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REDUCED-ORDER FLUIDIC MODEL FOR FLOW INSTABILITIES IN TWO-PHASE MICROFLUIDIC HEAT EXCHANGERS

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ABSTRACT

Two-phase microfluidic heat exchangers have the potential to provide high-heat flux cooling with lower thermal resistance and lower pumping power than single-phase heat exchangers. However, the process of phase change in two-phase heat exchangers can cause flow instabilities that lead to microchannel dryout and device failure [1-3]. Modeling these flow instabilities remains challenging because the key physics are highly coupled and occur over disparate time and length scales. This work introduces a new approach to capture transient thermal and fluidic transport with a reduced-order model consisting of fluidic, thermal, and phase-change submodels. The present study presents a reduced-order, transient, multichannel fluidic circuit submodel for integration into this proposed modeling approach. The fluidic submodel is applicable in flow regimes in which a thin liquid film exists around the bubble. Flow response to boiling is modeled considering bubble overpressure. An adaptive time step approach is used to treat the rapid flow response at short time scales after initial bubble vaporization. Using a seeded bubble technique for testing two-phase flow response, the model predicts a stability threshold at 0.015 W of localized superheating for two 100-micron square channels in parallel with a pump flow rate of 0.15 ml/min. Once integrated with the proposed reduced-order thermal and phase change models, this fluidic circuit model will yield criteria for stable two-phase heat exchanger operation considering factors such as pumping pressure, channel geometry, and applied heat flux that can be compared to experimental observations.

INTRODUCTION

Demand for high-heat flux cooling technologies for the microprocessor industry has motivated extensive research in two-phase microfluidic heat exchangers. Such devices have the potential to provide high-heat flux cooling at substantially lower pumping rates than those required for conventional liquid cooling. Furthermore, it may be possible to design the two-phase thermo-fluidic response of such devices to mitigate temporal and spatial hotspots, which are a growing concern in many industrial applications.

The performance of two-phase microfluidic heat exchangers, however, has been hindered by flow instabilities. As boiling occurs in a microchannel, the fluidic resistance increases; in a parallel channel configuration, this leads to a decrease in the flow rate to the two-phase channel, causing more vaporization of the working fluid. This excursive process can, depending on the flow supply characteristics, result in channel dryout and, consequently, device failure. Experimental evidence of this instability, as well as a related compressible region instability, were presented by Bergles and Kandlikar [1]. Their study suggested that all of the experimental studies of critical heat flux (CHF) in microchannels were conducted under unstable flow conditions, resulting in lower values for critical heat flux than would otherwise be achieved.

To date, the majority of research on two-phase microfluidic flow instabilities has been experimental studies. Chang and Pan [4] measured pressure oscillations in a microchannel heat sink with 15 parallel microchannels. They identified a narrow region of stable two-phase flow. Zhang et al. [5] demonstrated "eruption boiling" using flow visualization in single microchannels and measured transient pressure fluctuations up to 138 kPa. Hetsroni et al. [6] observed two evaporation modes under which a thin liquid film remained on the channel wall. Hardt et al. [7] conducted high-speed flow visualization in parallel channels and concluded that thin film evaporation influenced bubble expansion. Wu et al. [3] conducted flow instability experiments in a silicon microchannel heat sink consisting of eight parallel microchannels. They found that the observed liquid/two-phase alternating flow and liquid/two-phase/vapor alternating flow is caused by the vapor core reverse flow. Flynn et al. [8] conducted experiments in dual-channel devices and found that thermal conduction resistance between neighboring channels strongly influences the onset of flow instabilities.

Numerous experimental correlations have been developed for critical heat flux in microchannels. Qu and Mudawar [9] adapted mini-channel empirical models for microchannel boiling, finding that critical heat flux was largely independent from inlet subcooling. The resulting correlation provided results that closely matched experimental findings. Kennedy et al. [2] measured demand curves for boiling in uniformly heated microchannels and compared the onset of flow instability with widely used correlations.

