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FLOW STRUCTURE OF SUPERSONIC JET FROM A STRAIGHT MICRO-TUBE

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ABSTRACT

In the case of micro-channels, the boundary layer is formed on the walls and it plays a role of a wall of a converging and diverging nozzle. Then, the outlet Mach number is beyond unity even if the tube is straight. Therefore, in the present study, the successive incident and reflected shock waves on underexpanded state jet from a straight micro-tube whose diameter ranges from 150μm to 500μm were visualized by Schlieren and Shadowgragh methods. The stagnation pressure ranges from 597 to 963 kPa. The flow characteristics on supersonic jet at the micro-tube outlet were also obtained. Also, it is confirmed that Mach number at a straight micro-tube outlet is beyond unity since the shock wave generates from the needle and the Mach number fluctuates in the jet. The experimental correlation for the distance from the micro-tube outlet to the Mach disk as a function of the ratio of stagnation pressure to ambient back pressure was proposed and compared with available correlations in literature.

INTRODUCTION

Design and fabrication of MEMS have increased the need for understanding of fluid flow in micro flow devices such as micro-valves, micro-ejector, micro-heat exchangers and many other micro-fluid systems. In the case of gaseous flows in micro-channels, it is well understood that rarefaction (the slip on the surface), surface roughness and compressibility have significant effects on the results. Therefore numerous experimental and numerical investigations have undertaken to clarify flow characteristics as seen by the literature reviews in the publications by Turner et al. [1] and Asako et al. [2]. As can be seen from the review of the literature above, the focus of the research to date has been on thermal fluid transport phenomena for the inside of micro-channels.

Attention will now be focused on the nozzles, studies on the jet issued from sonic to supersonic nozzles have been investigated by numerical and experimental methods over the years, e. g., Addy [3], Cumber et al. [4] and Palmer and Hanson [5]. Matsuo et al. [6] investigated the effect of axisymmetric sonic nozzle geometry on characteristics of supersonic air jet. They found that the supersonic jet structure, sonic line and streamlines in supersonic jet are strongly influenced by the nozzle geometry.

In the case of conventional straight channel, the flow is choked and outlet Mach number is unity since the thickness of the boundary layer formed on the channel wall is relatively thin. On the other hands, in the case of straight micro-channel flow, the boundary layer is formed on micro-channel wall and its thickness becomes zero at the outlet of the channel. Then, it plays a role of a wall of a converging and diverging nozzle and the flow becomes supersonic at the micro-channel outlet. Recently, Hong et al. [7, 8, 9] conducted numerical investigations of flow characteristics of gaseous flow in micro-channels and micro-tubes with both no-slip and slip boundary conditions. They obtained f·Re correlations as a function of Mach number and Knudsen number for quasi-fully developed region. Also it was observed that the Mach number of flow at the exit is beyond unity and the flow is supersonic. This fact was also found by Masugi [10] who reported that there exits supersonic main flow in a planar duct. Mini channels were used for his experiments. Unfortunately this fact is not widely known. Also there seems to be no parametric study to investigate the flow characteristics at the micro-tube outlet. The correlation to predict the micro-tube outlet flow has not been investigated yet. It has been of importance in academic aspect, as well. This has motivated the present experimental study to characterize the jet flow in down stream region from the micro-tube outlet by Schlieren and Shadowgraph visualization. Also, the supersonic flow in the jet was identified since the shock wave generates on the needle.

NOMENCLATURE

A	cross sectional area per unit depth	m
D_h	hydraulic diameter	m
D	tube diameter	m
f	friction factor	-
i	specific internal energy	J/kg
Kn	Knudsen number	-
L	tube length	m
$\dot{m}$	mass flow rate	kg/s
Ma	Mach number	-
max	maximum value	
p	pressure	Pa
R	gas constant	J/kg K
Re	Reynolds number	
T	temperature	K
u, v	velocity components	m/s
x, r	coordinates	m
r_1	radius of tube	m
r_2	radius of ambient region	m
γ	specific heat ratio	
μ	viscosity	Pa s
ρ	density	kg/m^3
τ	shear stress	N/m^2

