right)^2 = g\left(\omega^*\right) \tag{6}$$

$$\frac{S_w}{S_p}\left(\frac{\rho_s}{h}\right)^2 U_c^4 = g\left(\omega^*\right) \tag{7}$$

$$\frac{S_w}{S_p}\left(\frac{U_c}{h}\right)^2 \rho_s E = g\left(\omega^*\right) \tag{8}$$

It can be observed that in the case of weak structural-fluid coupling the ratio between the power spectral density of the output and the power spectral density of the input must be independent of flow parameters and in particular of the flow velocity.

This means that the structural mass, stiffness and damping matrices are not affected by the flow speed thus, Eqs. 7 and 8 can be considered of minor importance. This hypothesis is consistent with the experimental and numerical studies performed on the considered and on similar systems [3,14].

In conclusion the only relation that is considered significant for the present analysis is that provided by Eq. (6). The functional dependence on the dimensionless frequency can be investigated from the direct analysis of experimental data.

DATA ANALYSES

The proposed dimensionless forms of the structural response, Eqs.1 and 6, are applied to the data sets previously described.

The power spectral densities of the plate responses are represented by their mean response over

- *8 points for plate 1,*
- *3 points for plate 2 and*
- *5 points for plate 3.*

Data for plate 4 are directly derived in terms of metric response eq (1) from Figure 18 of reference [11]. In the same paper it is stated that the plate velocity response acquired with a laser vibrometer was averaged over 75 points.

The reference power spectral densities of wall pressure fluctuations are represented by their average over

- *10 points for plates 1 and 2 and*
- *19 points for plate 3.*

Values for the convection velocity are usually obtained from time domain cross-correlation analyses or from the phase of the cross spectral densities. According to the results reported in [3,8,9,10], the values of U_c used for the present analysis are:

- 0.7 U for plate 1
- 0.65 U for plate 2
- 0.75 U for plate 3
- 0.62 U for plate 4

In Figure 3 the dimensional acceleration power spectral densities are shown for plates 1, 2 and 3. Data for plate 4 are not available in this form. Due to the difference between test conditions of the three data sets in terms of both fluid dynamic

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and structural parameters but with particular reference to flow velocity values, the dimensional acceleration responses exhibited different amplitudes especially if looking to the low-mid frequency range. In this region the sensible gap in the response level is related to the different values assumed by the ratio between the structural and the aero/fluid-dynamic wavenumbers.

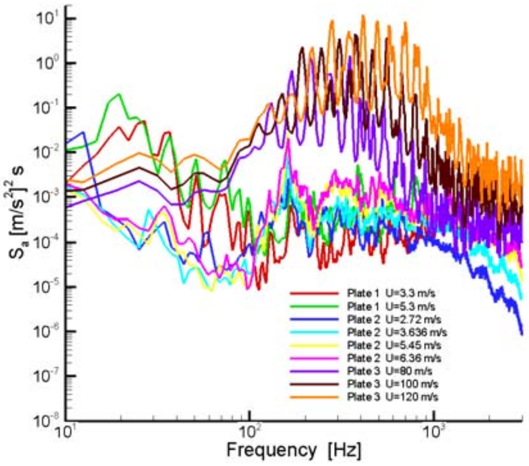

Figure 3 Dimensional acceleration power spectral densities.

In particular for the aerodynamic case (plate 3 and 4) structural wavenumbers are smaller than flow wavenumbers until the so called coincidence frequency. In this case the structural response is dominated for a large part of the frequency range by the convective components of the pressure field and in correspondence of the coincidence frequency reach its maximum value.

On the contrary, for Plate 1 and 2 the coincidence frequency is below 1 Hz thus, they receive energy mainly from the sub-convective component of the pressure field.

In this case the spectra amplitude, following the pressure PSD behavior, is nearly constant for a large part of the low-mid frequency range.

It is interesting to analyze what happens if the metric response of Eq.(1) is adopted as reported in Figure 4.

It is evident that the gap between different data sets is still large and that the proposed dimensionless form does not produce a collapse between different curves of the same data set either. As said before this dimensionless form for the structural response does not contain any information about the spatial characteristics of the fluid-structure interaction.

In Figure 5 the same experimental curves are represented in terms of power spectral density of the plate displacement. The same considerations hold for the contents of Figure 3.

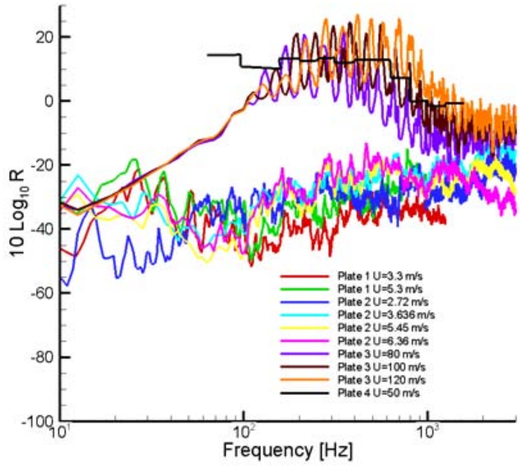

Figure 4 Metric responses.

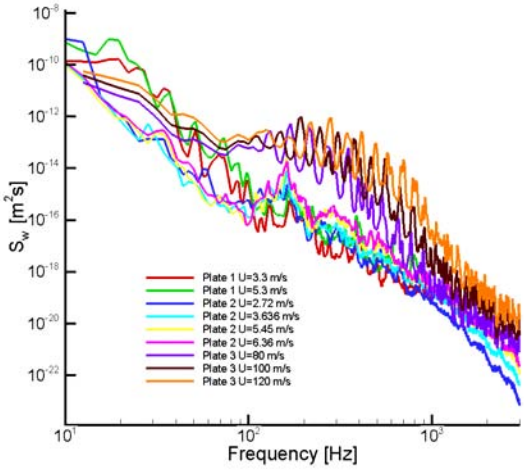

Figure 5 Dimensional displacement power spectral densities.