Because the relevant physics for this parallel channel instability occur over disparate time and length scales, complete, multichannel, transient thermal-fluidic modeling of microfluidic flow instabilities has remained unresolved. Thome et al. [10] developed a model for evaporation of an elongated bubble in a single channel. A study by Revellin and Thome [11] developed a theoretical model for critical heat in stable flow in heated round microchannels. The model solved continuity, conservation of momentum, conservation of energy, the Laplace-Young equation, and a semi-empirical interfacial wave equation. Results from the model compared well with experimental results for R-113 refrigerant as well as water in multiple geometries.

Previous work by Kandlikar [12] included numerical modeling of a single nucleated bubble in a microchannel. A numerical model based on the incompressible Navier-Stokes equations was used to model the thermal-fluidic response to a single nucleated bubble in a microchannel, with and without an inlet constriction. The results suggested that channels with increasing diameter in the direction of flow would help stabilize flow. Experimental results from Lu and Pan [13] support this conclusion.

Flynn [14] proposed a reduced-order thermal-fluidic model for two-phase microfluidic heat exchangers based on fluidic and thermal circuits. Flynn's work served as a foundation for the present modeling approach.

The present work develops a new, reduced-order modeling approach for these thermal-fluidic phenomena based on insight from these past studies. An adaptive, reduced-order fluidic model for microfluidic boiling is presented. By coupling this fluidic model with an appropriate reduced-order phase change and thermal model, the combined approach will provide insight into the dominant physics of instability regimes observed in past experimental work. Furthermore, the model may serve as a design tool for further development of two-phase microfluidic heat exchangers.

NOMENCLATURE

β = effective bulk modulus of fluid (Pa)
V = volume (m^3)
Q = volumetric flowrate (m^3/s)
P = pressure (Pa)
C = fluidic capacitance (m^3/Pa)
L = fluidic inductance (kg/m^4)
R = fluidic resistance (N/m^2-s)
r = radius of circular tube (m)
t = time (s)
μ = dynamic viscosity (N-s/m^2)
ρ = fluid density (kg/m^3)

Conceptualization of Thermal-Fluidic Model

The current work presents a reduced-order flow physics model for flow boiling in parallel square microchannels. This model has been developed as a component of a coupled thermal-fluidic model currently under research. The concept behind the reduced-order coupled thermal-fluidics model is to take advantage of the disparate time scales on which the thermal physics and fluidic physics of microscale boiling occur; the relevant thermal transport in these systems occurs over relatively long time scales compared to the fluidic response. Characteristic thermal diffusion time scales can be on the order of milliseconds or slower, even for highly transient hotspots on microfabricated test structures [15]. In industrial implementations, additional boundary resistances and heat exchanger mass would result in even longer thermal transport time scales. The fluidic response to bubble overpressure, however, is primarily characterized by the fluidic capacitance of the system. For systems with little to no upstream compressibility, fluidic response occurs at microsecond time scales. For systems with greater upstream compressibility, the time scale of fluidic response also depends on other flow physics such as viscous losses and fluidic inertia. While there may be microfluidic systems of interest in which the assumption of disparate time scales becomes invalid, it is necessary to conduct thermal and fluidic time scales before implementing this approach. Based on past experimental observation, however, it appears a significant proportion of observed phenomena satisfy this condition.

Figure 1 presents a conceptual outline of the thermal-fluidic model consisting of three coupled submodels: a fluidic circuit submodel, a thermal circuit submodel, and a phase-change submodel. In such a model, the fluidic submodel will first solve the single-phase flow response for a given microfluidic heat exchanger. Once heat is applied, the thermal submodel resolves the increase in temperature in the solid channel wall and in the flowing liquid working fluid. The thermal model also captures the conjugate heat transfer in the microchannel wall. As the temperature of the fluid surpasses the saturation temperature, the phase change model captures the resulting vaporization process and associated change in

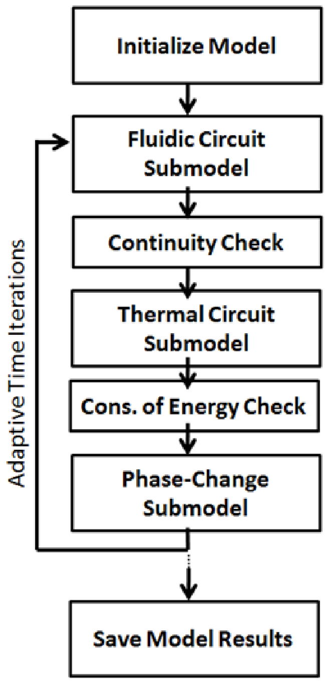

Figure 1: Flowchart of proposed reduced-order thermal fluidic modeling approach.