Subscript

amb	ambient value
ave	average value
CL	centerline
i	initial value
in	inlet
md	Mach disk
out	outlet
stg	stagnation value

EXPERIMENTAL SETUP

The experimental apparatus in the present investigation is shown in Fig.1. A micro-tube was fixed to the hole of a plexiglass disk by adhesive, and the disk was fasten to the chamber by holding a O-ring between them. The gas pressure was measured at the inlet chamber by a pressure transducer (Valcom, VPRNP-1700, max: 1700 kPa). PEEK tubes (Upchurch scientific, 1532) were used for the test micro-tube whose nominal diameters were 150, 250, 400, 500μm. The ratio of length to diameter was 200. A picture of the cross-section of the PEEK tube by a scanning electron microscope (KEYENCE, VE-7800) is shown in Fig. 2. The chamber is connected to flow meter (Kofloc, 3105-30SLM, accuracy: ±1.0% F.S.) which measures the flow rates, and the flow meter was connected to a data acquisition system (Eto Denki, CADAC21).

The exit of flow was open to the atmosphere. The ratio of stagnation pressure to atmospheric pressure was up to about 9.9 until to the pressure limitations of the regulator. Additionally, experimental setup for the investigation consisted of Schlieren photography of external flow. The Schlieren apparatus consist of a light source, converging lenses with 150, 50mm diameter, pin hole with 0.5mm diameter and a knife edge. As a light source, the Schlieren system employed the mercury lamp (OLYMPUS,U-LH100HGAPO, 100W). The high speed camera (Photron, APX RS) was used for the capture of photography. The spatial resolution of the image of the camera is 512×512 pixels and shutter speed is 400 ns. The experiments of Shadowgraph were also conducted without the knife edge.

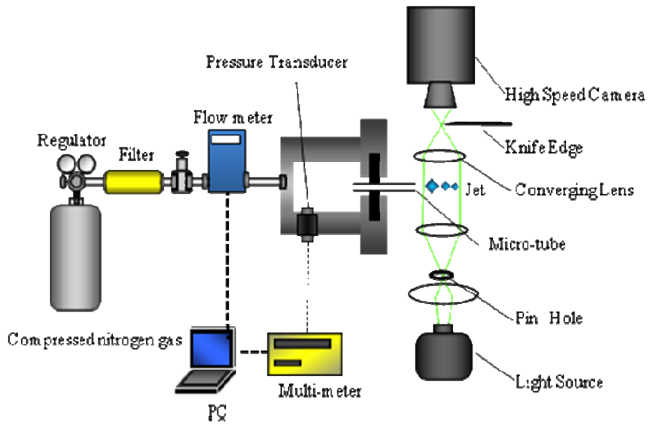


Fig.1 A scheme diagram of experimental setup

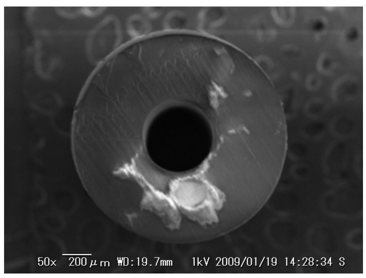
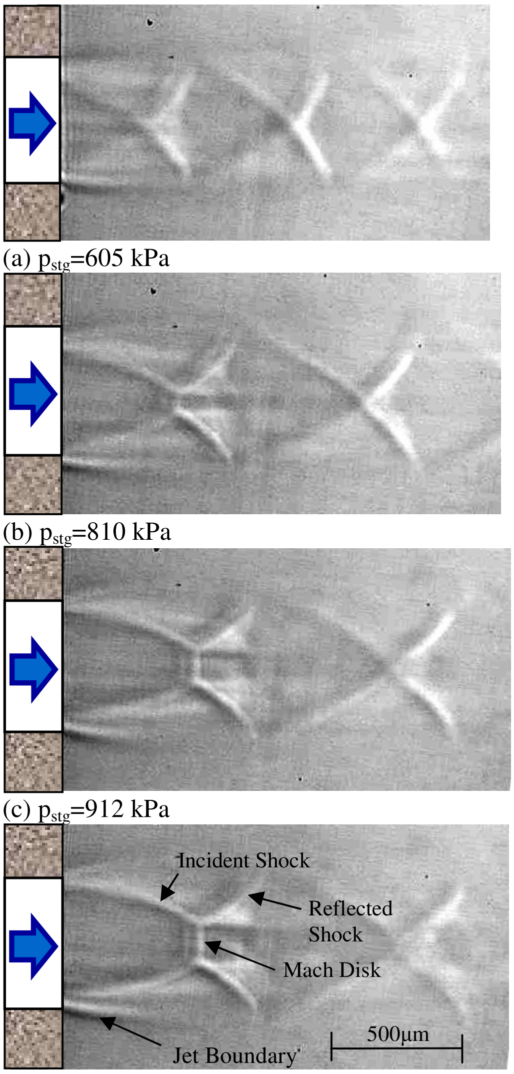