In Figure 6 the dimensionless representation of the plate responses provided by Eq.(6) is reported. It is evident an excellent collapse of the three curves relative to Plate 3; moreover the curve relative to Plate 4 shows the same trend in a quite similar non-dimensional frequency range and a complete superposition with the previous ones.

Curves relative to Plate 1 and 2 exhibit a good collapse with the other ones for values of the dimensionless frequency above the convective range.

The functional relation between the dimensionless response and the dimensionless frequency is well approximated by the simple relations:

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$$S_w^* = \begin{cases} 6.7\ 10^5 \left(\omega^*\right)^{-5} & \dfrac{\omega h}{U_c} < 0.2 \\ 1.0\ \ 10^6 \left(\omega^*\right)^{-3} & \dfrac{\omega h}{U_c} > 0.2 \end{cases} \qquad (9)$$

also shown for comparison in Figure 6.

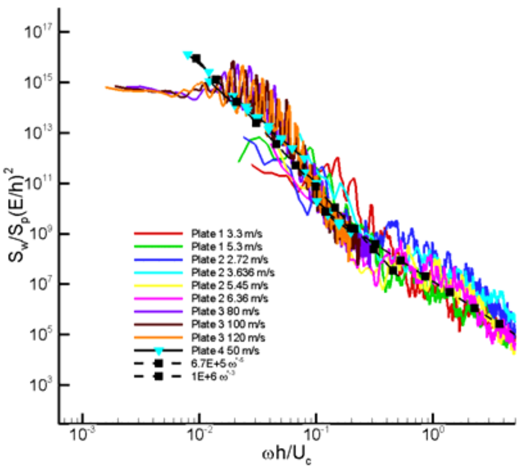

Figure 6 Dimensionless plate responses.

It should be evident that Eq.(6) can be used for both local and mean response. For all the results herein presented, the structural response has been considered a mean one, obtained through an average over the acquired locations. The validity of the mean response is dominated by the modal overlap factor of the system *i.e.* the product of the excitation frequency, the structural damping and the modal density. High values of this parameter (greater than unit value) are associated to a complete inhibition of the modal response. In principle, the dimensionless representation can be used for each acquired point, but this aspect was not addressed here.

As a final point, in order to investigate the validity of the assumption made in the previous section of weak fluid-structural coupling, relations 7 and 8 are used to scale plate spectra and the results reported in Figures 7 and 8. From the inspection of the figures it appear evident that the introduction of the flow speed does not produce a good collapse of the spectra. Additionally, if an higher power of U_c is used (eq.7) the gap between different curves tends to increase considerably.

A final note has to be given about the selected parameters for the material. Here, having a set of data acquired on plates all made by homogenous material, the obvious choice was to select the Young's modulus, E, and the mass density, ρ_S, as material constants.

In case of generic plates made of composite materials, sandwich or any other non-homogenous material or structural combination (i.e. sandwich plates), that choice for the elasticity has to be read as the elasticity modulus $\rho_S c_L^2$; the adoption of the longitudinal wave speed generalizes the present development to a generic material or combination of materials.

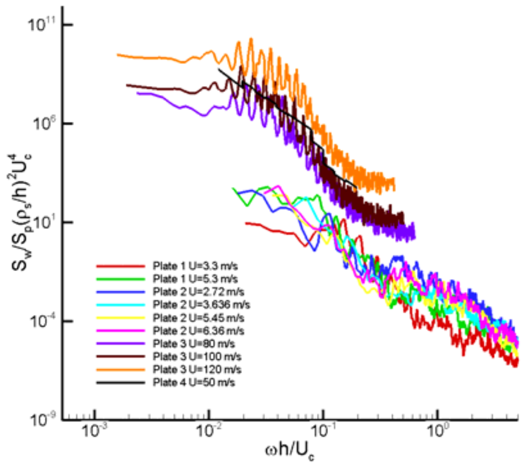

Figure 7 Dimensionless plate response

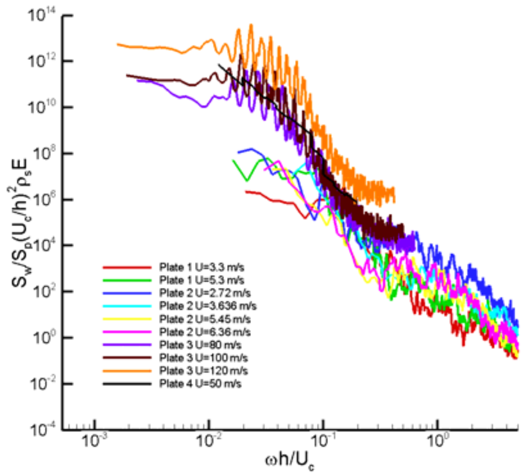

Figure 8 Dimensionless plate response

CONCLUDING REMARKS

In this work on the basis of a simplified dimensional analysis and a quite large amount of experimental data, scaling laws for the response of an elastic thin plate excited by the turbulent boundary layer are defined.

It was demonstrated that when the power spectral densities of the structural displacement can be represented in the plane $\left[\dfrac{\omega h}{U_c}; \dfrac{S_w}{S_p}\left(\dfrac{E}{h}\right)^2\right]$; all the presented experimental data collapsed in a single curve.

Simple analytical expressions for the dimensionless curve are finally provided.

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The work herein presented is at a very preliminary stage, but the approach deserves to be further investigated in view of the promising results.

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