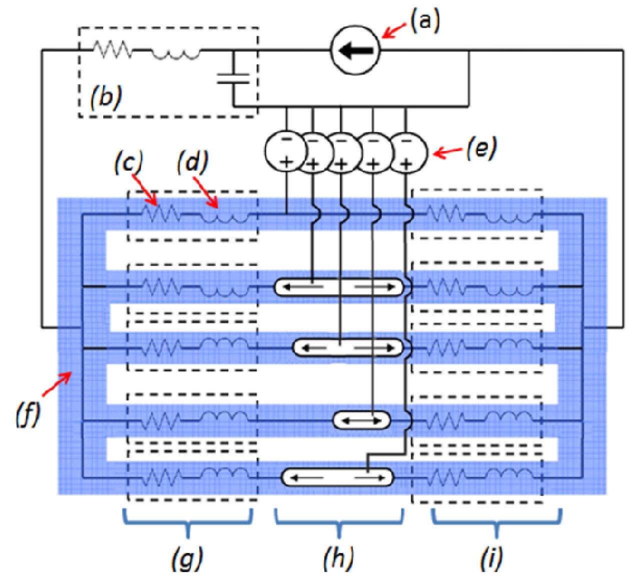

Figure 2: Representative fluidic circuit for five parallel microchannels, including (a) pump, (b) flow supply circuit elements, (c) fluidic resistance, (d) fluidic inductance, (e) pressure source to ascribe bubble pressure, (f) upstream manifold, (g) upstream region, (h) bubble region, and (i) downstream region. (Adapted from [14].)

pressure and temperature of the nucleated bubble. The fluid submodel resolves the resulting flow response, including flow redistribution and upstream compressibility effects. As the bubble expands upstream and downstream, the model continues to step between the three models to resolve the resulting physics. If the bubble reaches the channel outlet, the bubble pressure equilibrates with the outlet manifold pressure and evacuates the channel. For simplicity, the manifold is assumed to be large enough to evacuate the bubble without a significant additional pressure drop. If the bubble reaches the channel inlet, channel dryout occurs.

The present work focuses on the development of a reduced-order fluidic submodel that can be integrated into this thermal-fluidic modeling approach. The details of the reduced-order phase change model and thermal circuit model is the focus of ongoing research.

Methodology of Fluidic Model

To develop a reduced-order fluidic model capable of integration with the thermal and phase-change submodels, the microfluidic flow response has been modeled using an adaptive, transient fluidic circuit model. To do so, several simplifications are employed.

First, this model simplifies the bubble nucleation event by treating the bubble expansion in only the channel-wise dimension. Although the detailed physics of bubble nucleation involve important pressure and velocity gradients in the cross-section plane of the channel, the effect of these gradients in the context of flow stability are expected to be negligible relative to the effects of pressure gradients and velocity profiles in the channel-wise direction. Thus, it is reasonable to expect a one-dimensional treatment of the fluidic response could capture the key physics of flow instabilities after bubble nucleation and initial growth. Alternative modeling techniques that resolve the more complicated three-dimensional bubble physics are prohibitively computationally intensive when coupled with the proposed thermal model, so are deemed inappropriate for this application.

A second important simplification employed in this model is the treatment of surface tension. The current model is developed for flow boiling regimes in which solid-interface surface tension effects are highly localized around a nucleation site that is small relative to the bubble size. In such cases, the expanding bubble is enveloped by a thin film along the channel walls until channel dryout occurs. This simplification corresponds well with visualization studies that show the presence of a thin liquid film along the microchannel wall (e.g., [6, 7]). In alternative flow regimes, this thin film condition may not be met; for such cases, this model is invalid. This condition also prevents the use of the model for simulating flow response after channel dryout, such as rewetting after a temporary hotspot causes channel dryout. Such cases, however, are not of primary interest to characterizing the instability, so the simplification is deemed appropriate.