Fig.2 A picture of a cross section of PEEK tube (D=250µm)

RESULTS OF EXPERIMENT

Figures 3 and 4 show the pictures of the visualization by the Shadowgraph method. Theses are the results for D=500μm and D=400 μm, respectively. The flow at the outlet is under-expanded since the incident shock generated away from the micro-tube outlet. As can be seen in Fig.3 and 4, the incident shock wave and reflective shock wave and the Mach disk on the free jet boundary are more distinctive as stagnation pressure increases. The incident wave in the free jet boundary is generated and expansion and recompression successively occur near the micro-tube outlet.

Figures 5 and 6 also show the picture of the visualization by the Schlieren technique which measures the first derivative of density in the direction of the knife-edge. Then, the result is a set of lighter and darker patches corresponding to positive and negative jet density gradients in the direction to the knife-edge.



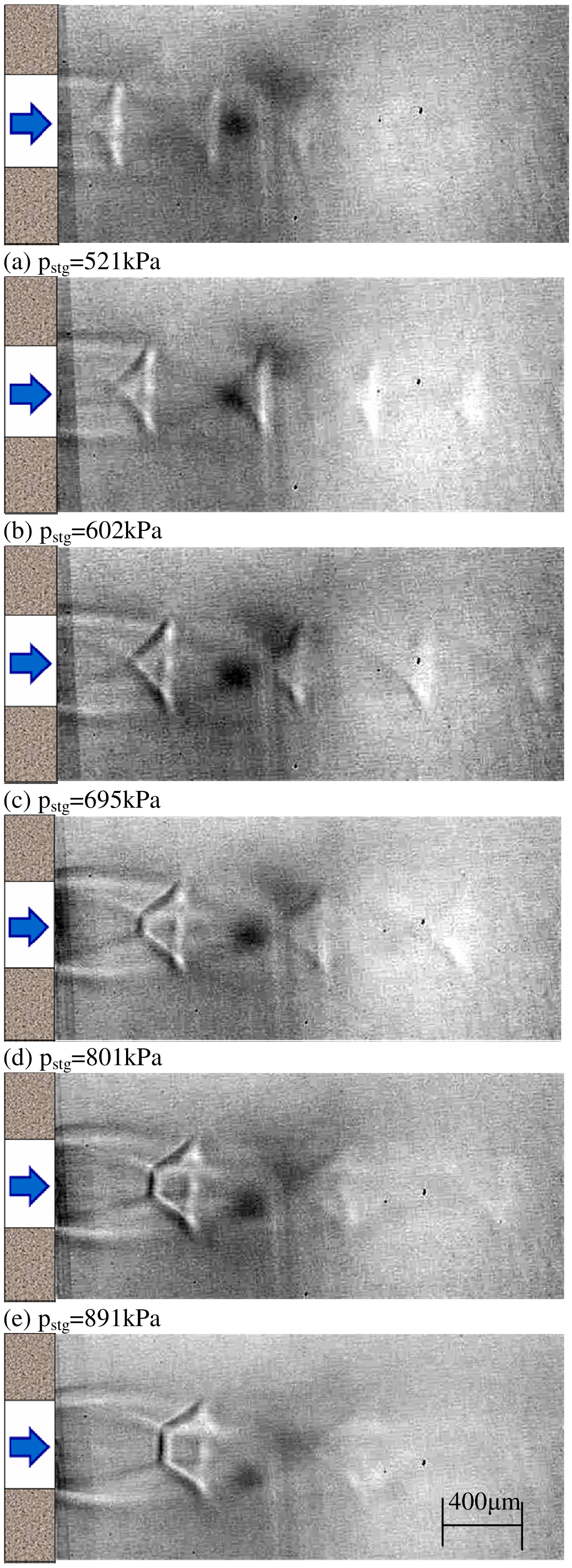
(a) p_{stg}=605 kPa

(b) p_{stg}=810 kPa

(c) p_{stg}=912 kPa

(d) p_{stg}=963 kPa

Fig. 3 Shadowgraph pictures of under expanded jets for D=500μm



(a) p_{stg}=521kPa

(b) p_{stg}=602kPa

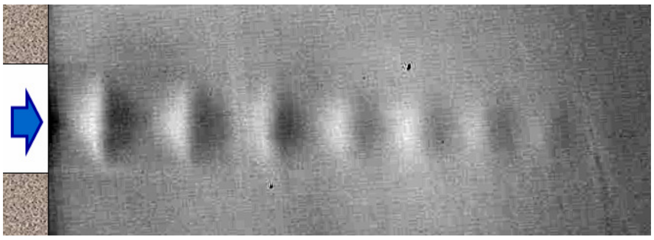
(c) p_{stg}=695kPa

(d) p_{stg}=801kPa

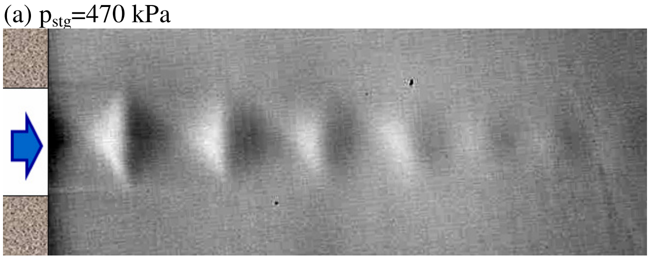
(e) p_{stg}=891kPa

(f) p_{stg}=997kPa

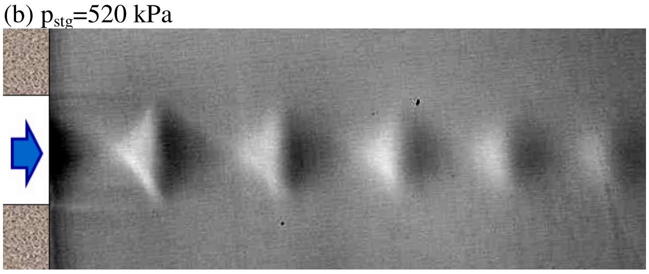
Fig. 4 Shadowgraph pictures of under expanded jets for D=400μm