Reducing the dimensionality of the flow response and imposing the thin film requirement facilitates modeling the bubble as a transiently varying pressure source in a fluidic circuit model. Figure 2 shows a schematic of the fluid circuit for a case of five parallel microchannels. The values of each element of the circuit are updated every iteration to reflect the local fluid properties and relevant thermal characteristics. The constant mass flow-rate pump is modeled as a current source and the delivery lines are modeled with a representative value for fluidic capacitance [16].

$$\frac{dP}{dt} = \frac{\beta}{V} Q \tag{1}$$

$$C = \frac{V}{\beta} \tag{2}$$

The capacitive term accounts for upstream compressible volumes such as compliant fluid lines, non-rigid contaminant filters, trapped gas regions in the supply line, and non-ideal pump response. For typical channel configurations and parameters, no additional fluidic capacitance occurs in the channel as long as the working fluid does not contain dissolved gas. The bubble compressibility is treated with the phase-change model.

The resistive and inductive circuit elements, respectively R and L, are calculated based on established solutions to single-phase flow in single-phase, laminar flow in circular tubes and are applied using the hydraulic diameter of the square microchannel cross section [16]:

$$\Delta P = \frac{8\mu l}{\pi r^4} Q \tag{3}$$

$$R_{laminar} = \frac{8\mu l}{\pi r^4} \tag{4}$$

$$\Delta P = \frac{\rho l}{\pi r^2} \left(\frac{dQ}{dt}\right) \tag{5}$$

$$L = \frac{\rho l}{\pi r^2} \tag{6}$$

To model the transiently varying pressure, which is closely coupled to the thermal submodel by phase change, a voltage source term is adaptively included between atmospheric pressure and the bubble pressure. This facilitates variation in the bubble pressure as a function of, for example, the rate of vaporization in accordance with the phase-change model.

One challenge of implementing such a technique, however, is resolving the initial rapid expansion of the bubble. To capture the bubble response at short times after initial vaporization, an adaptive, short-time-step simulation is conducted. The resulting flow rates and pressure response are then saved and compiled into a reference submodel for future tests. This method reduces computation time in future simulations by enabling longer time steps in thermal-fluidic modeling; however, it is important to note that the reference model is only valid for cases in which system parameters remain unchanged. For modeling a variety of systems with, for example, various channel geometries or pump rates, a library of short-time-scale solutions is required. For the system modeled, runtimes for creating a reference model were approximately 20 minutes on a standard personal computer. Developing a library of reference models, therefore, is not expected to be a significant challenge for implementation.

In the proposed thermal-fluidic model, phase-change criteria are coupled to the thermal model to capture initial bubble vaporization. To test the response of the current fluidic model to initial bubble vaporization in a manner representative of the fully coupled model, the model is set to initiate vaporization by treating a small fraction of the flow as a superheated region that has been vaporized. At very short time scales, this simulated vaporization process causes elevated pressures, which are subsequently reduced by the expansion of the bubble. Because the extent of fluid vaporization is a function of the amount of superheated fluid, the response to these “seeded” bubbles is expected to be representative of the flow response to a complete phase-change submodel.

This seeded bubble technique for testing the fluidic circuit is equivalent to applying a dramatically simplified phase-change model. In such a technique, the amount of liquid vaporized is determined from the amount of superheating calculated in the thermal model and the heat of vaporization of the working fluid. To determine the pressure of the bubble as it expands, the ideal gas equation is applied with the volume of the vapor region and the saturation temperature of the bubble. This bubble seeding technique results in an inappropriate representation of bubble pressure and flow response at short time scales because it neglects spatial gradients in the cross-section plane of the channel and ignores surface tension effects; as such, it is not advocated as a phase-change model for the proposed thermal-fluidic model.

During bubble expansion and departure, the model ascribes a constant wall thin film thickness, comparable to those found in the literature. Verification of the film thickness condition is conducted each time step. If the thin film is temporarily reduced due to bubble expansion, the model enforces the thin film condition by wicking liquid from neighboring regions.

Bubble response is also affected by the channel inlet and outlet. When the bubble reaches the outlet of the microchannel, the bubble pressure equilibrates with the pressure in the outlet channel manifold. If the bubble reaches the inlet manifold of the channel, the condition for channel dryout is met and the simulation is terminated. Because the model is only valid for flow regimes in which a thin film remains on the channel wall, the model does not resolve the rewetting process of a microchannel.

The model has been primarily developed in MATLAB® (The Mathworks™) and is coupled to WinSpice circuit solver (OuseTech), which was validated against solutions from a reference circuits textbook [17].

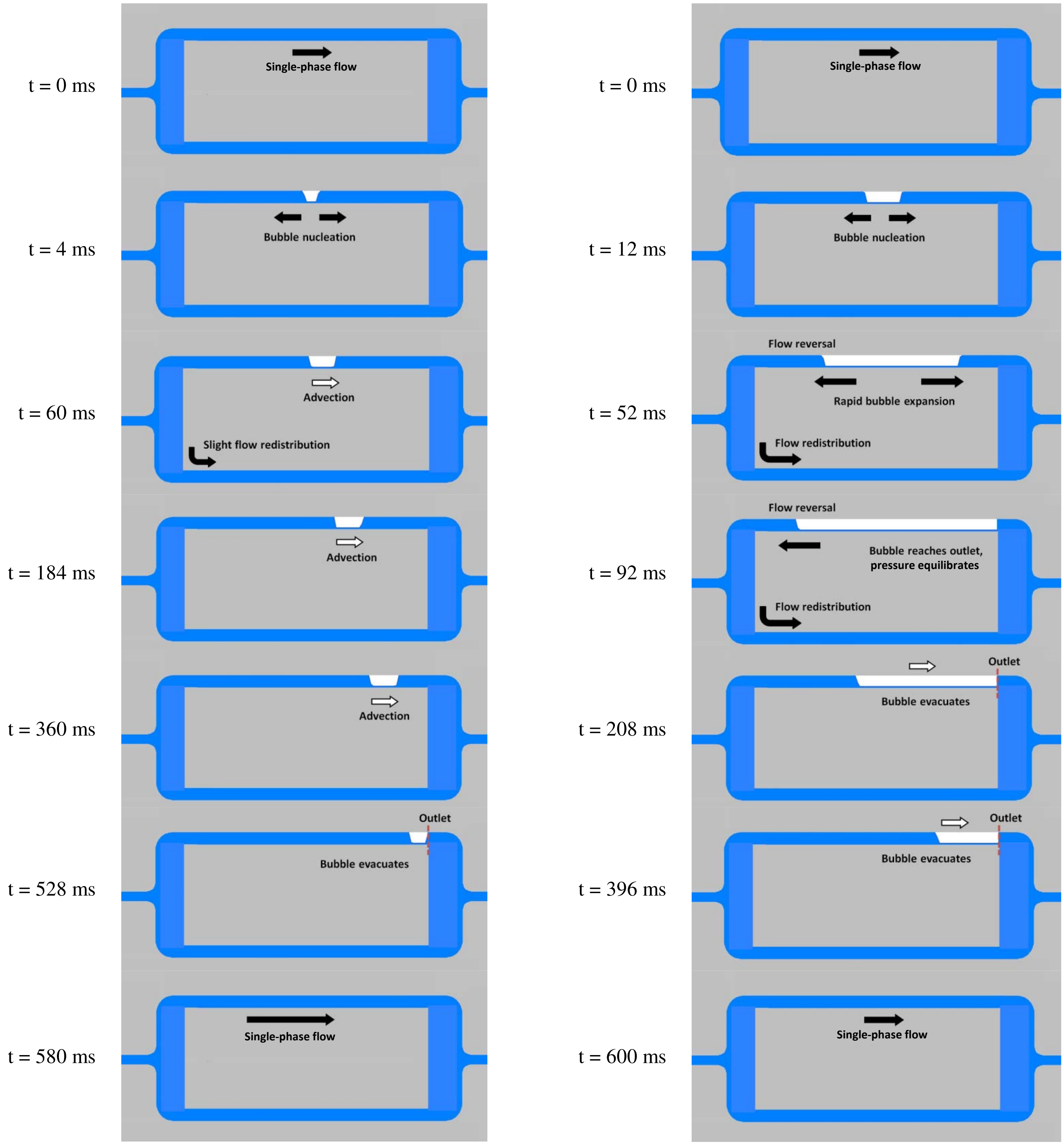

Figure 3: Time sequence of modeled flow response to a seeded bubble with equivalent latent heat of $8.7x10^{-4}$ J (left) and $2.1x10^{-4}$ J (right) in two parallel microchannels. Note: vapor not shown in outlet manifold region.