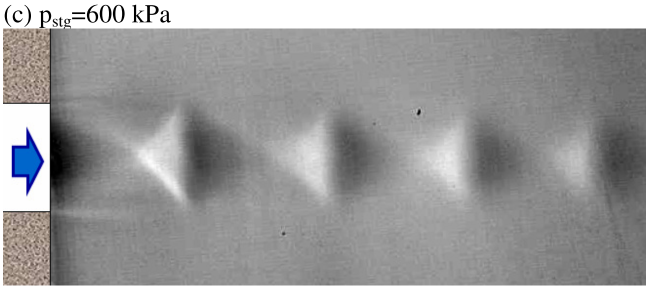
(a) p_{stg}=470 kPa



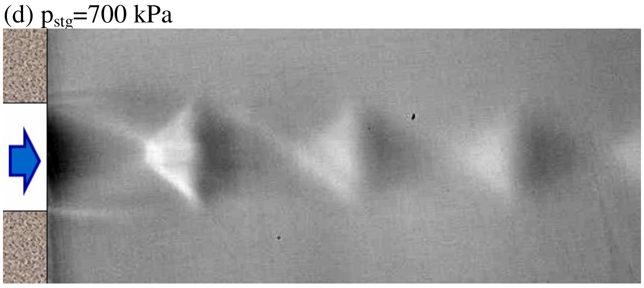
(b) p_{stg}=520 kPa



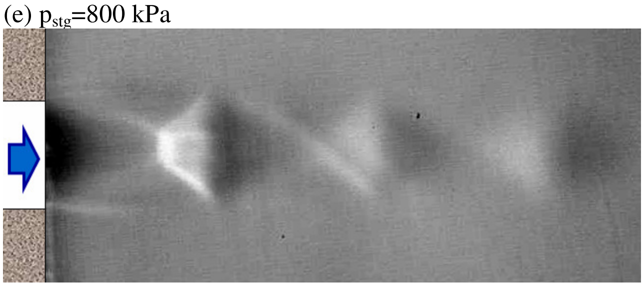
(c) p_{stg}=600 kPa



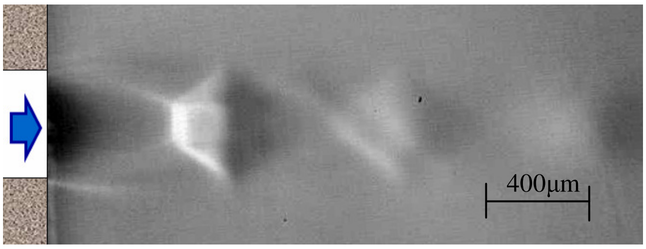
(d) p_{stg}=700 kPa



(e) p_{stg}=800 kPa

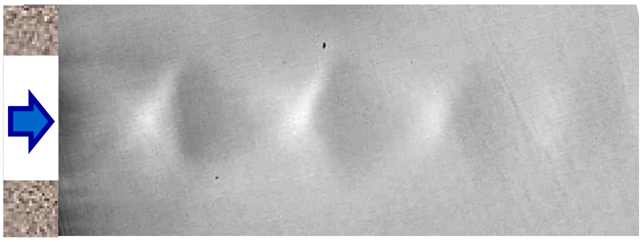


(f) p_{stg}=890 kPa

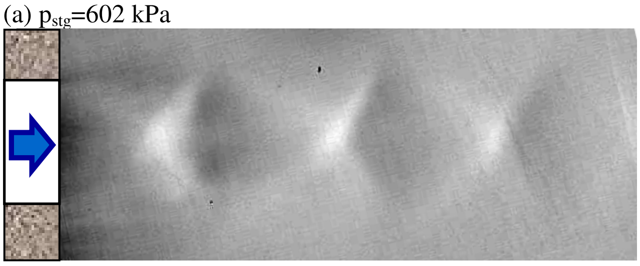


(g) p_{stg}=990 kPa

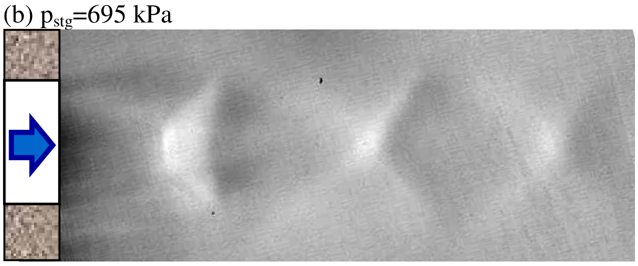
Fig. 5 Schlieren pictures of under expanded jets for D=400μm