Table 1: Selected simulation parameters.

Parameter	Value
Channel width [	100
Channel height [	100
Channel length [mm]	10
Number of channels	2
Channel-wise indices	100
Thin film thickness [	3
Pump flowrate [ml/min]	0.015

Results and Discussion

A two-microchannel channel system was chosen for testing to provide insight into flow instabilities while minimizing system complexity so valuable insight can be gleaned. The model was tested using a two-channel, parallel configuration. Table 1 shows the key parameters used in the model. The channels were modeled as 100-um square cross-sections with a length of 10 mm and water as the working fluid. The model can be later employed with additional channels for comparison with experimental studies that employ numerous parallel channels.

Figure 3 shows representative transient bubble response for two seeded bubbles. In this example, the pump flowrate is from left to right. On initial vaporization, the bubble expands rapidly upstream and downstream, driving reverse flow. The overall pressure drop increases in the system and the single-phase channel experiences accelerated flow. As the bubble expands and its pressure decreases, the reverse flow is reduced. If the bubble reaches the inlet, the channel experiences channel dryout. However, if the forward flow from the pump prevents the bubble from reaching the inlet, the upstream interface of the bubble is forced downstream until the bubble is expelled from the outlet.

One characteristic of flow response as it relates to microfluidic flow stability is the maximum length of the bubble. The "explosive boiling" regimes discussed in past work typically involved bubbles at least an order of magnitude as long as they are wide: this is distinct from more stable flow regimes in which the length of the bubble is of comparable scale to its diameter (e.g., [7]). Figure 3 shows a representative case of a relatively small bubble that is advected downstream; a second case shows a relatively large bubble that almost causes channel dryout. Figure 4 uses this bubble length metric to demonstrate variations in flow response for channels experiencing various degrees of flow vaporization. Channel dryout occurs for seeded bubbles of equivalent latent heat greater than 8.7 mJ. Given the bubble residency time of 580 ms, the threshold heat input is equivalent to 0.015 W of channel superheat.

Concluding Remarks

A new conceptual approach to modeling flow instabilities in parallel microfluidic heat exchangers has been presented to resolve the coupled thermal and fluidic physics of microfluidic flow boiling instability phenomena. A reduced-order

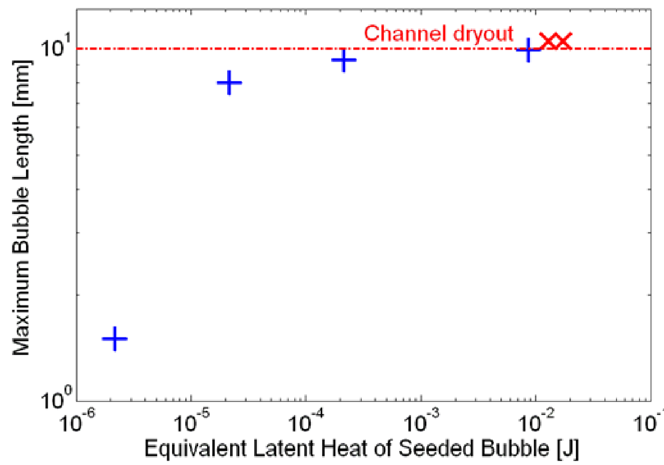

Figure 4: Effects of degree of vaporization on maximum seeded bubble length.

microfluidic flow model has been developed to capture the transient flow response to phase change at the microscale under "explosive boiling" conditions in which a thin film is maintained on the microchannel wall. The model captures flow response using a transient fluidic circuit. The rapid flow response to bubble formation is treated with a reference submodel and adaptive time steps.

The model has been tested under conditions that simulate the formation of a bubble in a non-uniformly heated microfluidic heat exchanger. The response indicates conditions under which flow reversal causes unstable flow conditions These conditions are shown to be dependent on the location of the bubble nucleation site. Furthermore, the bubble length is used to characterize the extent to which the flow response approaches channel dryout.

To date, the authors know of no alternative modeling approaches that have been demonstrated for resolving these instability phenomena over the disparate time and length scales on which they occur. The fluidic model presented in this study, when combined with additional phase-change and thermal modeling currently under development, may prove valuable for synthesizing our understanding of numerous experimental results and, ultimately, designing robust two-phase microfluidic heat exchangers.

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REFERENCES

[1] A.E. Bergles and S.G. Kandlikar, "On the Nature of Critical Heat Flux in Microchannels." vol. 127: ASME, 2005, pp. 101-107.

[2] J.E. Kennedy, G.M. Roach, Jr., M.F. Dowling, S.I. Abdel-Khalik, S.M. Ghiaasiaan, S.M. Jeter, and Z.H. Quershi, "The Onset of Flow Instability in Uniformly Heated Horizontal Microchannels." vol. 122: ASME, 2000, pp. 118-125.

[3] H.Y. Wu and P. Cheng, "Boiling instability in parallel silicon microchannels at different heat flux," *International Journal of Heat and Mass Transfer,* vol. 47, pp. 3631-3641, 2004.

[4] K.H. Chang and C. Pan, "Two-phase flow instability for boiling in a microchannel heat sink," *International Journal of Heat and Mass Transfer,* vol. 50, pp. 2078-2088, 2007.

[5] L. Zhang, E.N. Wang, K.E. Goodson, and T.W. Kenny, "Phase change phenomena in silicon microchannels," *International Journal of Heat and Mass Transfer,* vol. 48, pp. 1572-1582, 2005.

[6] G. Hetsroni, A. Mosyak, E. Pogrebnyak, and Z. Segal, "Explosive boiling of water in parallel micro-channels," *International Journal of Multiphase Flow,* vol. 31, pp. 371-392, 2005.

[7] S. Hardt, B. Schilder, D. Tiemann, G. Kolb, V. Hessel, and P. Stephan, "Analysis of flow patterns emerging during evaporation in parallel microchannels," *International Journal of Heat and Mass Transfer,* vol. 50, pp. 226-239, 2007.

[8] R. Flynn, D. Fogg, J.-M. Koo, C.-H. Cheng, and K.E. Goodson, "Boiling Flow Interaction Between Two Parallel Microchannels," *Proceedings of IMECE,* 2006.

[9] W. Qu and I. Mudawar, "Prediction and measurement of incipient boiling heat flux in micro-channel heat sinks " *International Journal of Heat and Mass Transfer,* vol. 45, pp. 3933-3945, 2002.

[10] J.R. Thome, V. Dupont, and A.M. Jacobi, "Heat transfer model for evaporation in microchannels. Part I: presentation of the model," *International Journal of Heat and Mass Transfer,* vol. 47, pp. 3375-3385, 2004.

[11] R. Revellin and J.R. Thome, "A theoretical model for the prediction of the critical heat flux in heated microchannels," *International Journal of Heat and Mass Transfer,* vol. 51, pp. 1216-1225, 2008.

[12] A. Mukherjee and S.G. Kandlikar, "The effect of inlet constriction on bubble growth during flow boiling in microchannels," *International Journal of Heat and Mass Transfer,* vol. 52, pp. 5204-5212, 2009.

[13] C.T. Lu and C. Pan, "Stabilization of flow boiling in microchannel heat sinks with a diverging cross-section design," *Journal of Micromechanics and Microengineering,* vol. 18, pg. 075035, 2008.

[14] R. Flynn, "Flow Boiling Instabilities in Microchannels," in *Ph.D. Defense Presentation* Stanford, Calif., 2008.

[15] J. Miler, R. Flynn, G. Refai-Ahmed, M. Touzelbaev, M. David, J. Steinbrenner, K.E. Goodson, "Effects of Transient Heating on Two-Phase Flow Reponse in Microfluidic Heat Exchangers," *Proceedings of the ASME/Pacific Rim 10th Technical Conference and Exhibition on Packaging and Integration of Electronic and Photonic Systems, MEMS, and NEMS (InterPACK), IPACK2009-89325, San Francisco, Calif., 2009*.

[16] A. Akers, M. Gassman, R. Smith, *Hydraulic power system analysis*. Boca Raton, Fla.: CRC/Taylor & Francis, 2006.

[17] J.O. Attia, "PSPICE and MATLAB for Electronics: An Integrated Approach," in *VLSI Circuits Series*, W.-K. Chen, Ed. New York: CRC Press, 2002.