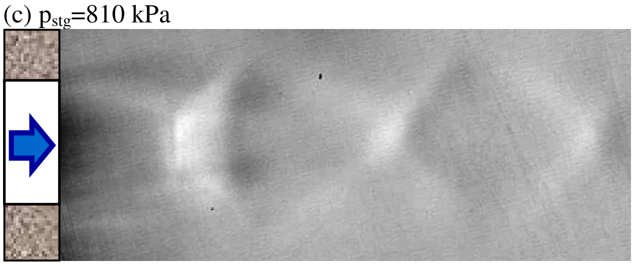
(a) p_{stg}=602 kPa



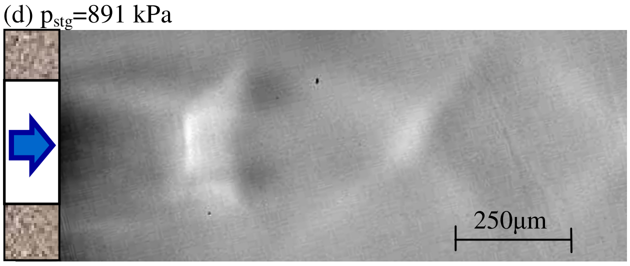
(b) p_{stg}=695 kPa



(c) p_{stg}=810 kPa



(d) p_{stg}=891 kPa



(e) p_{stg}=993 kPa

Fig. 5 Schlieren pictures of under expanded jets for D=250μm

The figures are the results for D=400μm and D=250μm, respectively. All the Schlieren pictures correspond to under expand supersonic. The density decreases at the tube exit, fluctuates and approaches asymptotically to the jet density. In the obtained Schlieren and Shadow pictures individual elements of the shock wave structure of the supersonic jet are clearly seen. Qualitatively similar Schlieren pictures are also obtained for D=150μm and 500μm.

ESTIMATION OF OUTLET PRESSURE

The micro-tube diameter and length, stagnation pressure and the distance of the micro-tube outlet to Mach disk normalized by the diameter, L_{md}/D are listed in Table 1. Note that the distance from the outlet to the Mach disk is increased with increasing the stagnation pressure.

Table 1 Tube diameter, length, p_{stg}, and p L_{md}/D

D (μm)	L (m)	p_{stg} (kPa)	L_{md}/D
150	0.03	665	0.70
		725	0.77
		787	0.83
		850	0.863
250	0.05	597	0.67
		657	0.80
		712	0.88
		759	0.95
		810	0.96
		857	0.99
		910	1.02
400	0.08	470	0.26
		517	0.39
		559	0.54
		597	0.56
		610	0.70
		665	0.81
		695	0.75
		720	0.9
		767	0.94
		801	0.89
		818	1.00
		869	1.05
		912	1.08
500	0.10	479	0.35
		565	0.52
		605	0.73
		682	0.81
		775	1.03
		810	1.01
		868	1.12
		912	1.10
		963	1.17

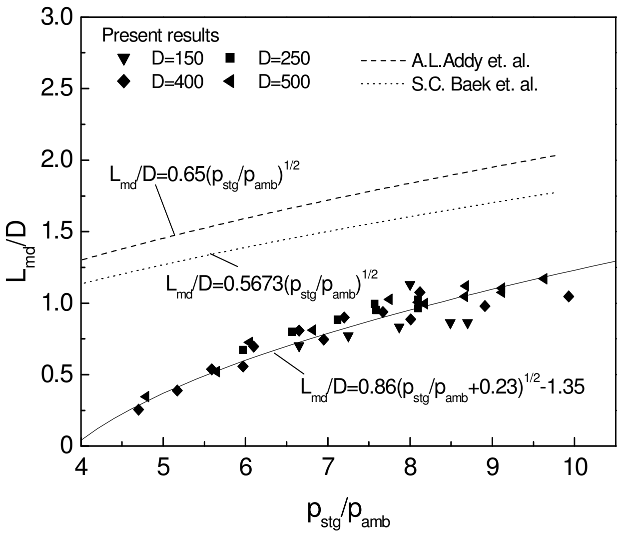


Fig. 7 The distance from tube outlet to Mach disk

Attention will now be turned to the under-expanded supersonic jet, the L_{md}/D normalized by the tube diameter plotted as a function of the pressure ratio, p_{stg}/p_{amb}, which is the ratio of the stagnation pressure to the ambient back pressure.

The values of L_{md}/D are plotted as a function of pressure ratio, p_{stg}/p_{amb} in Fig 7. In reference, the dashed line in the figure represents the L_{md}/D for axisymmetric sonic nozzle obtained by Addy [3] who performed experimental study on nozzle exit of various shapes. The doted line in the figure represents the experimental correlation proposed by Baek et al. [11] for the moist air jet. L_{md}/D obtained for the present study increases with increasing pressure ratio and tube diameter. The effect of tube diameter is relatively small. L_{md}/D is mainly a function of pressure ratio. Then, the solid line in the figure represents the empirical correlation for L_{md}/D and pressure ratio, p_{stg}/p_{amb} that is obtained by a polynomial curve fit as

$$L_{md}/D = 0.86\left(P_{stg}/P_{amb} + 0.23\right)^{1/2} - 1.35 \qquad (1)$$

L_{md}/D of a straight micro-tube is lower than that of a nozzle since the cross-sectional area ratio is formed by boundary layer.

ESTIMATION OF OUTLET MACH NUMBER

An oblique shock wave is generated in the supersonic flow when a sharp wedge is put in the flow as shown in Fig. 8. There is the following correlation between the shock angle, s, and the wedge angle, a [12].

$$\cot a = \tan s \left[\frac{(\gamma+1)\mathrm{Ma}^2}{2(\mathrm{Ma}^2 \sin^2 s - 1)} - 1 \right] \qquad (2)$$

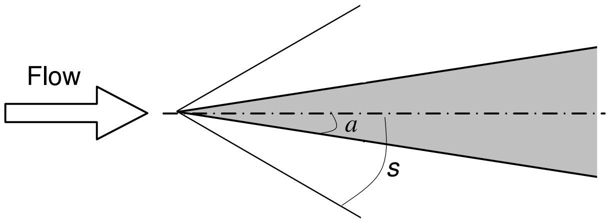


Fig. 8 Shock wave generated from a wedge

where Ma is the Mach number of the flow. If the shock angle and the wedge angle are specified, Mach number of the flow can be determined from Eq. (2). Note that smaller shock angles are associated with higher Mach number at the wedge.

In order to determine Mach number of the under expanded supersonic jet from a micro-tube, experiments were conducted with using a needle (SURUGA, M904-0006) whose point angle is 9.18°. The Shadowgraph technique was used to visualize the shock wave. Figure 9 (a) to (q) are Shadowgraph picture with different needle location. The estimated Mach number from the shock angle is indicated in the figure. These figures are the results for D=500μm and p_{stg}=990kPa. The shock angle generated from the needle differs according to the location of the needle. Mach number at the micro-tube outlet is beyond unity since the shock wave is generated from the top of the needle at the micro-tube outlet (see Fig. 9 (q)). Mach numbers obtained from Eq. (2) were plotted as a function of x/D in Fig. 10. As can be seen from Fig. 10, Mach number fluctuates in the jet and the location of the Mach disk is also indicated in the figure. The Mach disk was formed at x/D=1.3. In the downstream area of the Mach disk, Mach number is lower than unity since the shock wave is not generated from the needle in this area. The shock wave is not also generated for Ma<1.4 since the point angle of the needle is 9.18°. In the upstream area of the Mach disk, the shock wave appears again and Mach number is beyond unity until the micro-tube outlet.

CONCLUSION

(1) Mach number at a straight micro-tube outlet was beyond unity since the shock wave generates from the needle.
(2) The flow downstream of the micro-tube outlet is supersonic since shock cell were represented constantly.
(3) Jet pressure and density fluctuate from the tube outlet due to the presence of shock cells. The distance of Mach disk from the tube outlet increases with increasing pressure ratio, $p_{stg/pamb}$.

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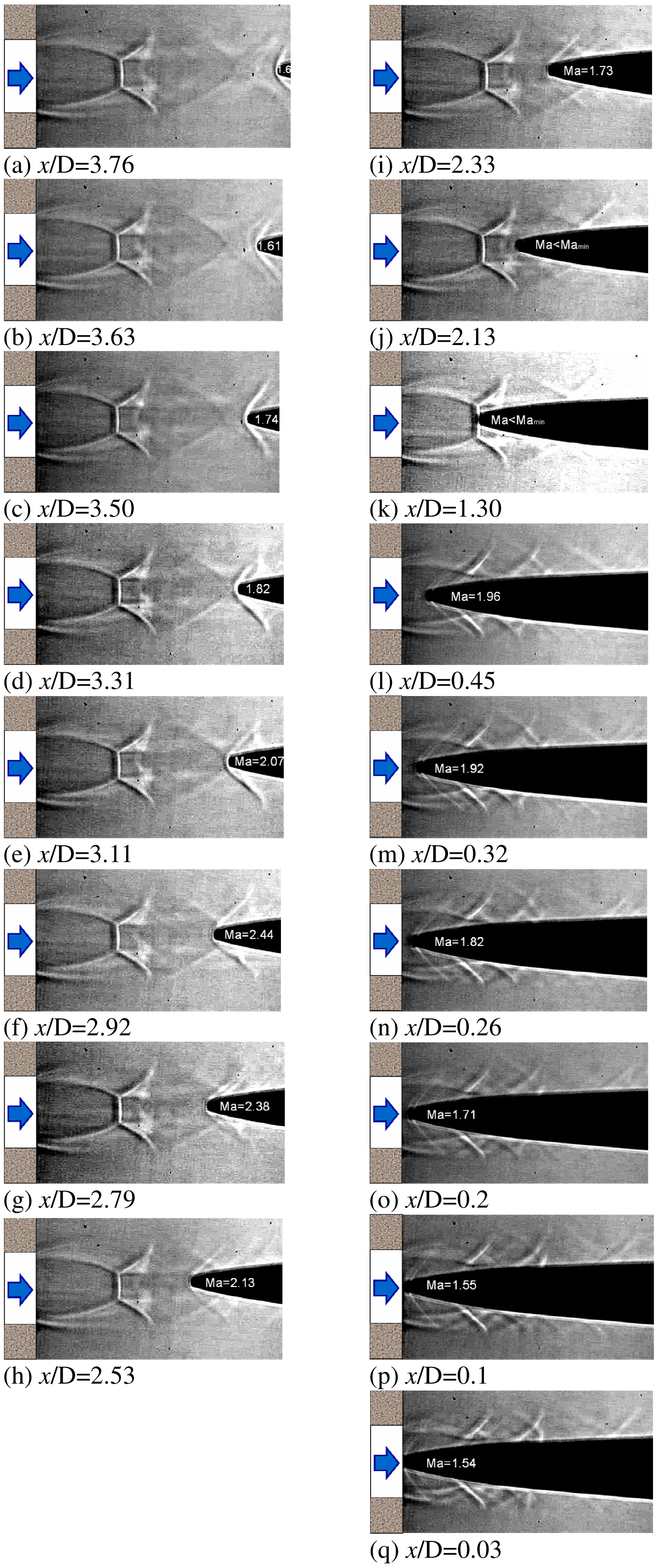

Fig. 9 Shadowgraph pictures of shockwave generated from the needle edge (p_{stg}=990kPa D=500μm)

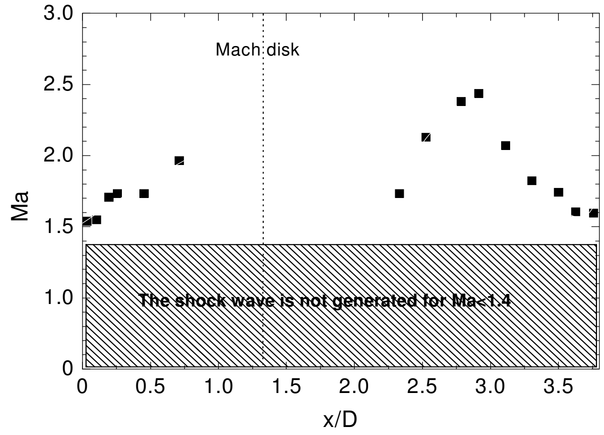

Fig.10 Mach number of jet

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